AN ADAPTIVELY SWITCHING ITERATION STRATEGY FOR POPULATION BASED METAHEURISTICS

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ABSTRACT

Population-based metaheuristics are iterative procedures that search for an optimal solution through exploration of the search space and exploitation of information by a group of search agents. The iteration strategy determines how the procedures are executed with respect to the population. Two types of iteration strategies are traditionally available. The first type which is the most commonly adopted strategy is the synchronous update. In the synchronous update, all the search procedures are executed as a group. The entire population needs to complete a particular procedure first before another procedure can be executed. The second type of traditional iteration strategy available is the asynchronous update. In asynchronous update, the procedures are executed as individual tasks and information is shared and used to guide the search for the optimal solution.

The two traditional iteration strategies have their own strengths and weaknesses. The agents in synchronous update are able to consider the performance of the entire population before their next search step is determined. Therefore, the agents from synchronous update is stronger in exploitation, as the entire population is drawn towards a similar reference point, which is typically the population's best performer. Meanwhile, an agent of asynchronous update is able to choose the reference point as soon as its fitness evaluation is finished. This update strategy improves the exploration of the population. Hence, selection of iteration strategy for a population-based metaheuristic can affect its overall performance.

The aim of this study is to investigate the role and importance of iteration strategy towards population-based metaheuristics and to propose a new class of alternative iteration strategies that i) balances exploration and exploitation, and ii) avoid premature convergence without introducing extra complexity through combination of the traditional iteration strategies.

Thus, a new class of iteration strategies which is a class of hybrid traditional strategies is proposed here. The strategies from this class are applicable for any population-based metaheuristics. The strategies are random switching, adaptive switching and adaptive switching with randomness. In the random switching strategy, the population randomly switches between the traditional strategies to cause disturbance to population diversity. The adaptive switching population, uses the information of the population's condition to determine when to switch its iteration strategy. Meanwhile, the adaptive switching with randomness, embed randomness to encourage more number of switching.

Experiments conducted using three parent algorithms namely particle swarm optimization (PSO), which is a popular population-based optimizer with population and individual memories, gravitational search algorithm (GSA), a memoryless young optimizer, and simulated Kalman filter (SKF), a newly introduced optimization algorithm that use population's memory to guide an agent's search, show that iteration strategy is an algorithm dependent parameter as well as function dependent. An iteration strategy is able to improve the performance of a parent algorithm and cause another parent algorithm to perform badly. The empirical analysis conducted here used the CEC2014's benchmark functions for single objective optimization problems.

ABSTRAK

Kaedah metahuristik populasi adalah prosedur-prosedur iteratif pencarian penyelesaian optimum melalui eksplorasi kawasan carian dan manipulasi informasi oleh sekumpulan ejen pencari. Strategi iteratif menentukan bagaimana prosedur-prosedur metahuristik populasi dijalankan. Terdapat dua jenis strategi iteratif yang sedia ada. Strategi pertama, iaitu strategi yang paling kerap diguna pakai adalah kemas kini segerak. Di dalam kemas kini segerak, kesemua prosedur dijalankan secara berkumpulan. Di mana seluruh populasi perlu menyelesaikan sesuatu prosedur terlebih dahulu sebelum prosedur lain dapat dijalankan. Jenis strategi iteratif sedia ada yang kedua adalah kemas kini tidak segerak. Di dalam kemas kini tidak segerak, prosedur-prosedur metahuristik adalah dijalankan sebagai tugasan-tugasan individu, dan informasi dikongsi serta digunakan bagi menentukan hala pencarian menghala ke arah penyelesaian yang optimum.

Kedua-dua strategi iteratif sedia ada mempunyai kelebihan dan kekurangan masing-masing. Ejen-ejen di dalam kemas kini segerak mampu mempertimbangkan pencapaian keseluruhan populasi sebelum menetapkan langkah pencarian seterusnya. Oleh itu, ejen-ejen dari kemas kini segerak mempunyai kekuatan dalam mengeksplotasi, ini disebabkan keseluruhan populasi adalah tertarik ke arah titik rujukan yang sama, iaitu ejen terbaik di dalam populasi. Sementara itu, setiap ejen di dalam kemas kini tak segerak berupaya menentukan titik rujukan mereka sejurus selepas penilaian kesesuaian penyelesaian. Strategi kemas kini ini menambah baik eksplorasi populasi. Oleh itu, pemilihan strategi iteratif bagi metahuristik populasi dapat mempengaruhi prestasi keseluruhannya.

Matlamat penyelidikan ini adalah bagi melihat peranan and kepentingan strategi iteratif terhadap metahuristik populasi dan mencadangkan suatu kelas baru strategi-strategi iteratif alternatif yang dapat i) mengimbangkan eksplorasi dan eksplotasi, dan ii)

mengelakkan penumpuan pramatang tanpa menambah kerumitan melalui kombinasi strategi iteratif sedia ada.

Maka, kelas baru strategi-strategi iteratif alternatif iaitu kaedah hibrid strategi-strategi sedia ada dicadangkan di sini. Strategi-strategi ini boleh diguna pakai bagi setiap metahuristik populasi. Strategi-strategi ini adalah; pensuisan rawak, penyuai pensuisan dan penyuai pensuisan terawak. Populasi yang menggunakan pensuisan rawak bertukar antara kedua-dua strategi iteratif sedia ada secara rawak bagi menimbulkan gangguan terhadap penumpuan populasi. Populasi yang mengunakan strategi iteratif penyuai pensuisan, bertukar antara kedua-dua strategi iteratif sedia ada menggunakan informasi mengenai keaadan populasi. Sementara itu, penyuai pensuisan terawak menggunakan kerawakan bagi menggalakkan pensuisan.

Eksperimen-eksperimen dijalankan menggunakan tiga algoritma induk iaitu, pengoptimuman kerumunan zarah (PSO), iaitu pengoptimum berdasarkan populasi yang terkenal yang menggunakan memori populasi dan individual, algoritma carian graviti (GSA), satu pengoptimum muda tanpa memori, dan simulasi penuras Kalman (SKF), satu pengoptimum yang baru sahaja diperkenalkan yang menggunakan memori populasi untuk memimpin pencarian ejen, menunjukkan bahawa strategi iteratif adalah tetapan yang bergantung terhadap algoritma dan juga fungsi permasahalaan. Suatu strategi iteratif mungkin boleh menambah baik satu algoritma induk manakala menyebabkan algoritma induk yang lain menjadi lebih teruk. Analisa empirikal yang dijalankan di sini menggunakan fungsi-fungsi penanda aras CEC2014 bagi masalah-masalah dengan satu objektif.

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A-GSA	:	Asynchronous Gravitational Search Algorithm
A-PSO	:	Asynchronous Particle Swarm Optimization
A-SKF	:	Asynchronous Simulated Kalman Filter
ASw-GSA	:	Adaptive Switching Gravitational Search Algorithm
ASw-GSA ^{r}	:	Adaptive Switching with Randomness Gravitational Search Algorithm
ASw-PSO	:	Adaptive Switching Particle Swarm Optimization
ASw-PSO ^{r}	:	Adaptive Switching with Randomness Particle Swarm Optimization
ASw-SKF	:	Adaptive Switching Simulated Kalman Filter
ASw-SKF ^{r}	:	Adaptive Switching with Randomness Simulated Kalman Filter
D^p	:	Population's position diversity
e_{fit}	:	Fitness error value
FES	:	Maximum number of fitness evaluation
fit_{ideal}	:	Ideal fitness value
fit^*	:	Fitness of the best solutions found
GSA	:	Gravitational Search Algorithm
PSO	:	Particle Swarm Optimization
RSw-GSA	:	Random Switching Gravitational Search Algorithm
RSw-PSO	:	Random Switching Particle Swarm Optimization
RSw-SKF	:	Random Switching Simulated Kalman Filter
SKF	:	Simulated Kalman Filter

LIST OF APPENDICES

CHAPTER 1: INTRODUCTION

1.1 Introduction

Optimization ensures that the best result is produced and limited resources is efficiently utilized. It is an important aspect in engineering. Its application can be seen in various engineering problems such as; selection of optimum values for PID controller parameters (Chaudhary, Raj, Kiran, Nema, & Padhy, 2013), VLSI circuit design (Ayob et al., 2010), antenna's direction of arrival predictor (Magdy, Mahmoud, & Ibrahim, 2013), selection of optimum weight for beamforming in wireless cellular communication system (Lazarus, Noordin, Ibrahim, & Abas, 2016), designing energy efficient power generators' schedule (Balci & Valenzuela, 2004), generation of optimum electric power distribution tree (Sabattin, Contreras Bolton, Arias, & Parada, 2012), and as noise canceller for EEG signal (Ahirwal, Kumar, & Singh, 2012). These are just a few of the numerous applications of optimization methods in engineering.

According to Talbi (2009), the optimization methods can be broadly classified as exact and approximate methods. Exact methods ensure ideal or optimal solution for given problems. However, depending on the complexity of the problem faced, the computational cost of these methods can be very expensive in terms of time and memory. In addition, exact algorithms are usually not robust to different type of problems and usually designed as problem specific algorithms (Dumitrescu & Stützle, 2003).

Approximate methods are more practical in solving optimization problems. Approximate methods do not just focus on finding optimal solutions but these methods also take computational constraints into consideration. The optimization algorithms belonging to this family can be categorized as approximation algorithms and heuristics algorithms. Approximation algorithms provide solutions that meet the minimum quality defined and suitable for optimization problems which require guaranteed quality of solution (Kuipers, Orda, Raz, Mieghem, & Van Mieghem, 2006). However, even though the quality of the solution produced by an approximation algorithm is within a guaranteed range, often the solution is far from optimal. Another disadvantage of approximation algorithms is problem dependency, hence, the algorithms are not robust to different optimization problems.

Heuristic algorithms, on the other hand, do not guarantee optimal solutions like exact algorithms nor solutions that meet the required range of quality like approximation methods. Instead, heuristics look for the best solutions possible using the allocated resources.

Metaheuristics and problem specific heuristics are subcomponents of heuristics approaches. In contrast to problem specific heuristics, metaheuristics are problem independent. Metaheuristics can be classified in numerous ways. One way to classify metaheuristics is population-based strategies and single agent-based strategies. In single agent-based metaheuristics, the search is done by iteratively updating the solution of a single agent. Whereas in population-based metaheuristics, a group of agents is used to search for optimal solution. Multiple candidate solutions are considered until the optimal solution is found. Population-based metaheuristics is the focus of this thesis. Figure 1.1 shows the classification of optimization methods as discussed above.

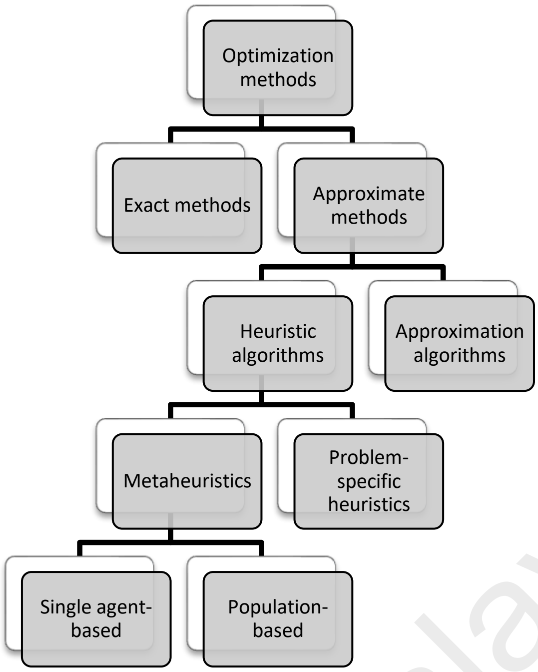

Figure 1.1: Classical Optimization Methods

1.2 Motivation

Every population-based metaheuristic searches for an optimal solution by updating its agents according to its unique set of search steps. During the execution of these search steps, information exchange happens between the members of the population. How the sequence of steps is conducted by an agent with respect to other agents is governed by an iteration strategy. Traditionally, the steps can either be executed independently, where an agent go through the steps without concerning itself with whether the other agents had gone through the same step as itself or not. Alternatively, the steps can also be executed as a group. In group execution, all agents need to perform each step together.

The iteration strategy is able to influence the agents' exploration and exploitation behavior. Thus, affecting the performance of the population in terms of the solution quality and the speed to reach an optimal solution (de Campos, Pozo, & Duarte, 2013; Engelbrecht, 2014; Liu, Sui, & Wang, 2009; Rada-Vilela, Zhang, & Seah, 2011b, 2013).

Despite its importance, not much in-depth research has been conducted to study the effect of the iteration strategy towards the performance of population-based optimization algorithms. This issue was also identified in (Engelbrecht, 2013b) as one of the aspects of particle swarm optimization (PSO) which is not sufficiently explored yet. This motivates the research conducted in this thesis. The research is conducted to systematically study the influence of the iteration strategy on population-based algorithms and also the possibility of manipulating the iteration strategy for performance enhancement. The findings are not only important for existing population-based metaheuristics but also for development of new population-based optimizers.

1.3 Objectives

The objectives of this thesis are listed as follow:

1. To identify and investigate the traditional iteration strategies available for population-based metaheuristics using three parent algorithms: PSO, gravitational search algorithm (GSA), and simulated Kalman filter (SKF). Any general patterns on the effect of the strategy towards the performance and search behavior of population-based algorithms are to be identified.
2. To propose a new class of iteration strategies with an embedded premature convergence avoidance mechanism. These characteristics are to be achieved without increasing the computational cost by using hybridization of the traditional strategies. The resulting new class of iteration strategies, namely, hybrid strategies are to be investigated.

1.4 Contributions

An extensive study of the effect of iteration strategies and their potential of improving population-based algorithms is conducted here using the three parent algorithms, PSO, GSA and SKF. Major contributions of this research are listed below:

1. This thesis identifies synchronous and asynchronous update strategies as the two traditional iteration strategies available. It is found that the effect of synchronous and asynchronous update strategies is algorithm dependent. While no significant difference is seen between synchronous PSO and asynchronous PSO, synchronous update is observed to be the best strategy for GSA and asynchronously updated SKF is seen to be significantly better than synchronously updated SKF. The effect of the synchronicity of the agents' position updates towards population diversity varies from one parent algorithm to another. No convergence is seen in asynchronously updated GSA whilst diversity is prolonged without preventing convergence in asynchronous SKF. On the other hand, no apparent difference is seen for diversity of synchronously updated PSO and asynchronously updated PSO. This contribution is reported in (Ab. Aziz, Mubin, Ibrahim, & Nawawi, 2014; Ab. Aziz et al., 2013; Ab. Aziz, Ibrahim, et al., 2014).
2. Three new iteration strategies from the hybrid class are proposed. The strategies combine the traditional update strategies so that premature convergence avoidance can be achieved through the iteration strategy of a population.
    a. The random switching iteration strategy randomly switches between the synchronous and asynchronous strategy. This strategy is able to significantly improve SKF.

b. The adaptive switching iteration strategy, switches the iteration strategy of a population based on switching indicator. The fitness of the best found solution is found to be a good choice of switching indicator that is applicable across all parent algorithms. The adaptive switching strategy is found to be able to significantly improve SKF. The contributions from the findings using this strategy are submitted for publication (Ab. Aziz, Ibrahim, Mubin, Nawawi, & Mohamad, n.d.)
c. The adaptive switching with randomness strategy, uses the condition of the population and some randomness to guide the most suitable time for switching. The randomness is found to be able to encourage more frequent switching which result in better performance. This strategy is found to be able to improve PSO, GSA and SKF. This finding is reported in (Ab. Aziz, Ibrahim, Mubin, & Sudin, 2017)

1.5 Thesis Outline

This thesis is divided into eight chapters. Chapter 2 presents the background necessary for this research which are the fundamentals of population-based metaheuristics, the parent algorithms and the benchmark functions used.

In chapter 3, existing works on premature convergence avoidance are reviewed. The works are categorized into five categories.

The traditional iteration strategies are presented and discussed in chapter 4. Two new asynchronous update algorithms, asynchronously updated GSA and SKF are proposed. The performances of the parent algorithms implemented using the traditional strategies are shown and studied.

The random switching iteration strategy is proposed in chapter 5. This strategy is then implemented by all parent algorithms and the performance is observed.

The adaptive switching iteration strategy is presented in chapter 6. The performances of the parent algorithms after adopting this new strategy is also shown in this chapter.

The last hybrid iteration strategy proposed, adaptive switching with randomness is discussed and its effect on the performance of the parent algorithms are analyzed and studied in chapter 7.

Finally, this thesis is concluded, its significance and also limitations are highlighted in chapter 8 together with suggestions for further research.

CHAPTER 2: THEORETICAL FUNDAMENTALS

2.1 Introduction

In this chapter, the background of this research is provided. The chapter starts with a discussion of the population-based metaheuristics and the principals of the algorithms, such as their iteration strategies, the importance of exploration and exploitation in ensuring good performance of the algorithms, and their relationship with the population diversity. This is followed by an introduction of the parent algorithms. The parent algorithms are the algorithms chosen to study the effect of iteration strategies and also the potential of the proposed strategies. Three parent algorithms are chosen, which are particle swarm optimization (PSO), gravitational search algorithm (GSA), and simulated Kalman filter (SKF). Finally, the benchmark functions used in this research are introduced.

2.2 Population-based Metaheuristics Algorithms

Metaheuristic algorithms implement approximate optimization procedures. These algorithms search for good quality solutions within acceptable computational time. The solutions found by metaheuristic algorithm are not guaranteed to be optimal, but rather are reasonably good solutions obtained without violating the given constraints. Metaheuristic algorithms can be categorized as population-based and single-solution based (Talbi, 2009). Population-based metaheuristics are the interest of this study. Population-based metaheuristics algorithms consist of group of agents. These agents search for an optimal solution through information sharing. The population does not have any central control.

There are four common steps shared among metaheuristic algorithms. The general steps of metaheuristic algorithms are shown in Algorithm 2.1.

1	:	Random initialization of possible solutions
2	:	Current solutions evaluation
3	:	Generation of next possible solutions
4	:	Repeat step 2&3 if stopping condition is not met, else end the algorithm and report the best found solution

Algorithm 2.1: General Steps of Metaheuristic Algorithm

The steps start with a random initialization of agents within the search space boundaries. This is followed by an evaluation of the quality of the solutions. The evaluation is done using a mathematical function. The function is formulated according to the problem to be solved. The solutions evaluation step is typically the most computationally expensive step of an optimization algorithm.

The next step is the generation of new possible solutions. This phase is what differentiates an algorithm from another. The generation follows certain rules which are derived based on the principles that inspired the particular algorithm. The principles determine how the information obtained from the previous search influences the determination of the new solutions. Many principles had inspired metaheuristics algorithms. For example, ants foraging behavior inspired ant colony optimization (Dorigo, Birattari, & Stutzle, 2006), animals flocking behavior has inspired the PSO algorithm (Kennedy & Eberhart, 1995), bat echolocation behavior inspired the bat algorithm (Yang & Gandomi, 2012), the Newton gravitational law that inspired the GSA (Rashedi, Nezamabadi-pour, & Saryazdi, 2009), the black hole phenomenon that inspired the black hole algorithm (Hatamlou, 2013), and Kalman estimator that inspired SKF (Z. Ibrahim et al., 2015).

Other than manipulation of the best solution, randomness or stochasticity is one of the fundamental components of metaheuristics. The randomness encourages exploration of the search space.

The last step of a metaheuristic algorithm is to evaluate the stopping condition. If the condition is satisfied then the algorithm is stopped and the best found solution is reported, otherwise the evaluation and generation procedures are repeated. The stopping condition is either one of the following conditions or combinations of these conditions;

i. The candidate solution obtains the ideal solution's quality, fit_{ideal}. In order to apply this stopping condition, the optimal solution's quality, i.e. fitness, need to be known.

ii. The fitness error value, e_{fit}, of the best solution found is within an acceptable value. The fitness error value, e_{fit}, is calculated by finding the difference between the fitness of the best solutions found, fit^*, with the ideal fitness, fit_{ideal};

$$e_{fit} = fit^* - fit_{ideal} \tag{2.1}$$

This stopping condition also requires knowledge of the fitness of the optimal solution, fit_{ideal}.

iii. The maximum number of iterations is reached; i.e. the maximum number of fitness evaluation, FES, is exceeded. No knowledge of the fitness of the ideal solution is required for this stopping condition.

Additionally, agents' diversity can also be used to determine when to terminate a population-based algorithm. The diversity indicates the spread of the agents in the search space. A stagnant diversity shows that the agents are no longer moving and exploring the search space. A small diversity indicates that the agents have clustered around a point, signifying convergence of the population. Either one of these observations can be used as the condition to stop the algorithm.

2.2.1 Iteration Strategy

From Algorithm 2.1, it can be seen that metaheuristic algorithms are iterative procedures where the procedures are executed repetitively until a stopping condition is met. In every iteration an algorithm strives to improve its candidate solutions.

Osman & Laporte (1996) defined metaheuristic; "*an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space, learning strategies are used to structure information in order to find efficiently near-optimal solutions*".

Yang and Karamanoglu (2013) defined; *"an algorithm is an iterative procedure whose aim is to generate new, better solutions from the current solution set so that the best solution can be reached in a finite number of steps, ideally, as few steps as possible"*.

Parejo, Ruiz-Cortés, Lozano, & Fernandez, (2012) defined metaheuristic *"an iterative process that guides the operation of one or more subordinate heuristics (which may be from a local search process, to a constructive process of random solutions) to efficiently produce quality solutions for a problem"*.

These definitions highlight that metaheuristics are iterative procedures. Therefore, the iteration strategy is one of the fundamental aspects of a population-based metaheuristic algorithm. Other aspects of metaheuristics mentioned are the importance of a balance of exploration and exploitation.

Traditionally, the iteration strategy of population-based algorithm can be categorized into synchronous and asynchronous update strategies. The strategy differentiates how the population goes through steps 2 and 3 of Algorithm 2.1 as well as influences the flow of the information within the population. In synchronous update strategy, the state of the whole population is known prior to new solutions generation. Hence, generation of new candidate solutions in synchronous update is done using same information. This strengthen the exploitation in the population-based algorithm that employs synchronous update strategy. On the other hand, lack of synchronicity in asynchronous update allows the population's candidate solutions to be updated using nonuniform information, which encourages exploration by the agents.

2.2.1.1 Synchronous Update Strategy

In synchronous update strategy, the execution of the metaheuristic algorithms' steps is group oriented, where the agents' evaluation in step 2 is carried out for the whole population prior to execution of step 3 by the entire population. This is the default iteration strategy of many members of the population-based optimization algorithms family. Algorithms such as PSO, GSA, SKF, ant colony optimization and bees algorithm (Pham, Ghanbarzadeh, & Koc, 2006), were introduced with a synchronous update strategy.

The general pseudocode of a synchronous population-based algorithm is shown in Algorithm 2.2. In synchronous update strategy, after initialization, step 2 of a metaheuristic algorithm, which is the performance evaluation, is executed for all agents. This is followed by the generation of the population's next possible solutions. The evaluation and generation processes are conducted within two separate loops.

1	:	Random initialization of possible solutions
		For $i = 1: number\ of\ agent$
2	:	Agent i^{th} evaluation
		End
		For $i = 1: number\ of\ agent$
3	:	Generate next solution for agent i^{th}
		End
4	:	Repeat step 2&3 if stopping condition is not met, else end the algorithm and report the best found solution

Algorithm 2.2: General Steps of Population-based Metaheuristics using Synchronous Update

2.2.1.2 Asynchronous Update Strategy

In asynchronous update strategy, the metaheuristics' steps are viewed as individual tasks. The agents within a population execute their optimization steps individually, independent of each other. After an agent completes its fitness evaluation, its new solution is immediately generated without the need to wait for other agents in the population to complete their evaluation.

The general pseudocode of sequential programming population-based algorithms with asynchronous iteration strategy is shown in Algorithm 2.3. Only one loop exists in the asynchronous update population-based algorithms. Steps 2 and 3 of a metaheuristic algorithm are conducted within the same loop. An agent is evaluated and updated before the next agent is evaluated and updated.

1	:	Random initialization of possible solutions
		For $i = 1: number\ of\ agent$
2	:	Agent i^{th} evaluation
3	:	Generate next solution for agent i^{th}
		End
4	:	Repeat step 2&3 if stopping condition is not met, else end the algorithm and report the best found solution

Algorithm 2.3: General Steps of Population-based Metaheuristics using Asynchronous Update

2.2.2 Metaheuristics and No Free Lunch Theorem

Even though many new or modified metaheuristic algorithms have been proposed, no universally the best algorithm exist (Yang, 2012c). An algorithm can be better for a set of problems and performs badly for another set of problems, which can be solved by another algorithm efficiently. This is known as the "no free lunch theorem" (Wolpert & Macready, 1997). The no free lunch theorem has motivated many researchers to keep proposing new optimizers or to keep improving existing algorithms.

2.2.3 Exploration and Exploitation

According to Cheng, Shi, & Qin (2011), Khajehzadeh et al. (2011), Talbi (2009), Yang, Deb, & Fong (2014), and Yang (2012b, 2013), the key to a good metaheuristics algorithm is a balance between exploration and exploitation.

Exploration is related to the diversification of the agents, while exploitation is agent's intensification of its search within an area, in order to refine candidate solution. Exploration helps the agents to ensure the search area to be extensively searched, so that area with good solution is not overlooked. On the other hand, exploitation allows the agents to fine-tune their search.

Without proper control of exploration and exploitation by the agents, an algorithm is prone to premature convergence or no convergence at all if focus is too strong on exploration. Premature convergence is a major concern in optimization, especially in solving multimodal problems. It may cause agents to be trapped within local optima thus reducing the chance to find a global optimum. One of the factor that causes premature convergence is due to the loss of diversity, leading to inefficient exploration and exploitation. When a population converges prematurely, its agents clustered within same subsection of the search area. Thus, if the optimal solution is not within this subsection, the chance of finding an optimal solution is minimal. Exploration is important in solving multimodal problems. However, exploitation is also important in fine tuning the candidate solution. The two fundamental components of metaheuristics, which are randomness and the best candidate solution manipulation, help in providing a balance between exploration and exploitation (Yang, 2012c). The randomness allows the agents to look for other candidate solutions instead of focusing on the current candidate solutions, while manipulation of the best candidate solution allows the agents to fine tune the best candidate solution so that possibly a better candidate can be found.

2.2.3.1 Diversity

Diversity is highly related to the distribution of agents in the search space. High distribution of agents allows exploration of the search space, while low distribution allows exploitation and intensification of the search within a subarea of the search space. Therefore, the information of agents' diversity can be used to analyze the exploration and exploitation state of a population. Diversity can also be used to control the agents' search. As a rule of thumb, high diversity is preferred in the early stage of the search when more exploration should be emphasized, while a reduction in diversity is desired as the search progresses. Reduction of diversity allows intensification, i.e. exploitation.

In (Cheng & Shi, 2011), L_1 normalized diversity measurements for PSO was presented. Three measurements were discussed, which are position diversity, velocity diversity, and cognitive diversity. The position diversity reflects the distribution of the solutions in the search space. When the solutions are highly distributed in the search space, the position diversity is higher, whereas, when they are distributed within a smaller area, the diversity is smaller. The velocity diversity shows the activity of the PSO's particles. The tendency of the swarm to expand its search is shown by high velocity diversity, while low velocity diversity shows reduced activity signifying the convergence of the swarm. The cognitive diversity represents the diversity of the best personal candidate solutions ($\boldsymbol{pBest_i}$) found by the particles. As the swarm converges, the particles shared almost the same $\boldsymbol{pBest_i}$, thus small value of cognitive diversity.

Among these three diversity measurements, position diversity is applicable for all type of metaheuristics. The other two are exclusively for PSO, as not all algorithms have a velocity and cognitive memory. The PSO's position diversity represents solutions spread within the search space, the position diversity is an attribute shared among all algorithms. Hence, the position diversity is adopted in this work.

The position diversity is calculated as follows. The calculation starts with the computation of the mean position, $\bar{x}^d$ for each d^{th} dimension of the population,

$$\bar{x}^d = \frac{1}{N}\sum_{i=1}^{N} x_i^d \tag{2.2}$$

In equation 2.2, agent i^{th} position in dimension d^{th} is represented as, x_i^d and N is presenting the number of agents in the population. Next, the diversity of the agents' position with respect to the mean position for every dimension, D^{pd}, is calculated,

$$D^{pd} = \frac{1}{N}\sum_{i=1}^{N} \left|x_i^d - \bar{x}^d\right| \tag{2.3}$$

Finally, the population's position diversity, D^p, is calculated as shown in equation 2.4

$$D^p = \frac{1}{D}\sum_{d=1}^{D} D^{pd} \tag{2.4}$$

where D represents the dimension size of the problem.

2.3 Introduction to the Parent Algorithms

Three population-based algorithms are chosen as the parent algorithms in this study. They are used to study the effectiveness of iteration strategy manipulation by the proposed strategies in improving population-based metaheuristics. The algorithms are PSO, GSA, and SKF. The PSO algorithm is a landmark algorithm for metaheuristics (Yang, 2012a), while GSA is a new algorithm proposed in 2009 which had gained interest among researchers from this field. Meanwhile, SKF is a newer addition to the population-based metaheuristics family.

PSO and SKF use memory in performing the search for an optimal solution, while GSA is a memoryless algorithm. The agents in PSO memorize their personal best experience and the best solution found among the neighborhood agents. In SKF, the memory is only used to remember the best performer of the entire population. The agents of SKF do not keep record of their personal best, whereas in GSA, the search for an optimal solution is only influenced by the current state of the population.

2.3.1 Particle Swarm Optimization

Particle swarm optimization (PSO), was introduced by Kennedy and Eberhart in 1995 (Kennedy & Eberhart, 1995). It is a nature inspired optimization algorithm. PSO is influenced by the flocking behavior of birds, where individual success is driven by an individual's own experience and social interaction. A bird improves its search by adjusting its flight pattern based on the information of the food source gained from its previous search and also the information of other food sources shared within its flock.

Similar to what is observed in nature, the search in PSO is carried out by a swarm of particles. The particles' search for an optimal solution is directed by individual's experience and neighbors' influence. The social interaction contributes to the success of PSO.

The PSO algorithm is simple but yet, a powerful optimizer. The simplicity and good performance has contributed to PSO's popularity. The original PSO was proposed for continuous single objective optimization. However, works had been carried out so that PSO has evolved to be a universal optimizer, where PSO is able to solve many other types of optimization problems, such as multiobjective optimization (K. S. Lim et al., 2013; Reyes-Sierra & Coello Coello, 2006), discrete optimization (I. Ibrahim et al., 2012; Kennedy & Eberhart, 1997; Pampara, Franken, & Engelbrecht, 2005), and dynamic optimization problems (C. Li & Yang, 2012; S. Yang & Li, 2010). PSO has also been successfully applied in various fields, such as robotics (Xue, Zhang, & Zeng, 2009), power distribution planning (M. Zhang, Cheng, Mei, & Dong, 2009), biomedical optimization (Eberhart & Hu, 1999; Z. Ibrahim et al., 2012; Mohamad et al., 2013), wireless sensor networks (Singh, Kumar, Saxena, & Priya, 2012), and financial planning (J. Sun, Xu, & Fang, 2006).

2.3.1.1 The Original PSO Algorithm

The PSO algorithm involves simple mathematical operations. Only multiplication, addition and subtraction are involved in PSO. Each particle of PSO has a position, $\boldsymbol{X}_i(t)$ and velocity, $\boldsymbol{V}_i(t)$, where,

$$\boldsymbol{X}_i(t) = \left(x_i^1(t), x_i^2(t), x_i^3(t), \dots, x_i^d(t), \dots, x_i^D(t)\right) \tag{2.5}$$

$$\boldsymbol{V}_i(t) = \left(v_i^1(t), v_i^2(t), v_i^3(t), \dots, v_i^d(t), \dots, v_i^D(t)\right)$$

$$i = 1,2,3, \dots, N \qquad\qquad d = 1,2,3, \dots, D$$

In equation 2.5, i is the particle index, while N is the number of particles, and t in the equation represents the iteration number. The dimension number is denoted by d and the number of dimensions is D. Typically, the particles' velocities and positions are randomly initialized according to the search space (Voglis, Parsopoulos, & Lagaris, 2012).

The particle's position represents a candidate solution while velocity is a particle's step size. A particle carries its search by iteratively updating these values. Particle *i*'s velocity at the d^{th} dimension in the t^{th} iteration, is updated using the following equations;

$$v_i^d(t) = v_i^d(t-1) + c_1 rand_1^d(t)\left(pBest_i^d(t) - x_i^d(t-1)\right) \qquad (2.6)$$
$$+ c_2 rand_2^d(t)\left(nBest^d(t) - x_i^d(t-1)\right)$$

This velocity update equation can be divided into three parts:

i. Momentum: $v_i^d(t-1)$

The momentum part is represented by the particle's previous velocity. It reflects how an individual tends to move towards the same direction it was previously moving. The momentum prevents the particle from abruptly changing its direction. This portion of the velocity is also commonly known as the inertia (Engelbrecht, 2007).

ii. Cognitive: $c_1 rand_1^d(t)\left(pBest_i^d(t) - x_i^d(t-1)\right)$

In PSO, the particles have memory. They are able to remember their previous success. The memory is one of the factors influencing a particle decision on its next move. This factor is known as the cognitive portion of the velocity or the particle's nostalgia (Kennedy & Eberhart, 1995).

The components of the cognitive part are the particle's best achievement, $pBest_i^d(t)$, its previous position, $x_i^d(t-1)$, the cognitive acceleration constant, c_1, and a random number, $rand_1^d(t)$.

iii. Social: $c_2 rand_2^d(t)\left(nBest^d(t) - x_i^d(t-1)\right)$

The last part of the velocity equation is the social part. The social part signifies the communication and information sharing that took place between the particle *i*th with the particles within its neighborhood. It represents how an individual within a swarm is likely to imitate the best performer among its neighbors, $\boldsymbol{nBest}$.

Two types of neighborhood structures are commonly used in PSO, which are global ($\boldsymbol{gBest}$) and local ($\boldsymbol{lBest}$). In global neighborhood, all members of the swarm are connected with each other. On the other hand, a particle is only connected to a number of its immediate neighbors for local neighborhood PSO. The particle's neighbors are depending on the topology of the neighborhood. In this thesis the PSO with global best neighborhood is adopted.

Similar to the cognitive part, the strength of the influence of the social part is also controlled by an acceleration constant, c_2, and a random number, $rand_2^d(t)$.

Typically, the acceleration contants, c_1 and c_2, are set such that, $c_1 + c_2 \leq 4$ (Parsopoulos & Vrahatis, 2002). Shi & Eberhart suggested the value for both factors to be equivalent to 2 (Shi & Eberhart, 1998). The two random numbers in equation 2.6, $rand_1^d(t)$ and $rand_2^d(t)$, are independent of each other as well as to the search dimension, and also iteration. These random numbers are drawn from uniform distribution between 0 to 1 and they contribute to the stochastic behavior of PSO and reduce the convergence speed of the particles (Kim, Chang, & Kang, 2013).

The next position of a particle is computed using equation 2.7.

$$x_i^d(t) = x_i^d(t-1) + v_i^d(t) \tag{2.7}$$

It can be seen that the particle's next move is launched from its last location. The step size is the velocity. Typically, the position is bounded according to the problem to be solved. This is to prevent the particles from wandering off to infeasible search space.

The flowchart of the original PSO algorithm is shown in Figure 2.1 and its pseudo code is shown in Algorithm 2.4. The algorithm starts with initialization of the population. Next, the performances of all the particles are evaluated. This is done within the first loop of this algorithm. In the second loop, the swarm's velocities and positions are updated using the information available for the best values. The fitness evaluation and particles' update are repeated until a stopping condition is met. The particles in the original PSO are updated using synchronous update strategy, thus the original PSO will be referred as synchronous PSO (S-PSO) from here on.

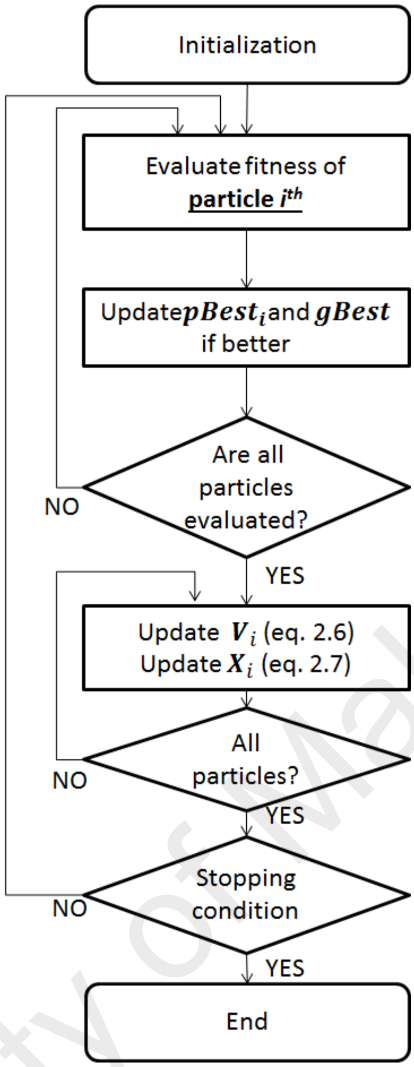

Figure 2.1: Flowchart of S-PSO

Initialization of swarm
Do{
    For every particles
        Evaluate fitness
        Update $\boldsymbol{pBest}$ and $\boldsymbol{gBest}$ if better
    End for
    For every particles
        Update $\boldsymbol{V}_i$, equation 2.6
        Update $\boldsymbol{X}_i$, equation 2.7
    End for
}**While** not stopping condition

Algorithm 2.4: Pseudo Code of S-PSO

2.3.1.2 Inertia Weight PSO

In this work, a global best ($\boldsymbol{gBest}$) PSO with decreasing inertia weight incorporated within its velocity equation is used. The inertia weight was introduced by Shi & Eberhart, (1998). It is used to help in balancing exploration and exploitation among the particles of PSO. A decreasing inertia weight is reported to contribute to better performance (Shi & Eberhart, 1998). Larger inertia weight during the early phase of the search allows bigger step size thus more exploration, while smaller inertia weight at a later phase encourages more exploitation through smaller step size. Inertia weight had been accepted as part of the standard PSO (Clerc, 2009).

PSO with inertia weight works in the same way as the original PSO with only an extra multiplier is added to the momentum part of the velocity equation. Equation 2.8 shows PSO's velocity with the inertia weight, ω;

$$v_i^d(t) = \omega v_i^d(t-1) + c_1 rand_1^d(t)\left(pBest_i^d(t) - x_i^d(t-1)\right) + c_2 rand_2^d(t)\left(gBest^d(t) - x_i^d(t-1)\right) \tag{2.8}$$

2.3.2 Gravitational Search Algorithm

GSA is inspired by the gravitational force phenomenon. Specifically, it is rooted on Newton's law of gravitation and second law of motion. It was proposed by Rashedi et al., (2009). The GSA's agents look for optimal solution within the search space using the attraction force exerted by themselves towards each other. The strength of the force is proportional to agents' masses and inversely proportional to their acceleration. An agent's mass is dependent on the quality of the solution proposed by the agent. The better the solution is, the bigger is the mass. Therefore, the highest pulling force in the entire population is exerted by the population's best performer.

Like PSO, the original GSA is a single objective optimization algorithm. However, with some modifications GSA has successfully been applied in various types of optimization problems, such as multi-objective (Nobahari, Nikusokhan, & Siarry, 2011, Hassanzadeh & Rouhani, 2010), multimodal problems (S. Yazdani, Nezamabadi-pour, & Kamyab, 2014), and binary optimization problems (Rashedi, Nezamabadi-Pour, & Saryazdi, 2010, Mirjalili, Wang, & Coelho, 2014).

GSA is found to be more superior to some well-established optimization algorithms (Rashedi et al., 2009), such as genetic algorithm (GA), and PSO. The main attraction of GSA is its simplicity which requires only two parameters tuning compared to other algorithms. However, GSA algorithm has a reputation to converge too fast thus lowering its performance (Nobahari et al., 2011).

2.3.2.1 The Original GSA

Gravity influences bodies existed within the universe. According to Newton's law of universal gravitational, the attraction force of two bodies towards each other is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Mathematically, the gravitational force, F_G, acting between body 1 and body 2 can be expressed in the following equation,

$$F_G = G\frac{M_1 M_2}{R^2} \tag{2.9}$$

where M_1 and M_2 are the masses of body 1 and body 2, respectively. The distance between the bodies is represented by *R.* While *G* is the gravitational constant.

Based on Newton's second law of motion, a moving body's acceleration, α, is directly proportional and in the same direction as the net force, F_{net}, acting on itself, but, inversely proportional to its mass, M. This is represented in equation 2.9,

$$\alpha = \frac{F_{net}}{M} \tag{2.10}$$

These two laws introduced by Newton are the essence of GSA. The GSA's optimization procedures start with random initialization of the agents within the search area. Each of the agents has mass. The agents' masses are calculated based on the fitness of the solutions. The fitness is evaluated using problem dependent fitness function. A fitter agent has a higher mass compared to agents that do not perform as good. Therefore, a fitter agent exerts a stronger attraction force.

Using similar notation as PSO, position of agent i^{th}, at t^{th} iteration is,

$$\boldsymbol{X}_i(t) = \left(x_i^1(t), x_i^2(t), x_i^3(t), \dots, x_i^d(t), \dots, x_i^D(t)\right) \tag{2.11}$$

$$i = 1,2,3, \dots, N \qquad\qquad d = 1,2,3, \dots, D$$

Similarly, $x_i^d(t)$ represents the position of agent i^{th} at t^{th} iteration in dimension d^{th}. The number of dimension is D, while N is the number of agents. Agent i^{th}'s fitness at iteration t is represented as, $fit_i(t)$. Its mass, $M_i(t)$, is calculated as follow;

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{2.12}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{2.13}$$

The $best(t)$ and $worst(t)$ notation in equation 2.12, represent the best and worst fitness among the agents in the population. In a minimization problem these values are selected as follows;

$$best(t) = min\{fit_1(t), fit_2(t), \dots, fit_N(t)\} \tag{2.14}$$

$$worst(t) = max\{fit_1(t), fit_2(t), \dots, fit_N(t)\} \tag{2.15}$$

The gravitational force acting on agent $i^{th,}$ $F_i^d(t)$ is calculated using equation 2.16;

$$F_i^d(t) = \sum_{j=1, j\neq i}^{N} rand_j^d(t) F_{ij}^d(t) \tag{2.16}$$

where $rand_j^d(t)$ is a random number in the interval [0,1], which is independent of agent, iteration, and dimension. $F_{ij}^d(t)$ is the gravitational force of agent j^{th} towards agent i^{th}. The weight of the force from the other agents toward agent i^{th} is not equal, but, rather randomly determined. $F_{ij}^d(t)$ is calculated as follow;

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \tag{2.17}$$

In equation 2.17, $R_{ij}(t)$ is the Euclidian distance between agent i^{th} and j^{th}. A small constant ε is added to avoid division by zero when the position of both agents overlapped. $G(t)$ is the gravitational constant at time t. The update equation of $G(t)$ is;

$$G(t) = G_0 \times e^{-\beta \frac{t}{T}} \tag{2.18}$$

G_o is the gravitational constant at the start of the search. According to the original work on GSA, the recommended value of G_o is 100 while β is set to 20. T is the total number of iteration.

$M_{pi}(t)$ and $M_{aj}(t)$ in equation 2.17 are passive and active gravitational mass of agent i^{th} and j^{th}, respectively. GSA assumes the passive and active gravitational mass to be equivalent. Thus, the relation between $M_{pi}(t)$ and $M_{ai}(t)$ used by GSA is;

$$M_{pi}(t) = M_{ai}(t) = M_i(t) \tag{2.19}$$

The agents' acceleration in GSA are subjected to Newton's law of motion, therefore, the acceleration of agent i^{th} over dimension d^{th}, $\alpha_i^d(t)$, can be calculated using the following equation;

$$\alpha_i^d(t) = \frac{F_i^d(t)}{M_i(t)} \tag{2.20}$$

The agents' velocities and positions are then updated using the equations below;

$$v_i^d(t) = rand_i^d \times v_i^d(t-1) + \alpha_i^d(t) \tag{2.21}$$

$$x_i^d(t) = x_i^d(t-1) + v_i^d(t) \tag{2.22}$$

The original GSA algorithm is shown in Figure 2.2 and Algorithm 2.5. The fitness of the whole population is evaluated first before best and worst values are identified. The generation of new agents' positions follows after these steps. Hence, the original GSA is a synchronous update algorithm, thus in this work it is known as synchronous GSA (S-GSA).

In the original GSA, an elitist strategy is proposed. According to this strategy, only $Kbest$ of top ranked agents proceed from an iteration to the next iteration. The $Kbest$ value is linearly decreased with time. Elitism is claimed to encourage exploitation.

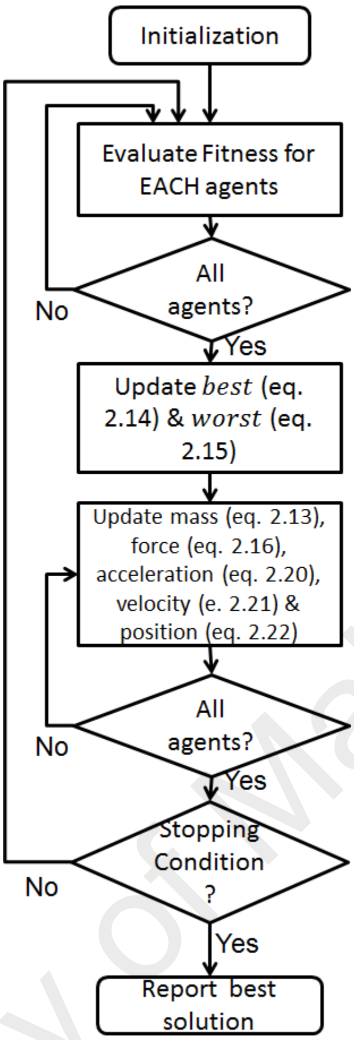

Figure 2.2: Flowchart of S-GSA

```
Initialization of agents
Do{
        For every agents
                Evaluate fitness
        End for
        Identify best(t) and worst(t) using equation 2.14 & equation 2.15
        For every agents
                Update mass, equation 2.13
                Update force, equation 2.16
                Update acceleration, equation 2.20
                Update velocity, equation 2.21
                Update position, equation 2.22
        End for
}While not stopping condition
```

Algorithm 2.5: Pseudo Code of S-GSA

2.3.3 Simulated Kalman Filter

SKF is a new addition to the population-based algorithm family. It was introduced in 2015 for continuous unimodal optimization problems (Z. Ibrahim et al., 2015). Unlike PSO and GSA, which are based on natural phenomenon, SKF was developed based on scalar Kalman filter. Kalman filter is a state estimator algorithm.

In SKF, population of agents works together to solve optimization problem by emulating Kalman filters. Each of the agent work like a Kalman filter. The agents go through prediction, measurement, and estimation process in every iteration. The best information obtained among the agents is shared during the measurement phase. The agents then use the simulated measurements to improve their estimation of the optimal solution for the problem considered. Prediction is carried out based on previously estimated value.

Since SKF is a new algorithm, very few work had been reported on SKF. Nonetheless, a binary SKF (BSKF) for binary optimization problems had been introduced in (Md Yusof et al., 2015) and a hybrid PSO-SKF is reported in (Muhammad et al., 2015). As a new algorithm, there are many areas of improvement that can be explored for the betterment of SKF, such as reducing the number of parameters in SKF, tuning the parameters' value, controlling the usage and flow of information shared among the agents, and changing the SKF's iteration strategy.

2.3.3.1 The Original SKF

Every agent of SKF is a Kalman filters. The possible solutions are stored as estimated states of the agents. Other than estimated values, each agent has its own measurement value. Given *N* number of agents and *D* dimensional problem, the estimated state, $\boldsymbol{X}_i(t)$, and measured values, $\boldsymbol{Z}_i(t)$, for agent i^{th} at t^{th} iteration are presented as;

$$\boldsymbol{X}_i(t) = \left(x_i^1(t), x_i^2(t), x_i^3(t), \dots, x_i^d(t), \dots, x_i^D(t)\right) \tag{2.23}$$

$$\boldsymbol{Z}_i(t) = \left(z_i^1(t), z_i^2(t), z_i^3(t), \dots, z_i^d(t), \dots, z_i^D(t)\right)$$

$$i = 1,2,3, \dots, N \qquad\qquad d = 1,2,3, \dots, D$$

The SKF algorithm starts with random initialization of the agents estimated values. The initialization depends on the search space of the problem to be solved.

Before any steps of Kalman filter begins, the fitness of current estimated values is evaluated. Once the evaluation is completed, the best solution of the current population, $\boldsymbol{X}_{best}(t)$, is identified. In minimization problem, $\boldsymbol{X}_{best}(t)$ stores a copy of estimated value of the agent with the lowest fitness value, while in maximization problem, the agent with the highest fitness value is stored as $\boldsymbol{X}_{best}(t)$. Next, fitness of $\boldsymbol{X}_{best}(t)$ is compared with $\boldsymbol{X}_{true}$. The $\boldsymbol{X}_{true}$ holds the best found solution from the start of the iteration. If $\boldsymbol{X}_{best}(t)$ offers a better solution than $\boldsymbol{X}_{true}$, then it is chosen as the new $\boldsymbol{X}_{true}$.

After fitness evaluation and $\boldsymbol{X}_{best}(t)$ and $\boldsymbol{X}_{true}$ identification, the prediction phase starts. In the prediction phase, the current predicted state, $\boldsymbol{X}_i(t|t+1)$, is assumed to be the estimated value;

$$\boldsymbol{X}_i(t|t+1) = \boldsymbol{X}_i(t) \tag{2.24}$$

After the prediction phase, the measured values of the agents are calculated. The dimensional wise calculation of measured value for dimension d^{th} of agent i^{th} is calculated as follow;

$$z_i^d(t) = x_i^d(t|t+1) + \sin(rand_i^d(t)\times 2\pi)\times\left|x_i^d(t|t+1) - x_{true}^d\right| \tag{2.25}$$

The $rand_i^d(t)$ is random value within the range of [0,1]. The term $\sin(rand_i^d(t)\times 2\pi)$ allows the agent to move either towards or away from $\boldsymbol{X}_{true}$ by maximum length of $x_i^d(t|t+1) - x_{true}^d$ from its current estimated value. This is the stochastic and random element of SKF. The randomness supports exploration by the agents.

The estimation phase follows the measurement phase. The estimated next value is updated using equation 2.26;

$$x_i^d(t+1) = x_i^d(t|t+1) + K(t)\times\left(z_i^d(t) - x_i^d(t|t+1)\right) \tag{2.26}$$

where $K(t)$ is the Kalman gain, which is calculated as follow;

$$K(t) = \frac{P(t|t+1)}{P(t|t+1) + R} \tag{2.27}$$

In equation 2.27, R is the measurement noise, which is suggested to be set to 0.5. The current transition error covariant estimate, $P(t|t+1)$, is calculated using current error covariant estimate, $P(t)$, and the process noise, Q.

$$P(t|t+1) = P(t) + Q \tag{2.28}$$

Q is suggested to be set to 0.5 and the initial error covariant, $P(0)$, is set to 1000. The current error covariant estimate is updated in estimation phase using equation 2.29;

$$P(t+1) = (1 + K(t))\times P(t|t+1) \tag{2.29}$$

In next iteration, the fitness of the new estimated values is then evaluated and the predict, measure, and estimate steps are repeated. These steps continue until stopping condition for the SKF algorithm is met.

SKF is introduced as synchronous update algorithm. All the phases of the algorithm are executed and completed as a group. This can be seen in Figure 2.3 and Algorithm 2.6.

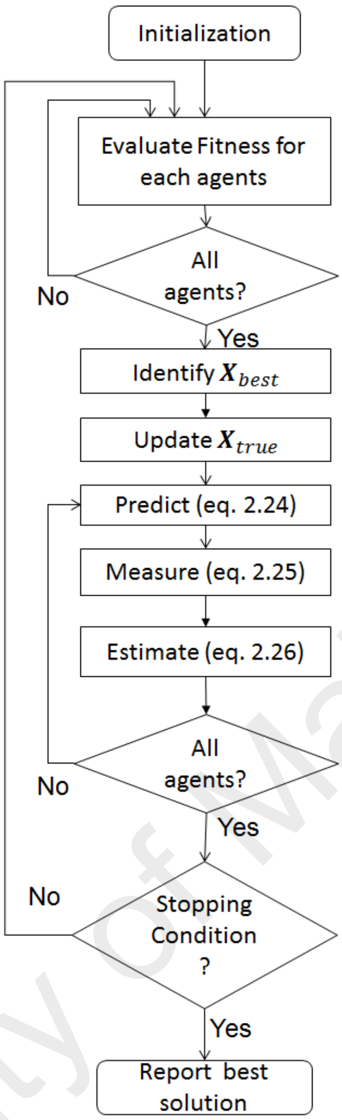

Figure 2.3: Flowchart of S-SKF

Initialization of agents
Do{
    For every agents
        Evaluate fitness
    End for
    Identify $\boldsymbol{X}_{best}(t)$
    Update $\boldsymbol{X}_{true}$
    For every agents
        Predict, equation 2.24
        Measure, equation 2.25
        Estimate, equation 2.26
    End for
}While not stopping condition

Algorithm 2.6: Pseudo Code of S-SKF

2.4 Benchmark Functions

The performance of the iteration strategies studied and proposed in this research is evaluated through benchmarking. Benchmarking is able to provide fair comparison of optimization algorithms (Oparaa & Arabasb, 2011). It can be achieved by measuring the averaged performance of the algorithms in solving a number of benchmark problems. The benchmark problems are artificial landscapes, designed in such a way that finding the optimal value is not easy. The number of problems need to be sufficiently enough so that fair observation can be made (Garden & Engelbrecht, 2014). Therefore, the CEC2014's single objective real-parameter numerical optimization test suite is used as the benchmark problems here.

There are 30 test functions in this test suite. Table 2.1 listed the 30 functions. All of the functions are minimization functions. The functions are designed as black box problems derived from 14 basic functions, which can be found in Appendix A.

The functions consist of three rotated unimodal functions, 13 simple multimodal problems which are either shifted only or shifted and rotated, six hybrid functions, and eight composition functions. Rotation and shifting of the functions change the location of the optimal solution. It is done so that the optimal solution is not located at the center of the search space thus solving the problems is more challenging. The hybrid functions are combination of more than one function, while the composition functions consist of unimodal, multimodal, and hybrid functions with local optima trap set at the origin which is the centre of the search area.

Table 2.1: Test Functions (Liang, Qu, & Suganthan, 2013)

Function Type	Function ID	Function	Ideal Fitness fit_{ideal}
Unimodal Function	f1	Rotated High Conditioned Elliptic Function	100
	f2	Rotated Bent Cigar Function	200
	f3	Rotated Discus Function	300
Simple Multimodal Function	f4	Shifted and Rotated Rosenbrock's Function	400
	f5	Shifted and Rotated Ackley's Function	500
	f6	Shifted and Rotated Weierstrass Function	600
	f7	Shifted and Rotated Griewank's Function	700
	f8	Shifted Rastrigin's Function	800
	f9	Shifted and Rotated Rastrigin's Function	900
	f10	Shifted Schwefel's Function	1000
	f11	Shifted and Rotated Schwefel's Function	1100
	f12	Shifted and Rotated Katsuura Function	1200
	f13	Shifted and Rotated HappyCat Function	1300
	f14	Shifted and Rotated HGBat Function	1400
	f15	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	1500
	f16	Shifted and Rotated Expanded Scaffer's F6 Function	1600
Hybrid Function	f17	Hybrid Function 1 (N=3)	1700
	f18	Hybrid Function 2 (N=3)	1800
	f19	Hybrid Function 3 (N=4)	1900
	f20	Hybrid Function 4 (N=4)	2000
	f21	Hybrid Function 5 (N=5)	2100
	f22	Hybrid Function 5 (N=5)	2200
Composite Function	f23	Composition Function 1 (N=5)	2300
	f24	Composition Function 2 (N=3)	2400
	f25	Composition Function 3 (N=3)	2500
	f26	Composition Function 4 (N=5)	2600
	f27	Composition Function 5 (N=5)	2700
	f28	Composition Function 6 (N=5)	2800
	f29	Composition Function 7 (N=3)	2900
	f30	Composition Function 8 (N=3)	3000
Search Range: $[-100, 100]^D$			

The CEC2014's benchmark functions are single objective functions. Single objectives problems have only one ultimately optimal solution. Functions' modality is one of the factors that influences the functions' difficulties. A multimodal function has several peaks, whereas a unimodal function has only a single peak. The multiple peaks in multimodal problems cause the optimal solution to be less obvious and increase ruggedness to the problem's landscape. Multimodality causes population-based optimizers to be prone to converge to local optima rather than the global optima. Even though a unimodal function does not have the multiple peaks, but, a unimodal function can have a large basin and valley with flatter slope which causes the optimal solution to be hard to find. Neutrality or flat section is one of the factor that influence a function's hardness (Malan & Engelbrecht, 2014).

Dimensionality of the functions also influences their complexity. The search space of a function grows exponentially with its dimensionality (Jamil & Yang, 2013).

The 3D map of the benchmark functions which are available as two dimensional problems are shown in Figure 2.4. The figures illustrate the complexity of these functions. From the figures, it can be observed the unimodal functions have large basin and valley with flat slope, while majority of the multimodal functions are highly multimodal with multiple local optima traps.

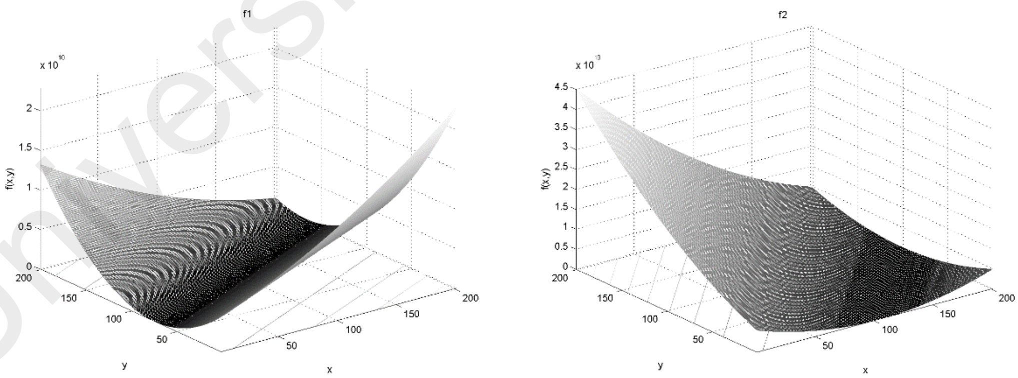

Figure 2.4: CEC2014's 3D Maps of Two Dimensional Problems

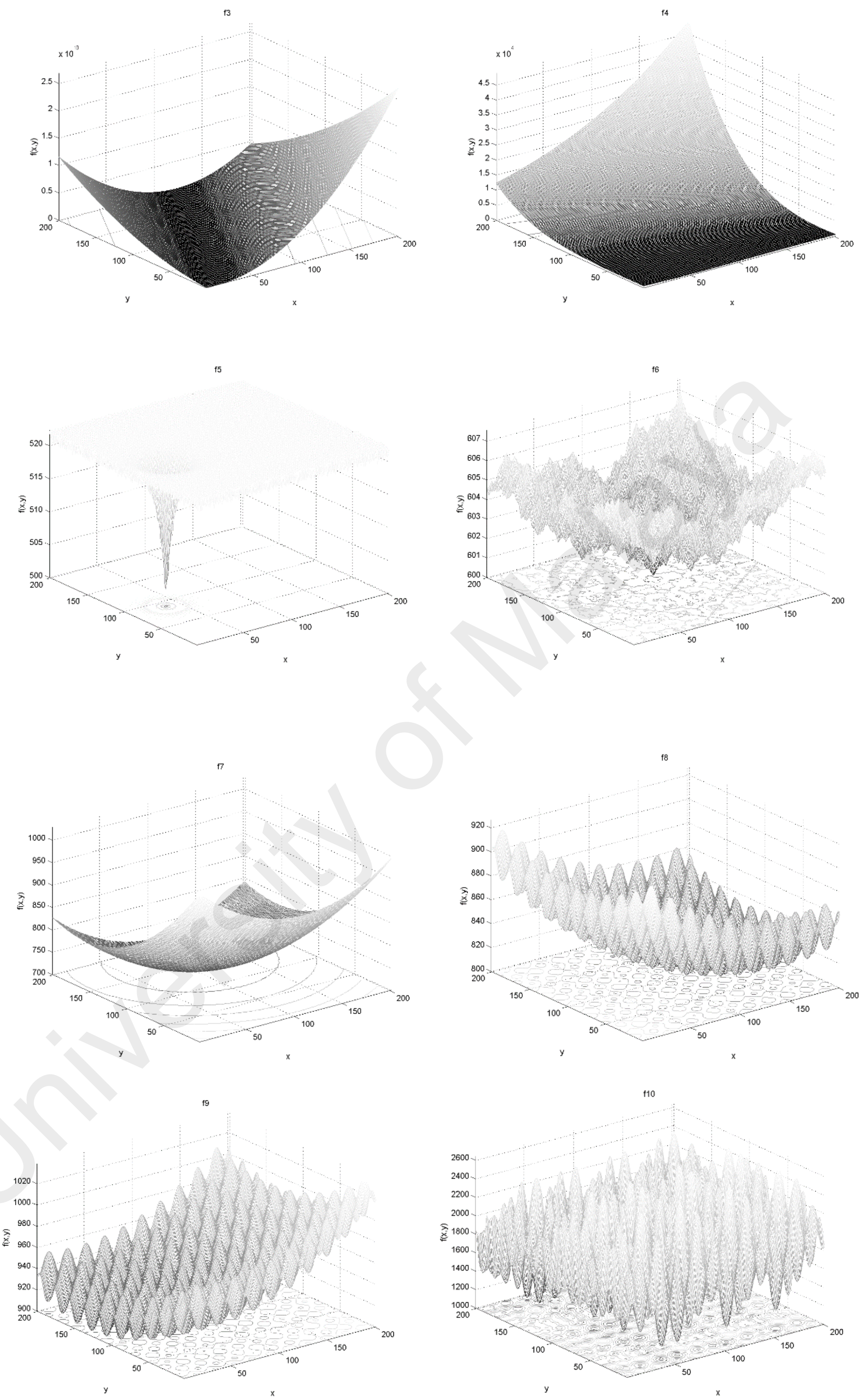

Figure 2.4: CEC2014's 3D Maps of Two Dimensional Problems (continued…)

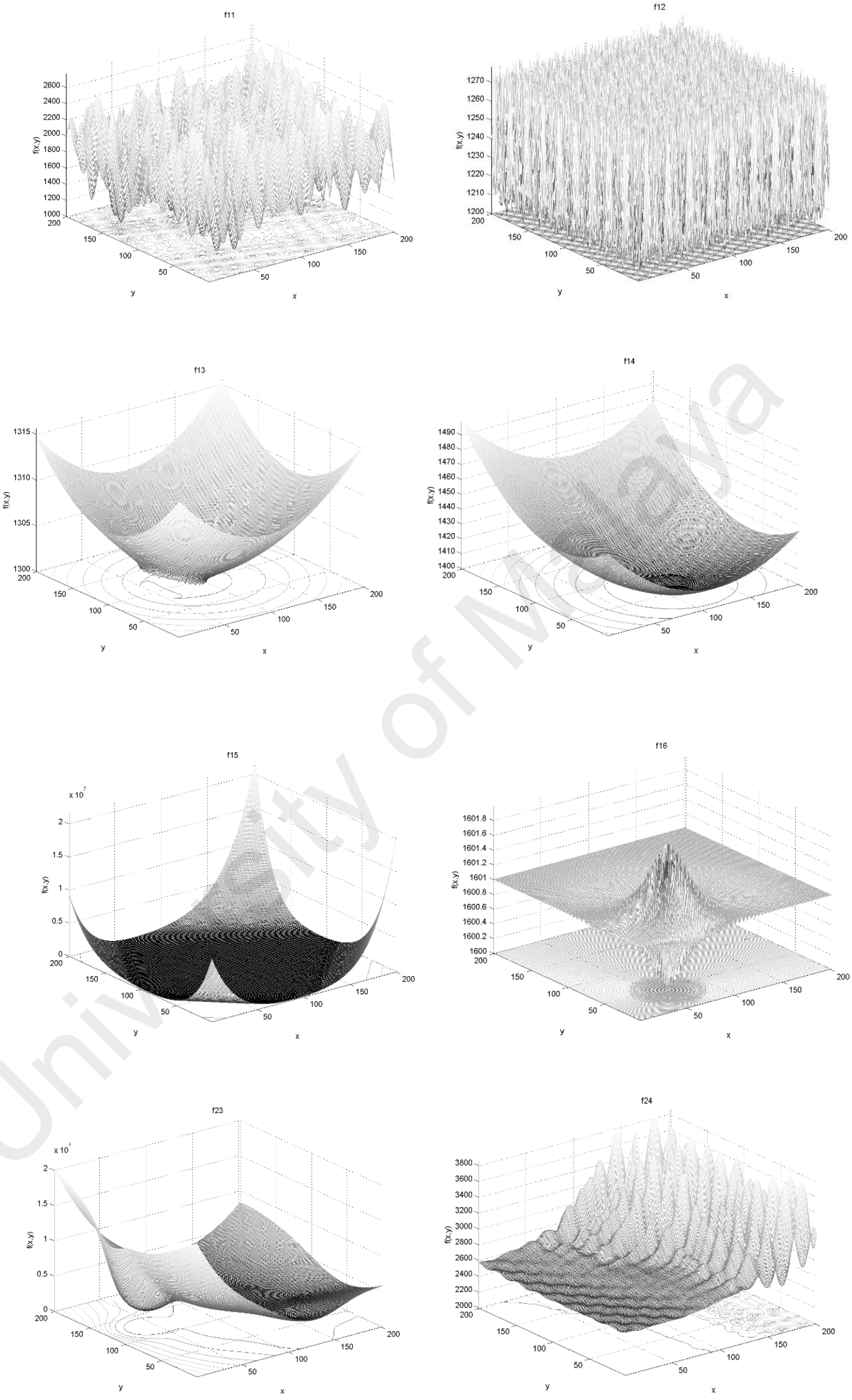

Figure 2.4: CEC2014's 3D Maps of Two Dimensional Problems (continued...)

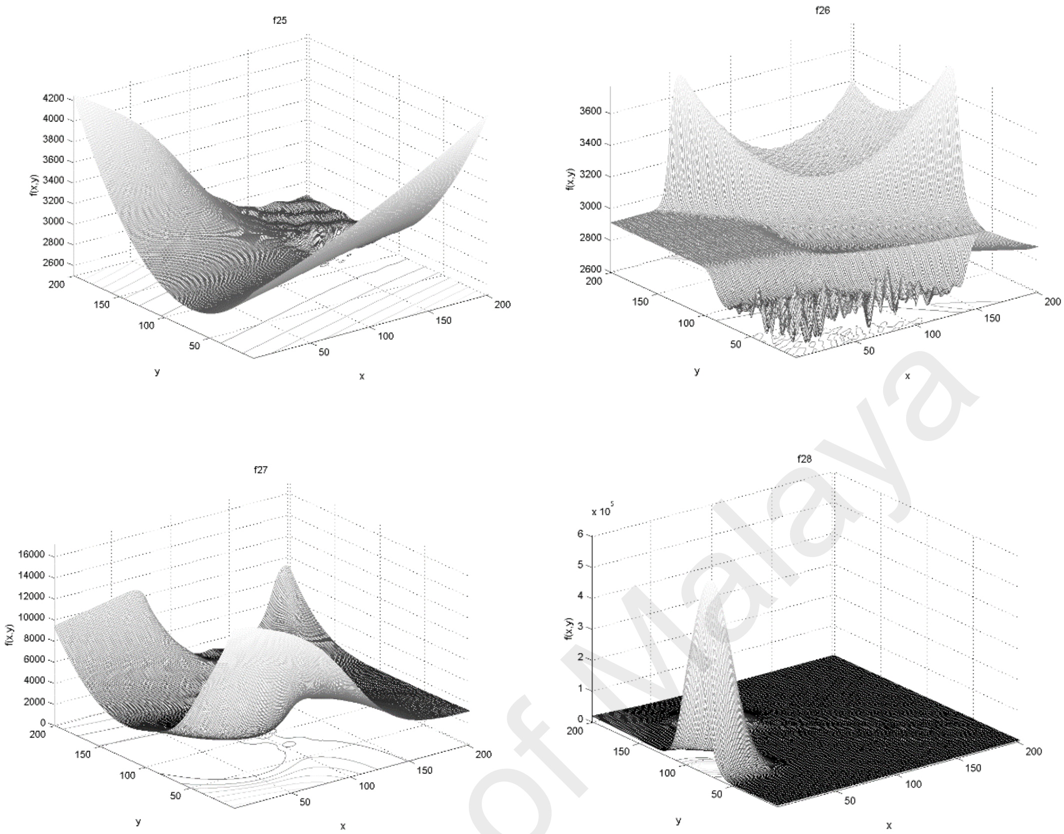

Figure 2.4: CEC2014's 3D Maps of Two Dimensional Problems (continued...)

2.5 Conclusion

The population-based metaheuristics is discussed in the first part of this chapter. The procedures of population-based metaheuristics are iterative process. Two type of traditional iteration strategies are available; synchronous and asynchronous update. The performance of population-based metaheuristics is highly influenced by the agents' exploration and exploitation. The exploration and exploitation of the population can be measured using agents' diversity.

In the second section of this chapter, the parent algorithms used in this study are reviewed. Three parent algorithms are selected, namely PSO, GSA, and SKF. PSO is a bioinspired algorithm. The search for optimal solution by the particles of PSO is performed by updating particles' velocities and positions. The agents in GSA search for optimal solution based on Newton gravitational law and law of motion, while SKF is inspired by Kalman filtering algoritnm.

The benchmark functions used in this work are reviewed in section 2.3. The functions are taken from the CEC2014's single objective real-parameter numerical optimization test suite. In total there are 30 functions consisting of unimodal, simple multimodal, hybrid and composite functions in the chosen test suite.

In the next chapter, works that had been conducted in overcoming the problem of premature convergence and controlling population's exploration and exploitation in the parent algorithms are categorized and reviewed. This is followed by the related works on the iteration strategy of the parent algorithms.

CHAPTER 3: LITERATURE REVIEW

3.1 Introduction

Population-based optimizers have the advantage of multipoint and diverse search points. However, the optimizers often lose this advantage due to premature convergence (Weise, Zapf, Chiong, & Nebro, 2009). This problem is reported in PSO (Jordehi, 2015; Nakisa, Nazri Ahmad, Rastgoo, & Abdullah, 2014; Nezami, Bahrampour, & Jamshidlou, 2013), genetic algorithm (Beheshti & Shamsuddin, 2013; Nicoară, 2009), and GSA (Han, Quan, Xiong, & Wu, 2015; Nobahari et al., 2011; Shang, 2013).

In multimodal problems, premature convergence by population-based optimizer is often caused by the agents' failure to escape from local optima. This causes the population to settle with a none optimal solution with poor performance. Therefore, mechanism to avoid and overcome premature convergence is important in improving population-based metaheuristics.

This chapter focuses on the methods that had been proposed to overcome the problem of premature convergence for the parent algorithms. Existing methods are reviewed and categorized. As shown in Figure 3.1, the works can be divided into five categories; step size manipulation, reinitialization, control of the information sharing, hybridization of multiple optimizers, and combination of methods from multiple categories.

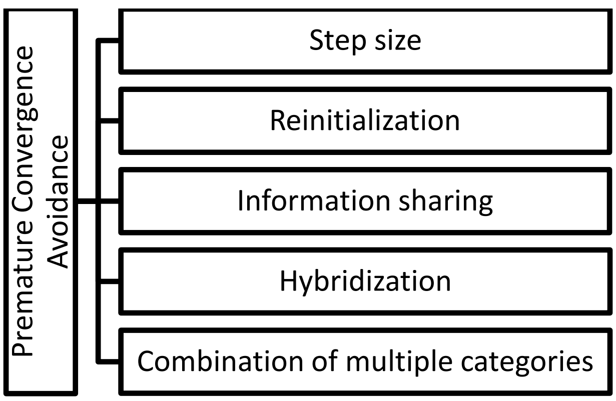

Figure 3.1: Categories of Premature Convergence Avoidance Methods

3.2 Existing Works on Premature Convergence Avoidance of the Parent Algorithms

3.2.1 Step Size

Step size is the rate of change from a current solution to the next solution. In PSO and GSA algorithms, the step sizes are the agents' velocities, while in SKF, the step size is influenced by the difference between the measured value and the predicted value. A big step size allows an agent to explore the search space by moving farther. On the other hand, smaller step size moves an agent to nearby area only, this encourages the agent to exploit the information within the area. Hence, controlling step size can be very beneficial in improving the performance of an optimizer. The step size can be controlled by manipulation of original parameters or introduction of new parameters.

The most effective and widely adopted parameter introduced to PSO is inertia weight (Shi & Eberhart, 1998). Inertia weight and acceleration constants, are capable of controlling the particles' step sizes, which contributes to a better performance. Ever since its introduction many works had been reported on variation of inertia weight for performance improvement. For example, adaptive weight was proposed in (Qin, Yu, Shi, & Wang, 2006) and a fuzzy based inertia weight was proposed in (C. Liu & Ouyang, 2010). In Sharma & Kaur (2015), constant inertia, random inertia, chaotic random inertia, and adaptive inertia were studied. Extensive survey on various inertia weight PSOs are presented in (Bansal et al., 2011; Harrison & Engelbrecht, 2016).

Another parameter that facilitates the control of particles' step size so that better performance is achieved is constriction factor (Clerc & Kennedy, 2002). Like the inertia weight, the constriction factor is a multiplier added to PSO's velocity equation. The inertia weight is only multiplied to the momentum part of the velocity, whereas constriction factor is multiplied to the entire original PSO's velocity, i.e.; without inertia weight. The constriction factor is able to control exploration and exploitation of the swarm. In (Eberhart & Shi, 2000), it was reported that constriction factor PSO is able to obtained good performance by clamping its maximum velocity according to the search space.

Acceleration constants can also be manipulated to improve PSO (Y. L. Zheng, Ma, Zhang, & Qian, 2003). In attractive-repulsive PSO, the signs within the velocity equation are inverted alternately to provide attractive and repulsion force according to the particles state of convergence (Riget & Vesterstrøm, 2002). Meanwhile in (Cheng & Shi, 2011), a new position update equation with additional parameter was introduced. The new parameter is able to control the PSO's swarm diversity.

In (Farivar & Shoorehdeli, 2016), Lyapunov particle dynamic is used in determining the GSA's agents acceleration. This additional computation was added to improve exploration and exploitation so that good performance can be achieved.

Momentum operator was introduced to GSA in (Ginardi & Izzah, 2014). The agents of this algorithm move to opposite direction when collision occurs. The collision is subjected to elastic collision. This able to preserve diversity and avoid premature convergence by the agents of GSA.

In (Abdul Aziz, Ibrahim, Ab. Aziz, & Razali, 2017), a parameter-less SKF is introduced. The parameter-less SKF is able to perform as good as the original SKF and lift the necessity of parameter tuning for optimal performance of SKF.

Methods from this category can be as simple as introduction of a new multiplier but nonetheless these methods introduce additional computation, which is caused by the new parameter or the additional procedures introduced to control the step sizes.

3.2.2 Reinitialization

Existing works on improvement of optimizers performance can also be categorized into reinitialization. In reinitialization, the search agents are redistributed within the search space so that the population is re-diversified.

Reinitialization can be done randomly or according to the condition of the population. In (Binkley & Hagiwara, 2008), median velocity is used to signal reinitialization, while in (Guo & Tang, 2009), the step length is chosen to determine when and which particles to be reinitialized. Radius of effect, which is a parameter used as a metric for reinitialization was proposed in (Budhraja, Singh, Dubey, & Khosla, 2013).

Cheng, Shi, and Qin, (2011), suggested two reinitialization methods; random and elitist. The random reinitialization randomly reinitialized the particle, while, elitist reinitialization maintains the particles with higher ranked performance and reinitialized the others. The reinitialization is conducted periodically.

GSA with disruption was proposed in (Sarafrazi, Nezamabadi-Pour, & Saryazdi, 2011). The disruption is similar to reinitialization method. Since GSA is memoryless, the present distance of masses is used to determine when to disrupt the agents' positions.

Reinitialization is a simple strategy, however, the population risk losing any good information found prior to the reinitialization. Currently no work involving reinitialization SKF had been reported.

3.2.3 Information Sharing

The performance of population-based optimization algorithms is contributed by collaboration of multiple agents via information exchange. Hence, proper control of the speed of the information sharing, type of the information shared, connectivity of the agents, and the origin of the information, can improve the algorithms' performance (Budhraja et al., 2013; Kennedy & Mendes, 2002; Mendes, Kennedy, & Neves, 2004; Nezami et al., 2013; Premalatha & Natarajan, 2009; Rada-Vilela, Zhang, & Seah, 2012; Riget & Vesterstrøm, 2002; Voglis et al., 2012). Among the methods proposed under this category are multiswarms, agents clustering, ranked based neighborhood, and various neighborhood topologies.

In (Van den Bergh, 2001), a guaranteed convergence PSO (GCPSO) was proposed. The velocity of a particle is updated using information of its neighborhood and personal best experience. This may cause the best performer of the swarm to stop moving and convergence of the swarm to a none optimum solution. Therefore, in GCPSO, the best performer of the swarm adopts different position update equation. The equation allows the best performer to explore its surrounding area and avoid stagnation and convergence of swarm towards a none optimum solution.

Particles in fully informed PSO as suggested in (Mendes et al., 2004) use information from all neighborhood particles. The neighborhood topology determines how diverse the source of information used by the particles. This method is computationally more expensive and more complex.

Kennedy in his work (Kennedy, 2000), suggested the particles of PSO to be divided into clusters. Through particle clustering, the performance of the swarm can be improved by stereotyping the particles to the best performer of their cluster. However, clustering requires additional computational cost.

Niching or partitioning the population into subpopulations is a popular approach for the improvement of population-based algorithms. It was used to improve the performance of PSO in (Brits, Engelbrecht, & Bergh, 2002; Passaro & Starita, 2008; Schoeman & Engelbrecht, 2004). Niche GSA was proposed in (S. Yazdani et al., 2014). The niche GSA also searches for multiple local optima as these can be good alternative solutions.

PSO with particles ranking was used in (W. H. Lim, Ashidi, & Isa, 2015; Ma, Zhang, & Xu, 2015). The rank of the particles is used to determine the importance of the information carried by a particle towards other members of the swarm.

The SKF's search for optimum solution is carried using information of the best solution found so far. No works on other method or modification of existing information sharing had been conducted for SKF.

3.2.4 Hybridization of Algorithms

According to the no free lunch theorem, no ultimate algorithm exists. An algorithm might outperform another algorithm in a particular case and performs badly in another. Hence, hybridization of two or more optimizers potentially can contribute to a high-performance optimizer. However, the hybrid algorithm can be more complex compared to the originals.

Many works on hybrid PSO had been proposed. GA operators are popular choice to be integrated with PSO. Selection is incorporated with PSO in (P. J. Angeline, 1998). In (Higashi & Iba, 2003; Janˇ Causkas, 2014; C. Li, Yang, & Korejo, 2008; Pant, Thangaraj, & Abraham, 2008; Premalatha & Natarajan, 2009), mutation is incorporated with PSO to improve its diversity, while in (Engelbrecht, 2013a, 2014, 2015; Wang, Wu, Liu, & Zeng, 2008) crossover operators are chosen.

A hybrid GSA with GA's selection and mutation operators was proposed in (G. Sun, Zhang, Yao, & Wang, 2016) and crossover in (Shang, 2013). The GA's operators are merged with GSA as an attempt to recover from premature convergence.

Hybridization of PSO and simulated annealing was suggested in (Basu, Deb, & Garai, 2014). The simulated annealing is applied periodically based on PSO's convergence to encourage local search within the neighborhood of $\boldsymbol{gBest}$. Simulated annealing was also chosen to be hybridized with GSA in (H. Chen, Li, & Tang, 2011). The simulated annealing is used to determine whether to accept or reject solution found by various local search operations.

Quantum mechanics had been hybridized with GSA in several works (Moghadam, Nezamabadi-Pour, & Farsangi, 2012, 2014). The quantum mechanics provides diversity to the population and avoid premature convergence. Quantum mechanics had also been combined with PSO in (dos Santos Coelho & Mariani, 2008; Huang, Wang, Yang, & Wu, 2009; Jia, Duan, & Yan, 2015; Mikki & Kishk, 2006).

Fuzzy logic is a popular choice to be hybridized with PSO, this is seen in various publications such as (Altinoz, Tanyer, & Yilmaz, 2012; Khan & Engelbrecht, 2012; Mubeen, Hemalatha, & Reddy, 2015). In (Saeidi-Khabisi & Rashedi, 2012), fuzzy logic was used to balance exploration and exploitation of GSA through parameter control.

Performance improvement through hybridization had also been reported for SKF (Muhammad, Ibrahim, Zakwan, & Azmi, 2016a, 2016b; Muhammad et al., 2015, 2017; Muhammad, Ibrahim, Mohd Azmi, et al., 2016). In these works, SKF is proposed to be hybridized with either PSO or GSA in its prediction state. The hybrid algorithms are able to improve the performance of SKF.

3.2.5 Using Combination of Multiple Categories

Here, works that used combination of two or more methods from the four categories that are previously reviewed are grouped as the fifth category. The works from this category are more complex and computationally more expensive due to the combination of multiple methods.

In (Suganthan, 1999), the performance of PSO is improved by controlling the information shared through dynamically changing neighborhood size while the step size is controlled using time varying inertia and acceleration factors. Similarly in (Yazawa, Motoki, & Yasuda, 2009), the performance of PSO was improved using information sharing structure and step size, where the particles are divided into clusters and their velocities are updated using a new equation. Zhan, Zhang, Li, & Chung, (2009), proposed combination of fuzzy adaptive inertia weight and acceleration constants together with elitist learning strategy for a better PSO algorithm. Combination of rank-based population, new social influence, and acceleration constants was proposed in an improved PSO (Ostadmohammadi Arani, Mirzabeygi, & Shariat Panahi, 2013).

In (B. Jiang, Wang, & He, 2011), an asynchronous PSO with relearning and hypermutation were chosen for improvement of PSO. The relearning process is initiated when a particle's best is not improved. The relearning gives the particle a second chance to improve its performance by forgetting and recalculating its velocity and position. On the other hand, hypermutation is applied to randomly chosen particles to enhance the exploration of the swarm.

PSO with opposition-based learning, reinitialization, and adaptive velocity were proposed in (Kaucic, 2013). Space transformation, which is a method similar to opposition-based learning was combined with disturbance operator in an enhanced PSO introduced by (Yu, Wu, Wang, Chen, & Zhong, 2012).

In (Mirjalili & Lewis, 2014) the adaptive $\boldsymbol{gBest}$-guided GSA was introduced, where the best found solution, $\boldsymbol{gBest}$, and new parameters are incorporated into the velocity equation, this helps in exploration and exploitation of the population. Hybrid of chaotic perturbation and memory of the population is found to be able to avoid premature convergence in GSA (S. Jiang, Wang, & Ji, 2014). No work from this category had yet been reported for SKF.

3.3 Conclusion

In this chapter, works that focuses on improvement of the parent algorithms through premature convergence avoidance are reviewed. The works are categorized into five categories, namely; step size based methods, reinitialization, information sharing, hybridization of several algorithms and combination of two or more of the previous four categories.

In the next chapter, the influence of existing traditional strategies towards the performance and behavior of the agents of the parent algorithms are studied. Asynchronous GSA (A-GSA) and asynchronous SKF (A-SKF) are proposed in the next chapter.

CHAPTER 4: TRADITIONAL ITERATION STRATEGIES

4.1 Introduction

In this chapter, existing works related to the iteration strategy of the parent algorithms are reviewed followed by discussion on the implementation of the parent algorithms using the traditional synchronous update and asynchronous update. From the literatures, it can be seen that not many work had been conducted focusing on this fundamental aspect of population-based metaheuristics and no work had been reported on the usage of iteration strategy for algorithm improvement. Asynchronous GSA (A-GSA) and asynchronous SKF (A-SKF) which are presented in this chapter are new concept to the respective parent algorithms. The performance of the parent algorithms implemented using synchronous and asynchronous update are reported in this chapter.

4.2 Literature Review

In PSO, synchronous update is the most commonly adopted iteration strategy. It is the iteration strategy of the standard PSO. From the limited number of works studying the effect of iteration strategy on PSO, it was reported that the iteration strategy does influence the performance of PSO. S-PSO allows the agents to have overview of the swarm's current performance before the next move is made, this allows better selection of $\boldsymbol{gBest}$. Therefore, Carlisle & Dozier (2001) recommended global neighborhood structure S-PSO. The good selection of $\boldsymbol{gBest}$ influences S-PSO to converge faster (Rada-Vilela, Zhang, & Seah, 2011a) and exploits.

Asynchronous PSO (A-PSO) was first discussed in (Carlisle & Dozier, 2001). In A-PSO, a particle's select its $\boldsymbol{pBest}_i$ and update $\boldsymbol{gBest}$ as soon as its fitness is evaluated. The particle's velocity and position update follow immediately after that. Therefore, the particles in A-PSO are updated with imperfect information of the swarm (Rada-Vilela et al., 2013), where in a single iteration of A-PSO, $\boldsymbol{gBest}$ can assume more than one value, thus, encourages exploration of the particles. Carlisle & Dozier, in their work suggested that instead of global neighborhood, the local neighborhood is better suited for A-PSO (Carlisle & Dozier, 2001). The lack of synchronicity in A-PSO solves the issue of idle particles faced in S-PSO (Rada-Vilela et al., 2011b), an advantage especially in parallel implementation of the algorithm.

In a more recent work by Engelbrecht (2013b), the performance of S-PSO and A-PSO is studied using benchmark of 59 functions. The findings show that iteration strategy is a problem dependant parameter for PSO algorithm and A-PSO is neither faster nor better suited for local neighbourhood than S-PSO.

Asynchronous update also enables the sequence of the particles to be updated to change dynamically. Also a particle is allowed to be updated more than once in single iteration or none at all (Dioşan & Oltean, 2006; Rada-Vilela et al., 2011b). For example, in random asynchronous PSO (RA-PSO) (Rada-Vilela et al., 2011b), the particles to be updated are chosen randomly with repetition allowed. The order of the particles to be updated is randomly chosen regardless of the particle number. Since the selection of the particles is done randomly, the information flow is different from an iteration to another iteration. Such differences can prevent the particles from being trapped in local optima, unlike the particles of S-PSO which are prone to be stagnant in local optima.

A PSO based on social psychology (BSPSO) (W. Liu et al., 2009), adopts the asynchronous update in its iteration strategy. BSPSO incorporates mutation in the algorithm and the effect of neighbourhood information is controlled based on the age of the swarm. The combination of asynchronous update with mutation enhances exploration. The αPSO which is developed based on asynchronous update mechanism was introduced in (Takahama & Sakai, 2005). The αPSO algorithm is tailored for constrained optimization problems. In αPSO, the particle's best and neighbourhood's best are updated based on whether the constraints are met or not. No justification was given on why asynchronous update is chosen over synchronous update.

Asynchronous update is popular among parallel implementation of PSO (Akat & Gazi, 2008; Koh, George, Haftka, & Fregly, 2006; Venter & Sobieszczanski-Sobieski, 2005; Xue et al., 2009). It allows full utilization of the parallelization feature and the computational ability can be fully exploited using asynchronous update strategy.

In asynchronous multiswarms PSO (de Campos et al., 2013), asynchronous update is used between the multiswarms. The multiswarms are implemented among parallel processors. The information of best member of each swarm is shared using asynchronous communication. The asynchronous strategy among the swarms allows the swarms to carry search process independently and avoid local optima traps.

PSO with deliberate loss of information (PSO-DLI) was proposed in (Voglis et al., 2012). There are two loops in PSO-DLI, one for velocity and position update, the second loop is for particles evaluation. However, in PSO-DLI, not all particles are evaluated in an iteration. Particles are randomly selected to be excluded from performance evaluation phase. For these selected particles, only one loop is executed. This is similar to A-PSO. The deliberate loss of information is proposed due to the fact that in most situations, even though improvements are recorded, most of the time the improvements are marginal. The marginal improvements hinder the particles from exploiting the information gained from the previous bests. The loss of information in DLI-PSO contributes to more efficient exploitation of information. PSO-DLI shares a similarity with RA-PSO, where several particles are dropped from evaluation phase. However, in contrast to DLI-PSO these particles are not allowed to move in RA-PSO.

A similar approach is proposed in PSO with neighbourhood-based budget allocation (Souravlias & Parsopoulos, 2014). The algorithm used asynchronous update with ring topology and number of fitness evaluation as the stopping condition. In this algorithm, some of the particles are evaluated more frequently than others. The particles selected for evaluation are based on the performance and also the diversity of the neighbourhood. A particle within fitter neighbourhood has higher probability to find better solution by refining its search using the information shared by its neighbours. The evaluation of neighbourhood fitness adds extra computation for this variation of PSO algorithm.

No work has been reported that focus on the iteration strategy of GSA and SKF. Prior to this research, GSA and SKF were only implemented as synchronously updated population-based metaheuristics.

As a conclusion, based on the works reported for PSO, asynchronous update is chosen due to two reasons:

i. Ability to improve exploration through adoption of more than one reference points based on the latest information shared.
ii. Its suitability for parallel implementation.

4.3 The Parent Algorithms in Asynchronous Update Mechanism

4.3.1 Asynchronous PSO, A-PSO

The concept of asynchronous update in PSO was introduced by (Carlisle & Dozier, 2001). A particle in A-PSO, is able to move without the need to wait for the other members of the swarm. As a nature inspired algorithm, this approach is more natural compared to synchronous update. In nature, the individuals are able to move independently without the need to synchronize their movement with others.

The A-PSO algorithm is illustrated in Figure 4.1 and Algorithm 4.1. A-PSO starts with the initialization of the members of the swarm. There is one loop per iteration in A-PSO where a particle position is evaluated and compared with $\boldsymbol{pBest_i}$ and $\boldsymbol{gBest}$, this is immediately followed by the particle's velocity and position update. After a particle completed these steps, another particle is then selected to go through the same process. The stopping condition is compared at the end of an iteration. If it is met, then the algorithm is ended.

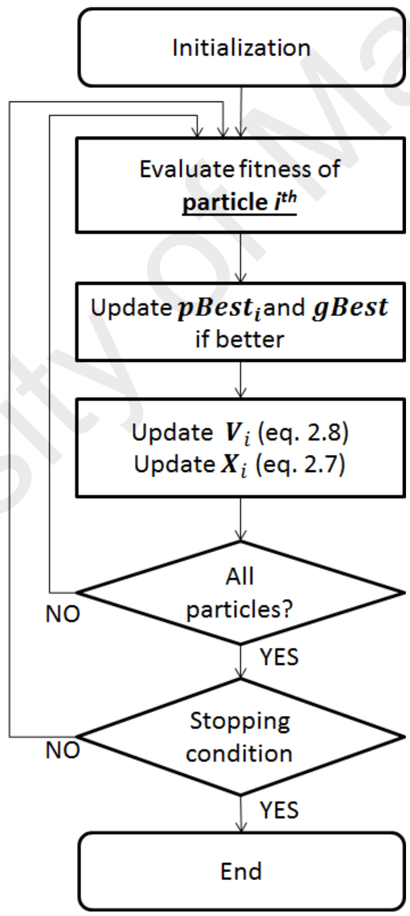

Figure 4.1: Flowchart of A-PSO

```
Initialization of swarm
Do{
    For every particles
        Evaluate fitness
        Update pBest and gBest if better
        Update V_i, equation 2.7
        Update X_i, equation 2.6
    End for
}While not stopping condition
```

Algorithm 4.1: Pseudo Code of A-PSO

4.3.2 Asynchronous GSA, A-GSA

Since its introduction, only synchronous update GSA had been reported by other researchers. Nonetheless, as a population-based algorithm, GSA has the potential to be implemented asynchronously. This study is the first to consider asynchronous-GSA (Ab. Aziz et al., 2013).

In the iteration of A-GSA, an agent's position update phase begins as soon as its performance is evaluated. The agent does not need to wait for the entire population to be evaluated. Hence, after its own evaluation, $best(t)$ and $worst(t)$ are identified using whatever information available. Therefore, the position is updated using mixture of information from updated positions and old positions. This mixture of information is believed to encourage more exploration by the agents.

As a memoryless algorithm, the asynchronous update in A-GSA causes the best and worst agents to change more frequently compared to S-GSA. The frequent change, hypothetically increases the population diversity. The algorithm of A-GSA is shown in Figure 4.2 and Algorithm 4.2. In contrast to S-GSA, the evaluation and update process of A-GSA are conducted within one loop.

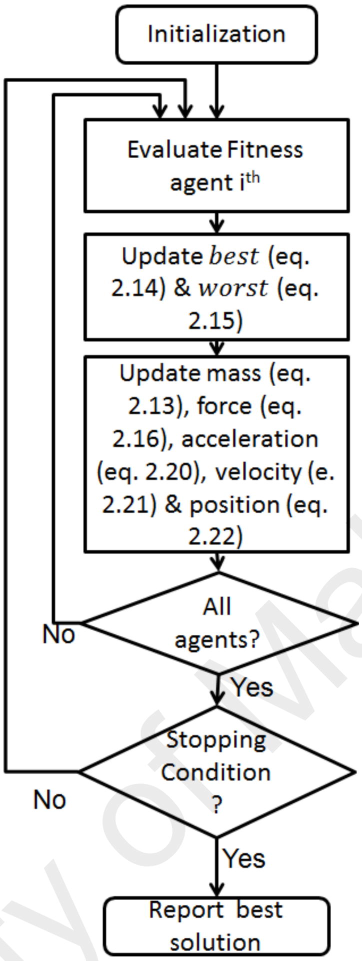

Figure 4.2: Flowchart of A-GSA

Initialization of agents
Do{
    For every agents
        Evaluate fitness
        Identify $best(t)$ and $worst(t)$ using eq. 2.14 & eq. 2.15
        Update mass, equation 2.13
        Update force, equation 2.16
        Update acceleration, equation 2.20
        Update velocity, equation 2.21
        Update position, equation 2.22
    End for
}While not stopping condition

Algorithm 4.2: Pseudo Code of A-GSA

4.3.3 Asynchronous SKF, A-SKF

This thesis is the first to consider asynchronous update for SKF. In asynchronous update mechanism, an agent is able to proceed with the Kalman filter's procedures; predict, measure and estimate, as soon as its own fitness is evaluated.

Similar to S-SKF, A-SKF starts with random initialization of the population according to the problem's search space. However, unlike S-SKF, the steps within the iteration are individually executed for A-SKF. Therefore, in an iteration of A-SKF, as soon as an agent is evaluated, its fitness is compared with $\boldsymbol{X}_{true}$. If the agent has found a better solution, then the $\boldsymbol{X}_{true}$ is immediately updated according to the estimated value of the agent. Thus, in A-SKF, $\boldsymbol{X}_{best}(t)$ is not needed.

After the $\boldsymbol{X}_{true}$ comparison, the agent's state is immediately predicted. This is followed by the agent's measurement and state estimation. When an agent completed its Kalman filter's procedures, next agent is selected to go through the same steps. The A-SKF algorithm is presented in the flowchart in Figure 4.3 and Algorithm 4.3.

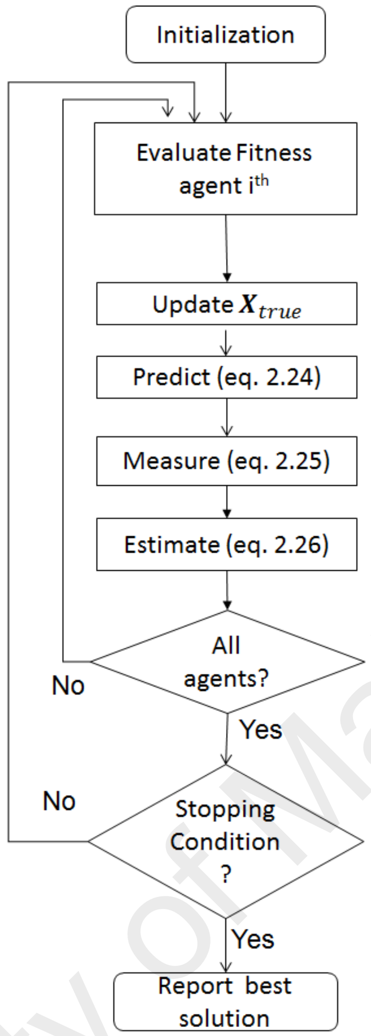

Figure 4.3: Flowchart of A-SKF

Initialization of agents
Do{
    For every agents
        Evaluate fitness
        Update $\boldsymbol{X}_{true}$
        Predict, equation 2.24
        Measure, equation 2.25
        Estimate, equation 2.26
    End for
}While not stopping condition

Algorithm 4.3: Pseudo Code of A-SKF

4.4 Experiment, Results and Discussion

4.4.1 Experimental Parameter Setting

The parameter settings used for the experiments conducted are as follow (the literatures following every parameter's value used the same setting for respective parameters):

- Population size, $N = 100$ (M. Li, Zhao, Weng, & Han, 2016; Z. Li, Wang, Yan, & Li, 2015; Rahnamayan, 2007)
- Dimension size, $D = 30$ (Astudillo, Melin, & Castillo, 2015; Cui, Li, Lin, Chen, & Lu, 2016; Kuo & Zulvia, 2015; M. D. Li, Zhao, Weng, & Han, 2016; M. Li et al., 2016; Rashedi et al., 2009; Y.-J. Zheng, 2015)
- Maximum function evaluation, $FES = 10000 * D$ (Cui et al., 2016; Kumar & Soman, 2016; X. Li & Yin, 2015; Liang et al., 2013; Piotrowski, 2015)
- Number of independent run, $T = 30$ (Cui, Li, Lin, Chen, & Lu, 2015; D. Chen et al., 2015; Doğan & Ölmez, 2015; Kuo & Zulvia, 2015; Piotrowski, Napiorkowski, & Rowinski, 2014; Rahnamayan, 2007; Rashedi et al., 2009)

The setting for the parameters unique for each parent algorithms are listed in Table 4.1.

The performance of the algorithms, S-PSO, A-PSO, S-GSA, A-GSA, S-SKF and A-SKF are measured using the fitness error value (equation 2.1). The average fitness error value from the total number of run is then statistically analyzed using non-parametric statistical analysis procedures. According to García, Molina, Lozano, & Herrera, (2008), for comparison of metaheuristics algorithms, non-parametric tests are more appropriate compare to parametric tests. Often, the data from experiments involving metaheuristics algorithms do not meet the normal data distribution condition for validity of parametric test. Therefore, non-parametric tests are more suitable.

The pairwise Wilcoxon signed rank test is used to compare the performance of the parent algorithms implemented using the two traditional iteration strategies. The Wilcoxon signed ranks test identifies if significant difference exists between two algorithms being compared. The significance level used in Wilcoxon test range from 1% to 10%. Significance level indicates rigidness of a claim. A smaller value of significance level shows the more rigid is the claim made in acknowledging the significance of the difference between two algorithms being analysed. All the algorithms tested here; S-PSO, A-PSO, S-GSA, A-GSA, S-SKF and A-SKF are later tested and ranked according to Friedman test. If the p-value of Friedman test indicates significant difference exist, the results are then compared using Holm procedure with significance level of 5%.

The change of populations' behaviour towards the iteration strategy is observed using position diversity (equation 2.4).

Table 4.1: Initial Parameters According to Parent Algorithms

Algorithm	Parameter	Value	Literature
PSO	Inertia Weight	0.9-0.4, linearly decreasing	(Eberhart & Shi, 2000)
	V_{max}	[-100,100]	(Eberhart & Shi, 2000)
	c_1 and c_2	2	(Shi & Eberhart, 1998)
GSA	G_o	100	(Rashedi et al., 2009)
	β	20	(Rashedi et al., 2009)
SKF	Q	0.5	(Z. Ibrahim et al., 2015)
	R	0.5	(Z. Ibrahim et al., 2015)
	$\boldsymbol{P}_i(0)$	1000	(Z. Ibrahim et al., 2015)

4.4.2 Fitness Error Value

PSO- Figure 4.4 shows the S-PSO's and A-PSO's fitness error value over iteration for unimodal functions. Both S-PSO and A-PSO exhibit almost similar trend, where the error decrease exponentially and start to stabilize when the iteration reaches about 1500 iterations. The same trends are observed for simple multimodal functions as shown in Figure 4.5, hybrid functions in Figure 4.6, and composite functions in Figure 4.7 where the pattern of the error rate of both S-PSO and A-PSO are closely matched to each other. For some simple multimodal functions, which are f5, f6, f8, f9, f10, f11, f12 and f15, as well as f27 and f28, which are composite functions, instead of exponential decrement the error rate decreases gradually.

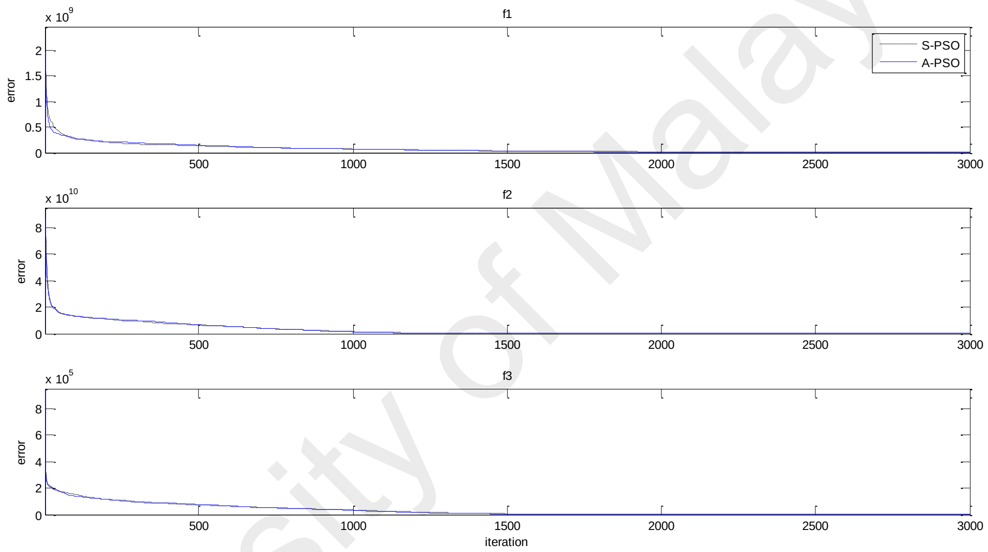

Figure 4.4: Fitness Error Rate of Unimodal Functions for S-PSO and A-PSO

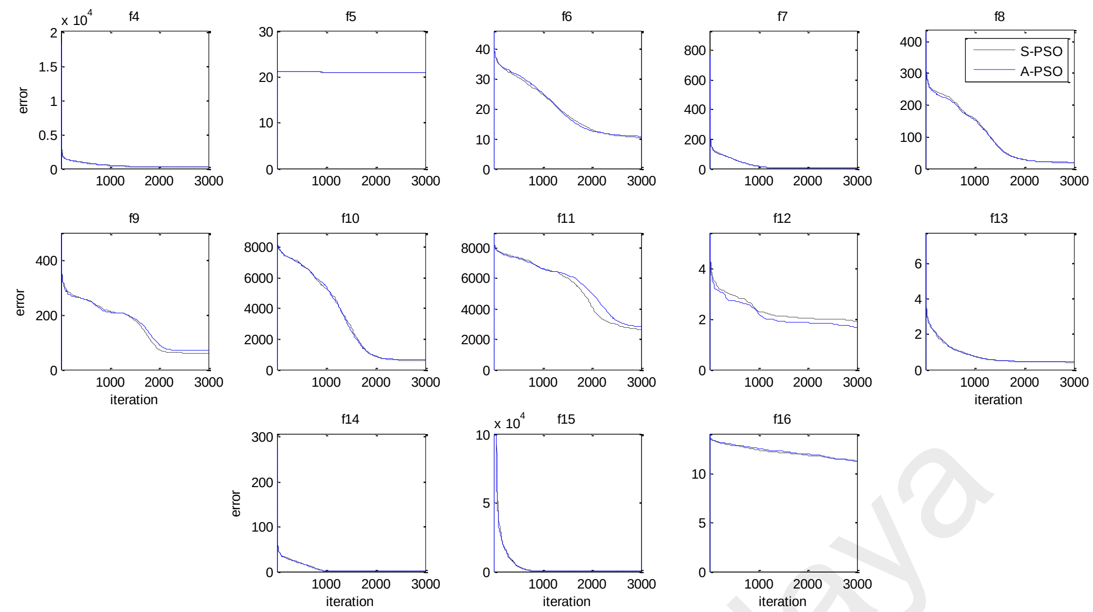

Figure 4.5: Fitness Error Rate of Simple Multimodal Functions for S-PSO and A-PSO

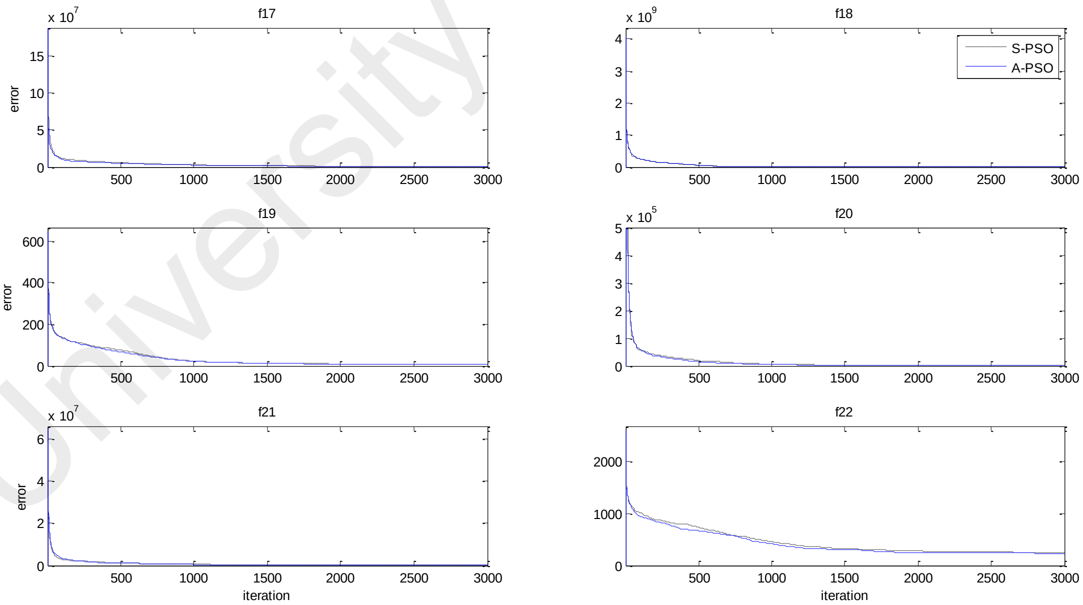

Figure 4.6: Fitness Error Rate of Hybrid Functions for S-PSO and A-PSO

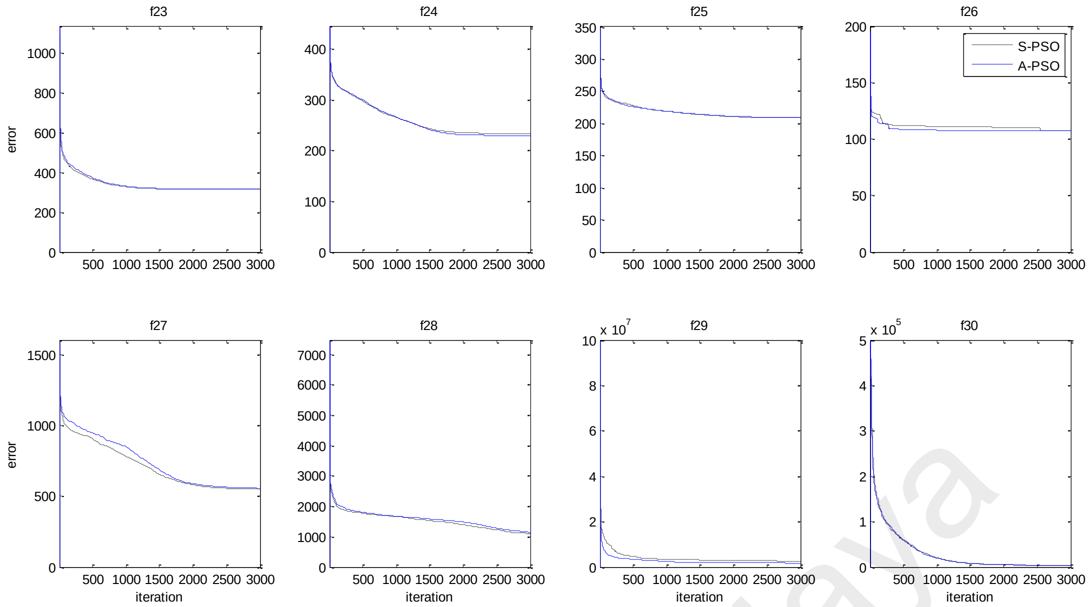

Figure 4.7: Fitness Error Rate of Composite Functions for S-PSO and A-PSO

The distribution of the best solutions found by S-PSO and A-PSO for unimodal functions are shown in Figure 4.8, while Figure 4.9 represents the simple multimodal functions, the hybrid functions are in Figure 4.10 and lastly the boxplots in Figure 4.11 are for composite functions. From the boxplots, it can be seen that the data are not uniformly distributed. The lines within the boxes show median values, while the circles out of the boxes show the outliers. The outliers represent the out of ordinary results. Lesser outliers and smaller box are desirable as it indicates stable performance. The test functions used are minimization functions, hence box with lower position indicates a good performance.

S-PSO has greater number of extreme outliers for the unimodal functions. These outliers are the factors that contribute to large average error of S-PSO for unimodal functions. For the simple multimodal and composite functions, the iteration strategy with lower box has a better performance. Both S-PSO and A-PSO do not have outliers for the tests involving the simple multimodal functions. A-PSO performs better for hybrid functions where S-PSO is observed having more outliers and higher box.

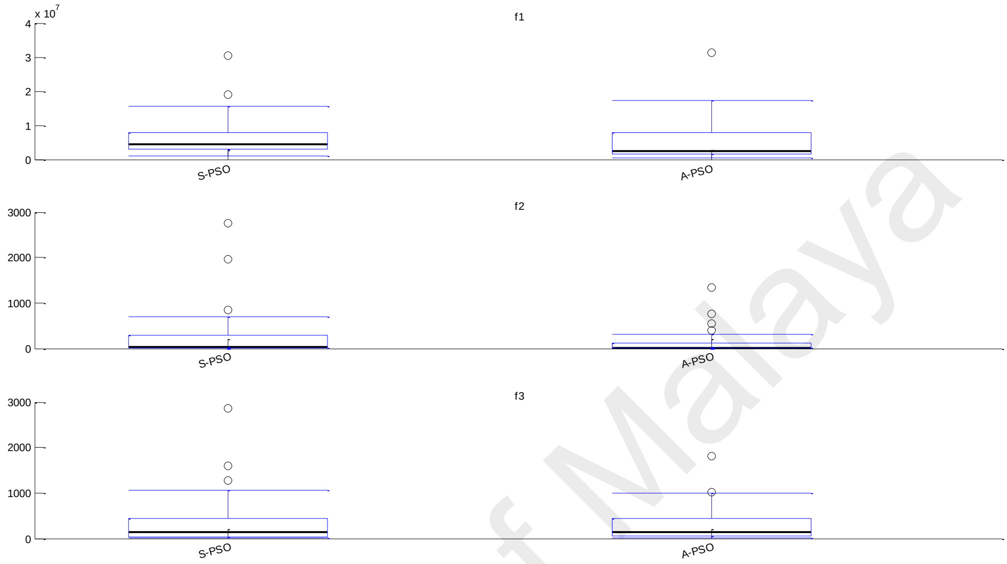

Figure 4.8: Fitness Error Distribution of Unimodal Functions for S-PSO and A-PSO

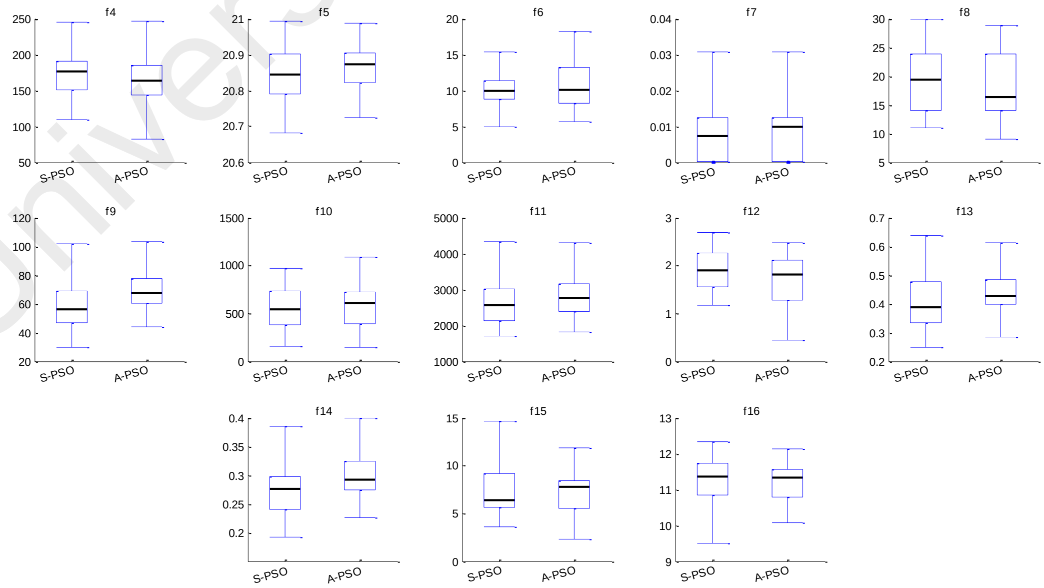

Figure 4.9: Fitness Error Distribution of Simple Multimodal Functions for S-PSO and A-PSO

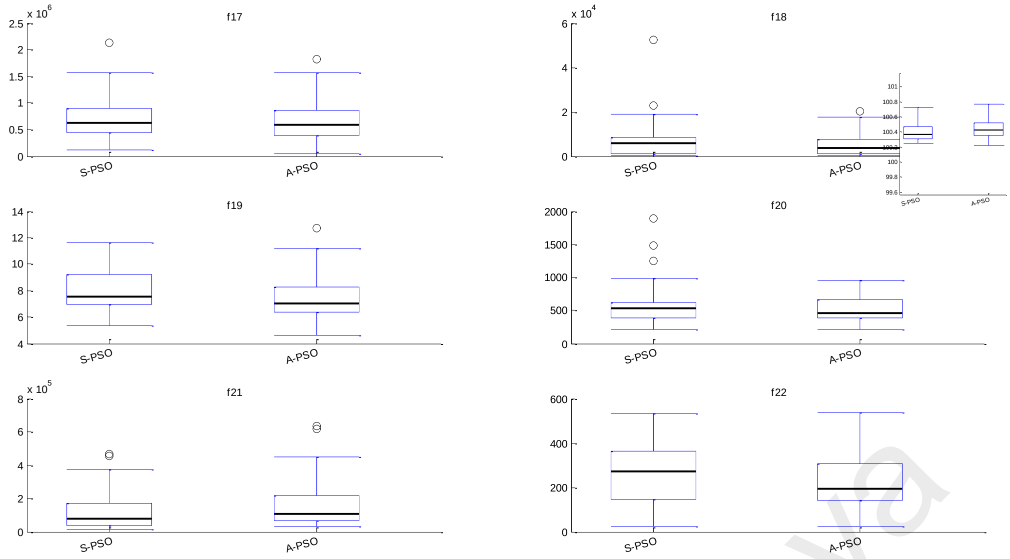

Figure 4.10: Fitness Error Distribution of Hybrid Functions for S-PSO and A-PSO

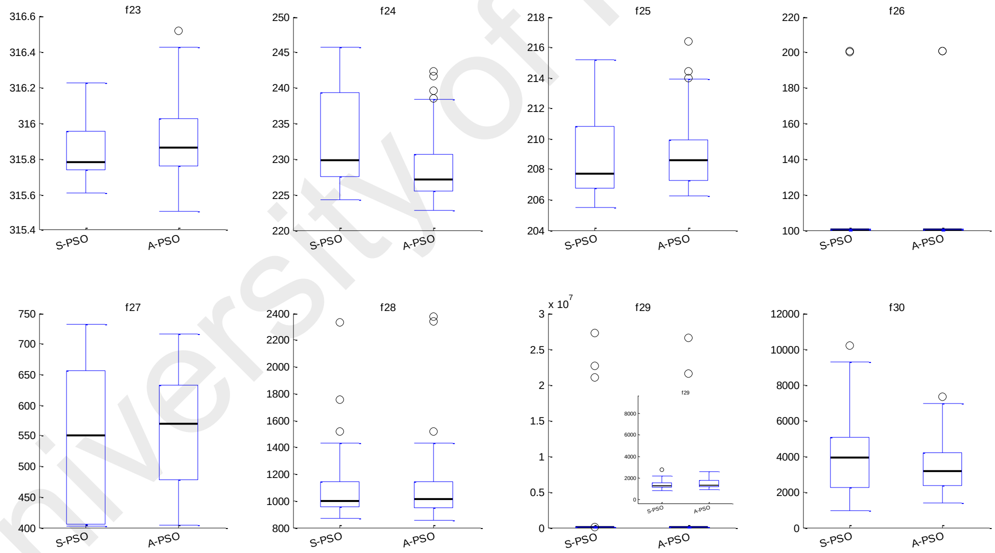

Figure 4.11: Fitness Error Distribution of Composite Functions for S-PSO and A-PSO

GSA **-** The rates of fitness error value over iteration for S-GSA and A-GSA are shown in Figure 4.12 to Figure 4.15. For all functions, the rate reduced exponentially for both S-GSA and A-GSA. But, A-GSA stopped at a higher fitness error value and sooner than

S-GSA. For function f16, f26, and f27, S-GSA performed poorly compared to A-GSA. S-GSA fails to escape from local optima trap in these functions thus its fitness error rate prematurely settled at a higher value than A-GSA's.

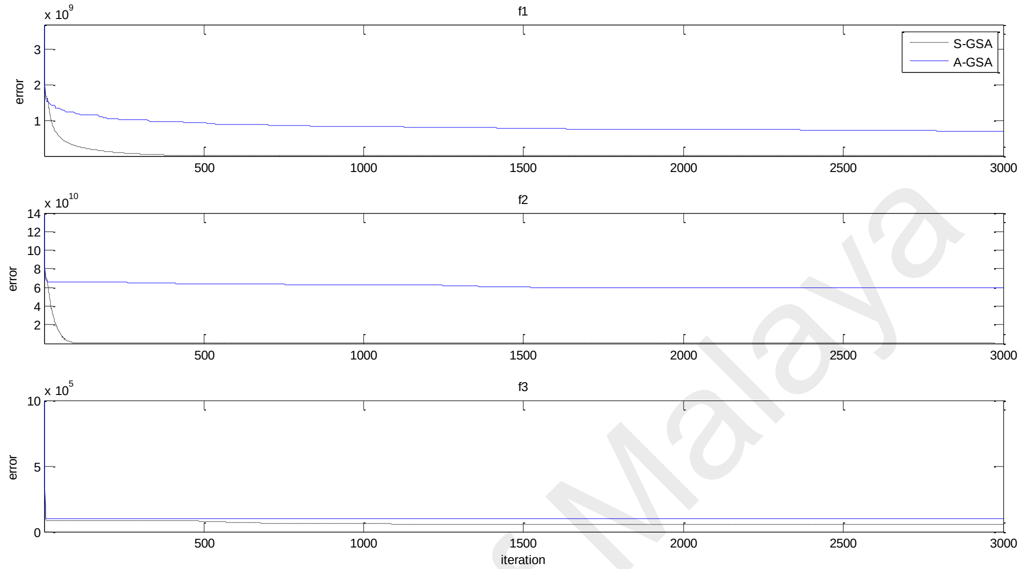

Figure 4.12: Fitness Error Rate of Unimodal Functions for S-GSA and A-GSA

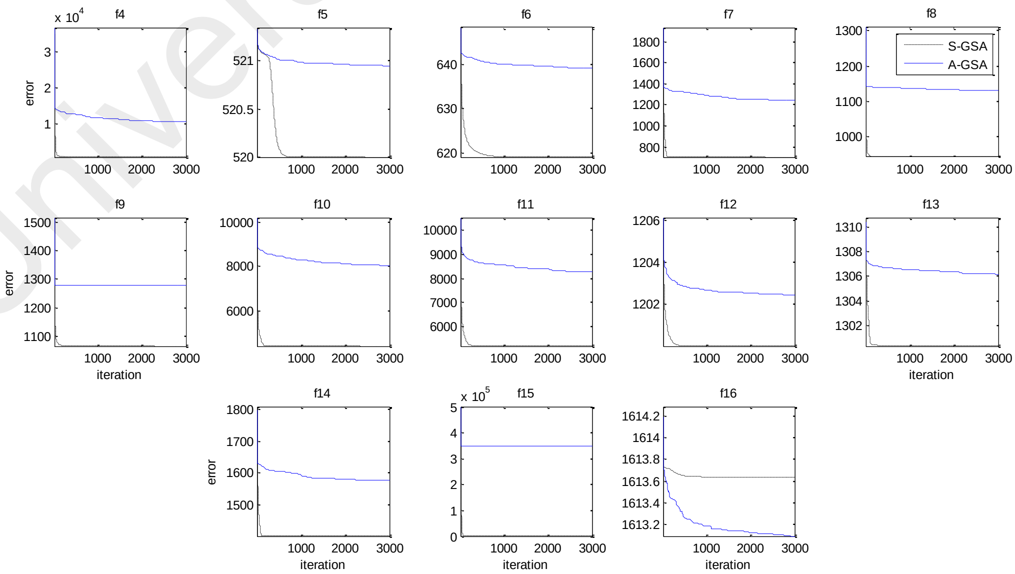

Figure 4.13: Fitness Error Rate of Simple Multimodal Functions for S-GSA and A-GSA

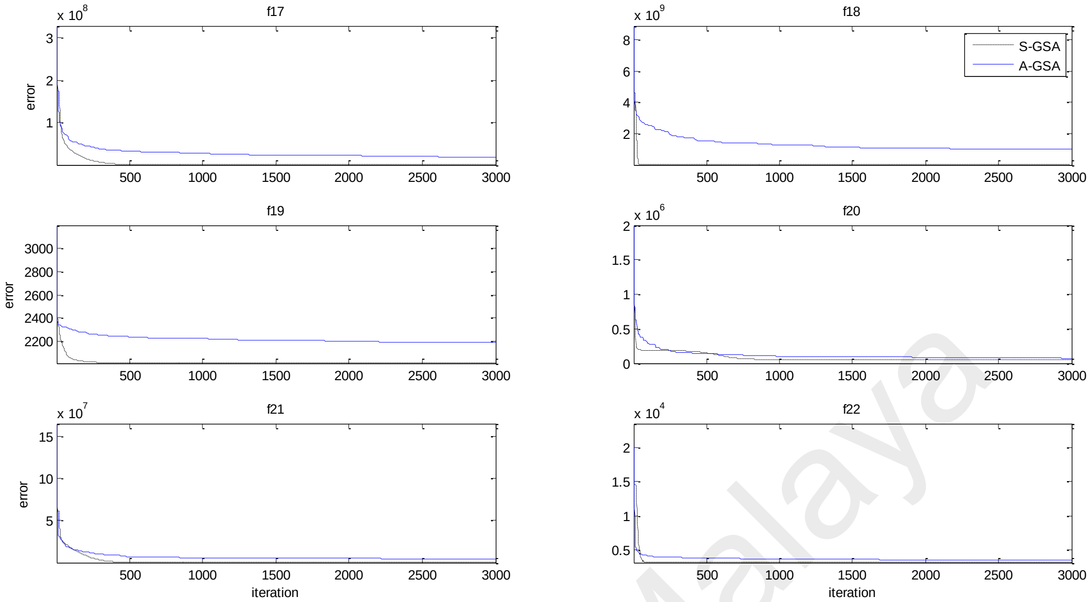

Figure 4.14: Fitness Error Rate of Hybrid Functions for S-GSA and A-GSA

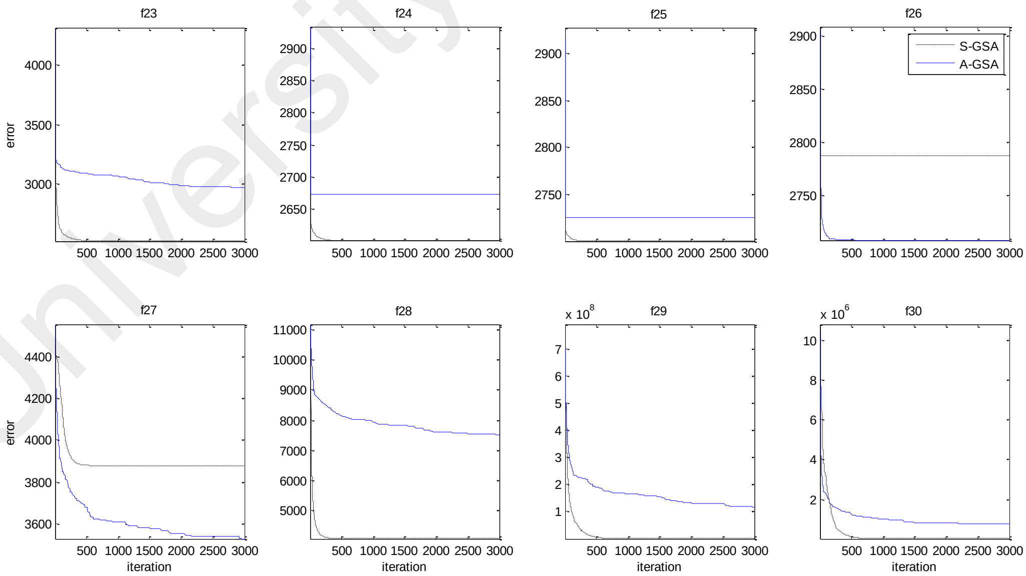

Figure 4.15: Fitness Error Rate of Composite Functions for S-GSA and A-GSA

The boxplots in Figure 4.16 to Figure 4.19 show non-normal distributions of the solutions found by both S-GSA and A-GSA in all categories of the benchmark functions. The boxplots for A-GSA are located higher and with wider spread in majority of the functions compared to S-GSA. This indicates poorer performance. However, for function f16, f26, and f27, the boxplots of S-GSA for these functions are higher and wider than A-GSA, indicating A-GSA is performing better for these functions. This is in line with the no free lunch theorem, even though S-GSA is seen to perform better in majority of the problems, but for the three problems A-GSA is able to provide good solution.

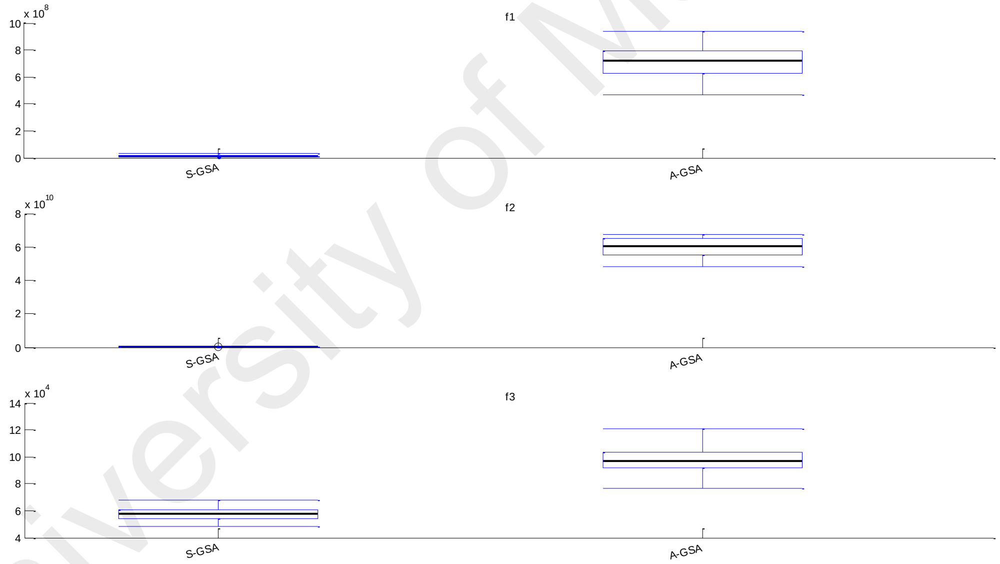

Figure 4.16: Fitness Error Distribution of Unimodal Functions for S-GSA and A-GSA

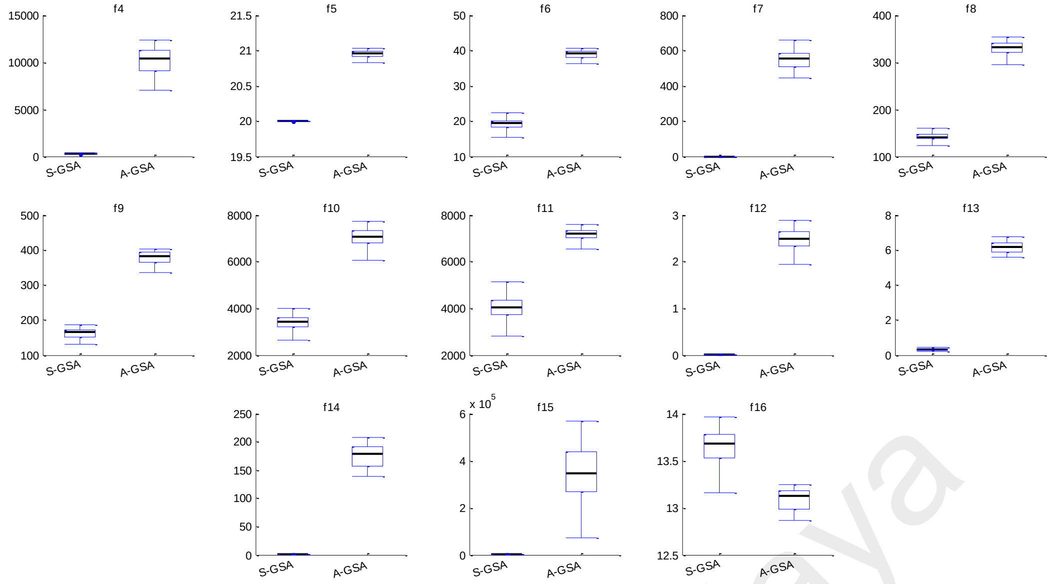

Figure 4.17: Fitness Error Distribution of Simple Multimodal Functions for S-GSA and A-GSA

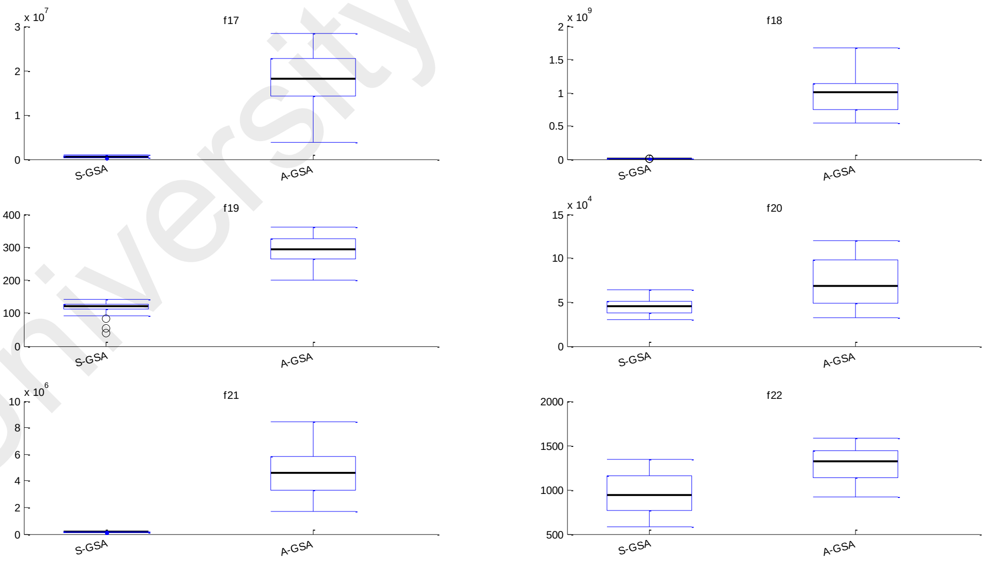

Figure 4.18: Fitness Error Distribution of Hybrid Functions for S-GSA and A-GSA

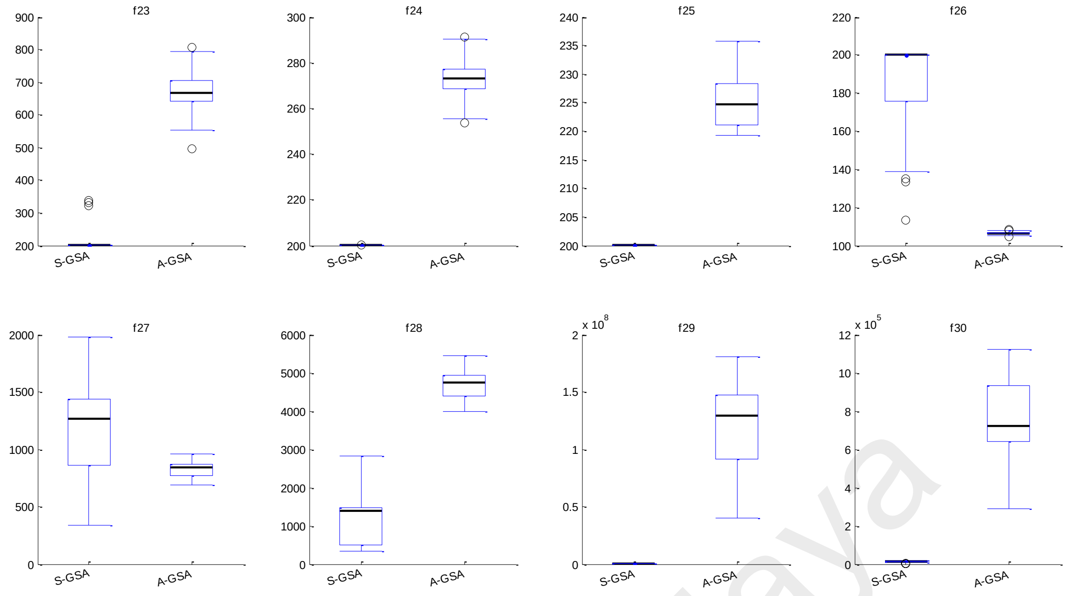

Figure 4.19: Fitness Error Distribution of Composite Functions for S-GSA and A-GSA

***SKF* -** The rate of fitness error value for S-SKF and A-SKF can be observed in Figure 4.20 to Figure 4.23. For both S-SKF and A-SKF, the fitness error rate decreased exponentially, but S-SKF's fitness error decreased more rapidly than A-SKF's. In several functions, namely f6, f9, f11, f12, f16, f25, and f28, it can be seen that S-SKF distinctly settled at a higher error value.

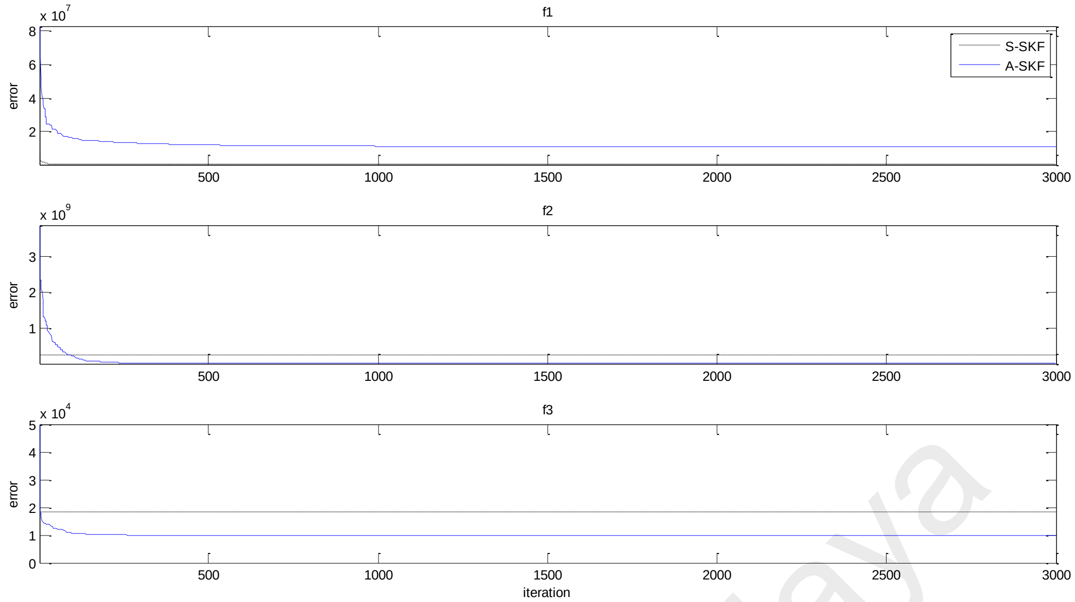

Figure 4.20: Fitness Error Rate of Unimodal Functions for S-SKF and A-SKF

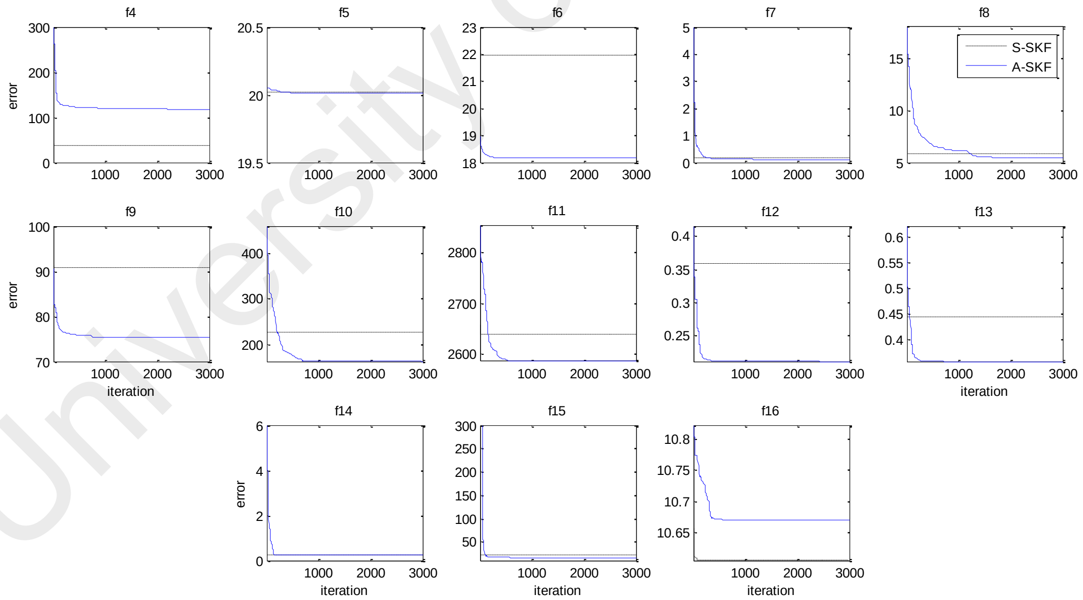

Figure 4.21: Fitness Error Rate of Simple Multimodal Functions for S-SKF and A-SKF

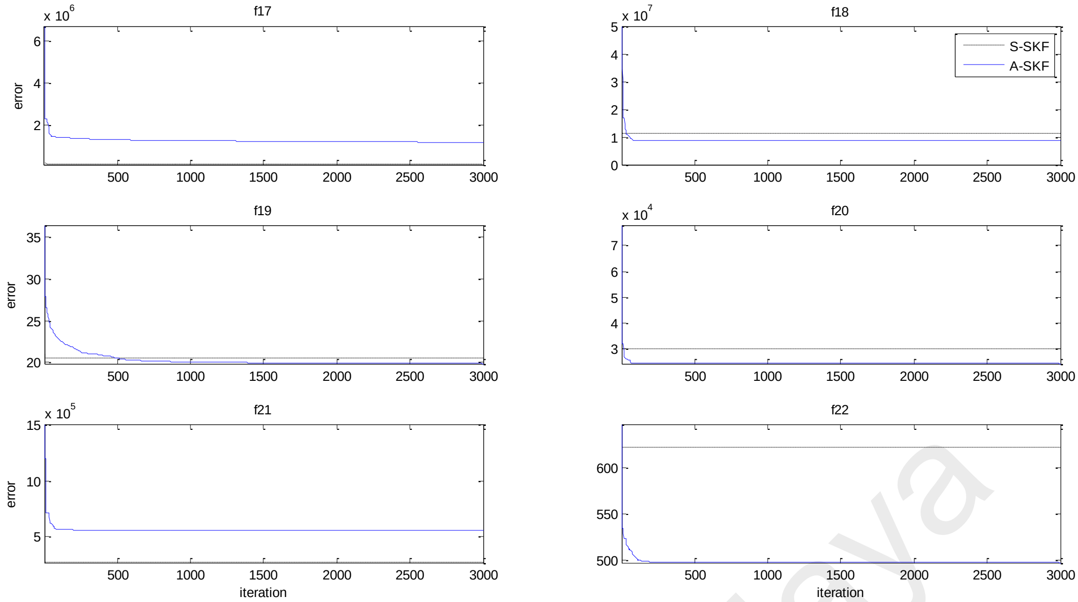

Figure 4.22: Fitness Error Rate of Hybrid Functions for S-SKF and A-SKF

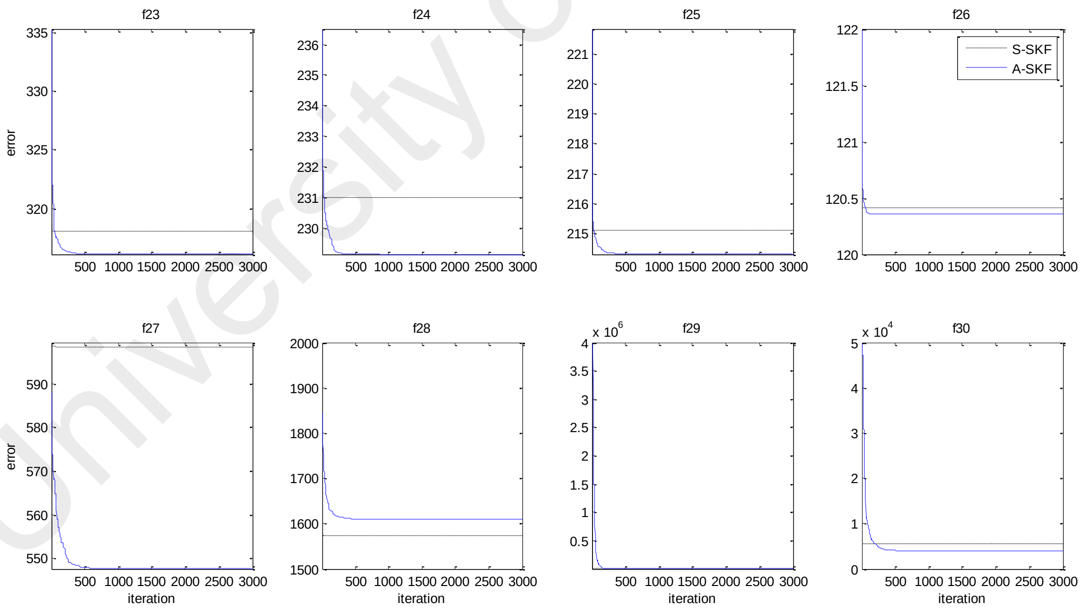

Figure 4.23: Fitness Error Rate of Composite Functions for S-SKF and A-SKF

The boxplots for S-SKF in Figure 4.24 to Figure 4.27 are at higher position than A-SKF. S-SKF's boxplots have bigger distribution than A-SKF's. These boxplots illustrate the inconsistency in the solutions' quality found by S-SKF compared to A-SKF. S-SKF also produced more outliers in unimodal, hybrid, and composite functions. There are no outliers for both S-SKF and A-SKF for the case of simple multimodal functions.

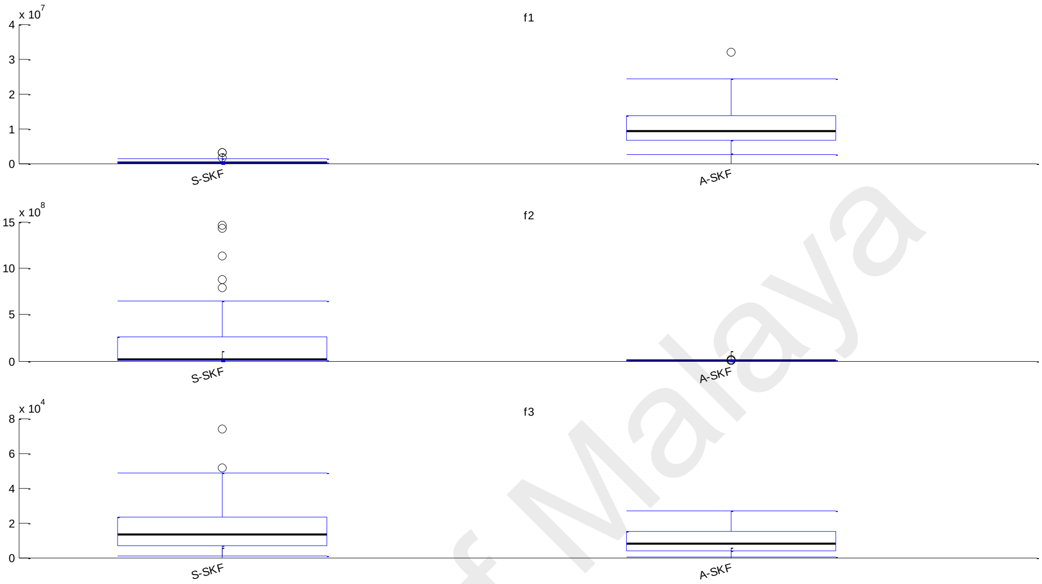

Figure 4.24: Fitness Error Distribution of Unimodal Functions for S-SKF and A-SKF

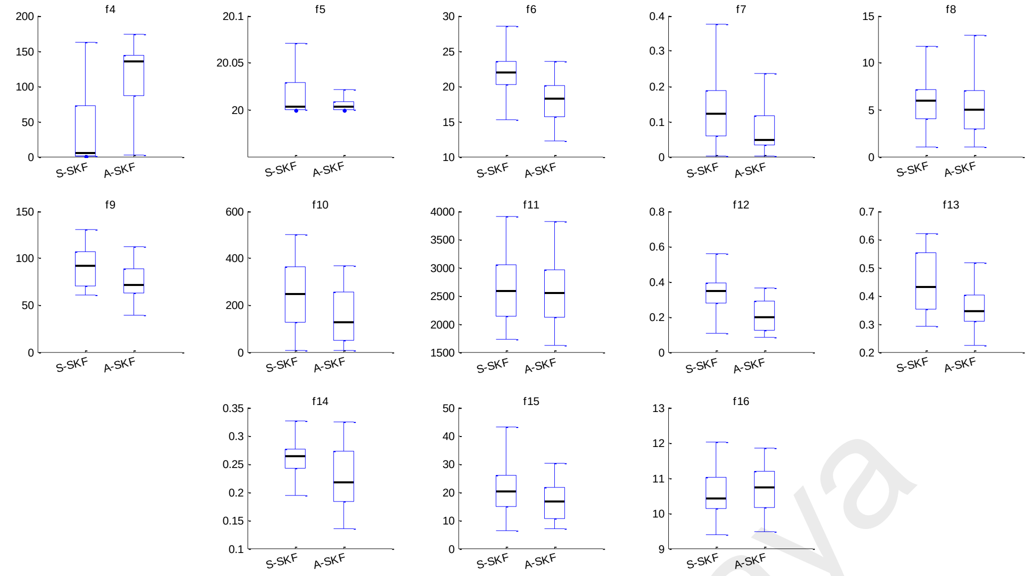

Figure 4.25: Fitness Error Distribution of Simple Multimodal Functions for S-SKF and A-SKF

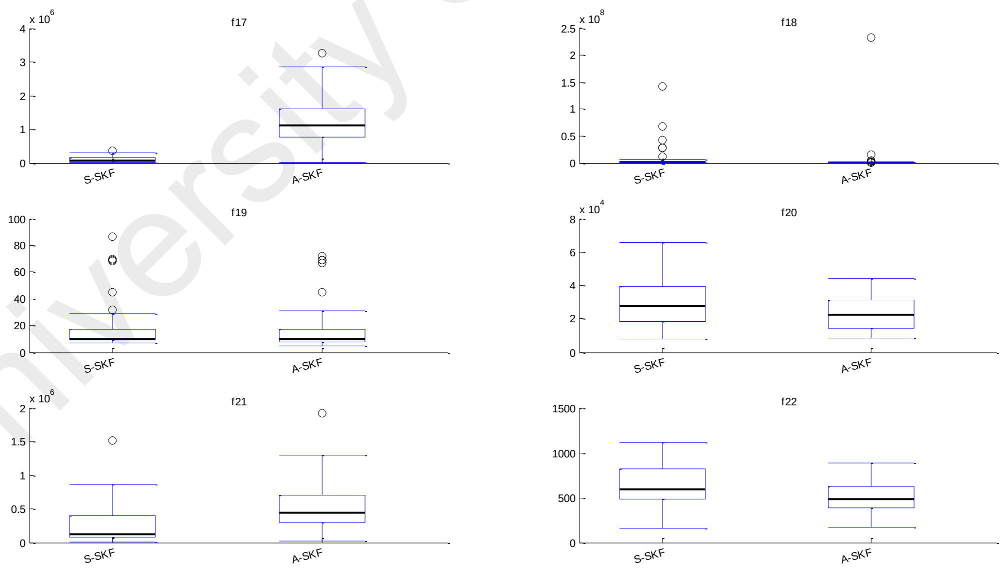

Figure 4.26: Fitness Error Distribution of Hybrid Functions for S-SKF and A-SKF

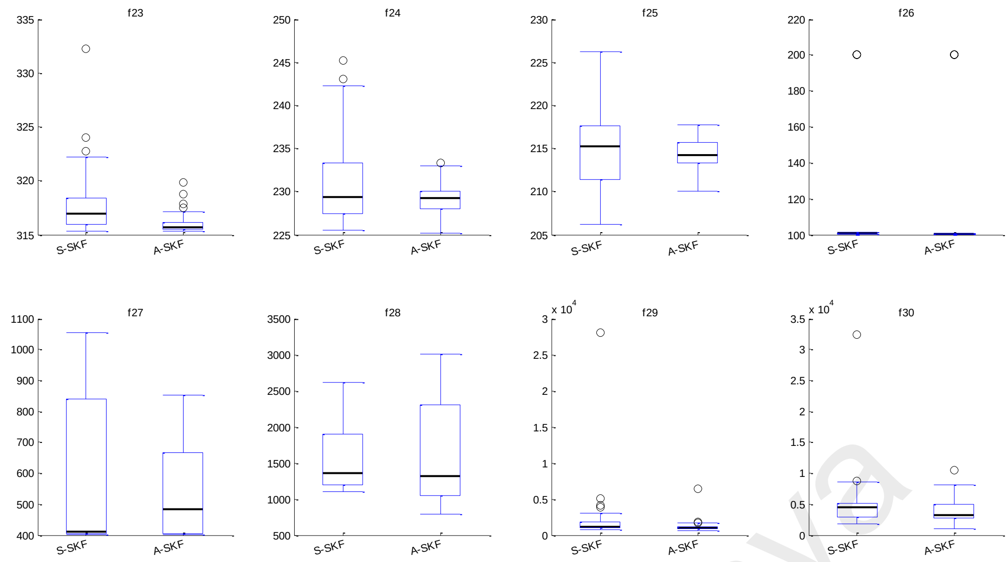

Figure 4.27: Fitness Error Distribution of Composite Functions for S-SKF and A-SKF

4.4.3 Statistical Analysis

PSO- The averaged fitness error value for the benchmark functions of S-PSO and A-PSO from the 30 runs are tabulated in Table 4.2. The best results which are the smallest value for each test function are highlighted with **boldface**. The shading is used to differentiate the different type of the benchmark functions. It can be seen that S-PSO is better than A-PSO in 13 functions, while A-PSO outperforms S-PSO in the remaining 17 functions. This is aligned with the findings of (Engelbrecht, 2013b) where the author conclude that the best iteration strategy is function dependent.

Analysis according to the type of the test functions shows that, A-PSO has better fitness error values than S-PSO in all the unimodal functions used. It is also better than S-PSO for hybrid functions with exception for f21. S-PSO has smaller fitness error values and better performance for more than half of the simple multimodal functions (7 out of 13 functions) and composite functions (5 out of 8 functions).

Table 4.2: Average Fitness Error of S-PSO and A-PSO

Function ID	Average e_{fit}		Function ID	Average e_{fit}	
	S-PSO	A-PSO		S-PSO	A-PSO
f1	6.670E+06	**5.200E+06**	f16	1.126E+01	**1.122E+01**
f2	2.879E+02	**1.389E+02**	f17	6.780E+05	**6.340E+05**
f3	3.663E+02	**2.945E+02**	f18	7.474E+03	**4.828E+03**
f4	1.746E+02	**1.608E+02**	f19	8.054E+00	**7.416E+00**
f5	**2.085E+01**	2.086E+01	f20	6.018E+02	**5.209E+02**
f6	**1.033E+01**	1.071E+01	f21	**1.360E+05**	1.660E+05
f7	1.058E-02	**9.766E-03**	f22	2.559E+02	**2.294E+02**
f8	1.917E+01	**1.857E+01**	f23	**3.158E+02**	3.159E+02
f9	**5.871E+01**	6.879E+01	f24	2.329E+02	**2.293E+02**
f10	**5.584E+02**	6.090E+02	f25	**2.087E+02**	2.091E+02
f11	**2.639E+03**	2.839E+03	f26	**1.071E+02**	1.071E+02
f12	1.893E+00	**1.658E+00**	f27	**5.512E+02**	5.556E+02
f13	**4.086E-01**	4.446E-01	f28	**1.103E+03**	1.142E+03
f14	**2.850E-01**	3.454E-01	f29	2.370E+06	**1.600E+06**
f15	7.404E+00	**7.254E+00**	f30	3.970E+03	**3.391E+03**

The average fitness error value in Table 4.2 are used for statistical analysis using the Wilcoxon signed rank test. The statistical table for Wilcoxon signed rank test is shown in Appendix B. The statistical values of the Wilcoxon signed rank test are tabulated in Table 4.3, where R+ is the sum of rank where the first algorithm out performs the second and R- is the opposite. The findings show that although A-PSO is slightly better than S-PSO, but, the statistic value of 165 is bigger than the critical value of 152, therefore, both S-PSO and A-PSO are statistically on par with each other.

Table 4.3: Wilcoxon Signed Rank Test Statistical Values for S-PSO and A-PSO

	$R+$	$R-$
S-PSO vs A-PSO	165	300

GSA- Table 4.4 listed the average fitness error value of S-GSA and A-GSA from the 30 runs of the experiment. The results show that given the CEC2014 benchmark functions, synchronous update strategy is the better iteration strategy for GSA. S-GSA has better average error for 27 functions. On the other hand, A-GSA has better performance for only three functions.

Table 4.4: Average Fitness Error Value of S-GSA and A-GSA

Function ID	Average e_{fit} S-GSA	Average e_{fit} A-GSA	Function ID	Average e_{fit} S-GSA	Average e_{fit} A-GSA
f1	**1.300E+07**	7.110E+08	f16	1.363E+01	**1.309E+01**
f2	**8.603E+03**	5.940E+10	f17	**5.310E+05**	1.840E+07
f3	**5.784E+04**	9.770E+04	f18	**3.817E+02**	9.810E+08
f4	**3.017E+02**	1.013E+04	f19	**1.153E+02**	2.924E+02
f5	**2.000E+01**	2.095E+01	f20	**4.521E+04**	7.100E+04
f6	**1.907E+01**	3.895E+01	f21	**1.550E+05**	4.760E+06
f7	**0.000E+00**	5.439E+02	f22	**9.562E+02**	1.300E+03
f8	**1.405E+02**	3.285E+02	f23	**2.130E+02**	6.697E+02
f9	**1.624E+02**	3.781E+02	f24	**2.000E+02**	2.726E+02
f10	**3.370E+03**	7.018E+03	f25	**2.000E+02**	2.249E+02
f11	**4.058E+03**	7.155E+03	f26	1.868E+02	**1.064E+02**
f12	**4.870E-04**	2.450E+00	f27	1.179E+03	**8.293E+02**
f13	**3.017E-01**	6.146E+00	f28	**1.257E+03**	4.703E+03
f14	**2.433E-01**	1.751E+02	f29	**2.001E+02**	1.170E+08
f15	**3.659E+00**	3.470E+05	f30	**1.096E+04**	7.470E+05

The average fitness error value for the 30 functions are used for Wilcoxon signed rank test. The statistical value of the Wilcoxon sign ranked test is shown in Table 4.5. The statistic value of 23 is lower than 109, this shows that significant difference exists with significance level down to 1%. Since $R+ > R-$, therefore, S-GSA is significantly better than A-GSA.

Table 4.5: Wilcoxon Signed Rank Test Statistical Values for S-GSA and A-GSA

	R+	*R*−
S-GSA vs A-GSA	442	23

SKF- The averaged fitness error value for S-SKF and A-SKF are tabulated in Table 4.6. From the tabulated values, asynchronous update is seen to be the better iteration strategy for SKF in majority of the functions. In particular, A-SKF is better than S-SKF in 2 out of the 3 unimodal functions, 12 out of 13 simple multimodal functions, 4 out of 6 hybrid functions, and 7 out of the 8 composite functions.

Table 4.6: Average Fitness Error Value of S-SKF and A-SKF

Function ID	Average e_{fit}		Function ID	Average e_{fit}	
	S-SKF	A-SKF		S-SKF	A-SKF
f1	**4.860E+05**	1.100E+07	f16	**1.060E+01**	1.067E+01
f2	2.450E+08	**1.290E+06**	f17	**1.050E+05**	1.170E+06
f3	1.841E+04	**9.901E+03**	f18	1.150E+07	**8.560E+06**
f4	**3.646E+01**	1.177E+02	f19	2.050E+01	**1.985E+01**
f5	2.002E+01	**2.001E+01**	f20	2.984E+04	**2.415E+04**
f6	2.195E+01	**1.817E+01**	f21	**2.610E+05**	5.550E+05
f7	1.635E-01	**8.444E-02**	f22	6.217E+02	**4.973E+02**
f8	5.878E+00	**5.473E+00**	f23	3.181E+02	**3.161E+02**
f9	9.087E+01	**7.526E+01**	f24	2.310E+02	**2.292E+02**
f10	2.263E+02	**1.620E+02**	f25	2.151E+02	**2.143E+02**
f11	2.640E+03	**2.585E+03**	f26	1.204E+02	**1.204E+02**
f12	3.592E-01	**2.099E-01**	f27	5.985E+02	**5.476E+02**
f13	4.443E-01	**3.567E-01**	f28	**1.574E+03**	1.610E+03
f14	2.593E-01	**2.273E-01**	f29	2.477E+03	**1.189E+03**
f15	2.192E+01	**1.640E+01**	f30	5.438E+03	**3.848E+03**

Wilcoxon sign ranked test carried using the average fitness errors in Table 4.6, shows that A-SKF is significantly better than S-SKF. The statistical value of 122 is smaller than

137, hence the level of significance is equivalent to 5%. The Wilcoxon's statistic values are listed in Table 4.7.

Table 4.7: Wilcoxon Signed Rank Test Statistical Values for S-SKF and A-SKF

	$R+$	$R-$
S-SKF vs A-SKF	<u>122</u>	343

Multiple Comparisons Among Algorithms– The performance of the six algorithms is compared using Friedman test. The algorithms ranks are tabulated in Table 4.8. A-PSO is ranked the best among the six algorithms followed by S-PSO, A-SKF, S-GSA, S-SKF and A-GSA. The Friedman's p-value is 7.59×10^{-11}, thus the null hypothesis of on par performance is rejected, significant difference exists between algorithms.

Table 4.8: Average Rankings of Friedman Test

Algorithm	**Ranking**
A-PSO	2.6833
S-PSO	2.8167
A-SKF	2.8833
S-GSA	3.3
S-SKF	3.55
A-GSA	5.7667
p-value: 7.59×10^{-11}	

Holm procedure shows that with significance level of 5%, A-GSA is worse than the other algorithms. This is aligned with the findings of Wilcoxon signed rank test, where GSA is found not to benefit from asynchronous iteration strategy. The statistical values from the Holm procedure are shown in Table 4.9.

Table 4.9: Statistics of Holm Test

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
15	A-PSO vs. A-GSA	6.486616	0	0.003333
14	A-GSA vs. A-SKF	6.00357	0	0.003571
13	S-PSO vs. A-GSA	5.934564	0	0.003846
12	S-GSA vs. A-GSA	5.037479	0	0.004167
11	A-GSA vs. S-SKF	4.692446	0.000003	0.004545
10	A-PSO vs. S-SKF	1.79417	0.072786	0.005
9	A-PSO vs. S-GSA	1.449138	0.147299	0.005556
8	S-SKF vs. A-SKF	1.311125	0.189816	0.00625
7	S-PSO vs. S-SKF	1.242118	0.214193	0.007143
6	S-GSA vs. A-SKF	0.966092	0.333998	0.008333
5	S-PSO vs. S-GSA	0.897085	0.369673	0.01
4	S-PSO vs. A-PSO	0.552052	0.580912	0.0125
3	A-PSO vs. A-SKF	0.483046	0.629063	0.016667
2	S-GSA vs. S-SKF	0.345033	0.73007	0.025
1	S-PSO vs. A-SKF	0.069007	0.944984	0.05

4.4.4 Population's Diversity

***PSO* -** The rate of S-PSO's and A-PSO's position diversity over iteration is plotted and observed in Figure 4.28 to Figure 4.31. The diversity of both S-PSO and A-PSO decreases gradually as the iteration progress.

Despite reports of A-PSO converges at a slower rate than S-PSO (Rada-Vilela et al., 2013), the results of the tests conducted show that in almost all functions from all categories, both S-PSO and A-PSO converged at similar rate. Since the particles from both variations of PSO have similar diversity behaviour, this results in performances that are on par with each other.

The strong usage of memory, $\boldsymbol{pBest}$ and $\boldsymbol{gBest}$ lessen the effect of asynchronous update in PSO. Even though, an agent is able to update its position as soon as its fitness is evaluated, its search direction is strongly influenced by $\boldsymbol{pBest}$ and $\boldsymbol{gBest}$. The agent is steered towards different direction only if $\boldsymbol{pBest}$ and $\boldsymbol{gBest}$ are changed.

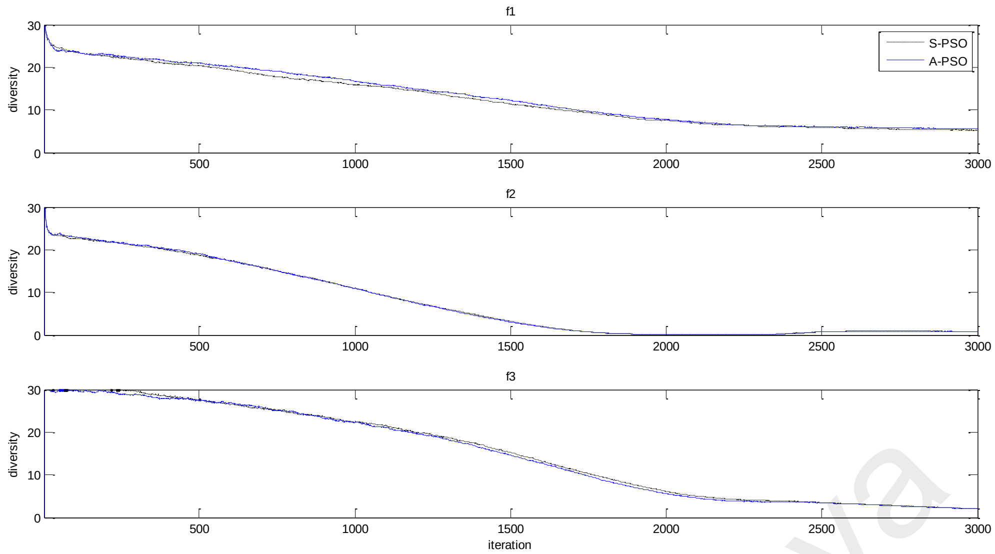

Figure 4.28: Rate of Position Diversity of Unimodal Functions for S-PSO and A-PSO

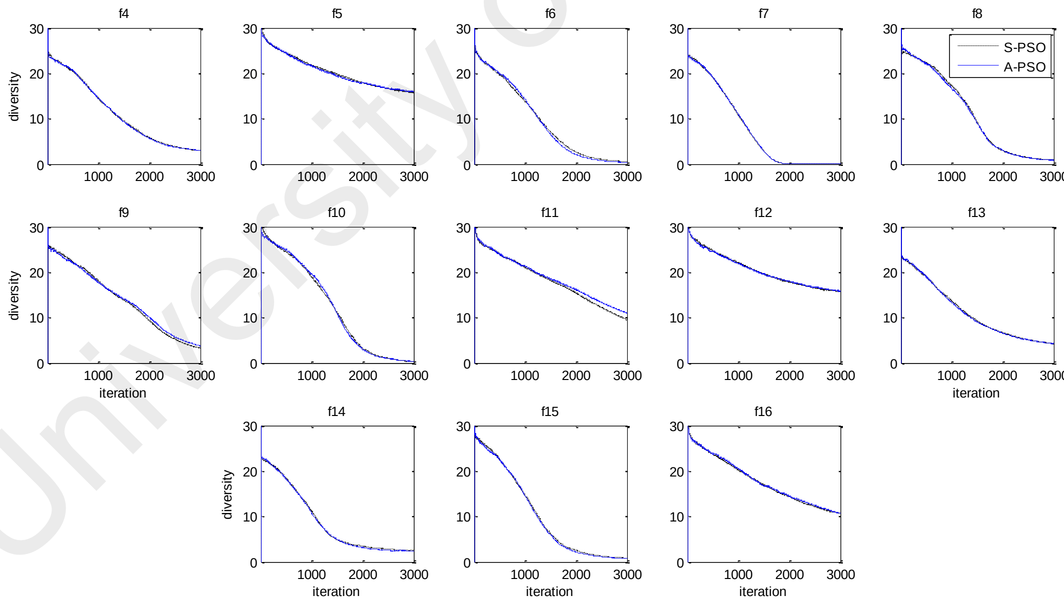

Figure 4.29: Rate of Position Diversity of Simple Multimodal Functions for S-PSO and A-PSO

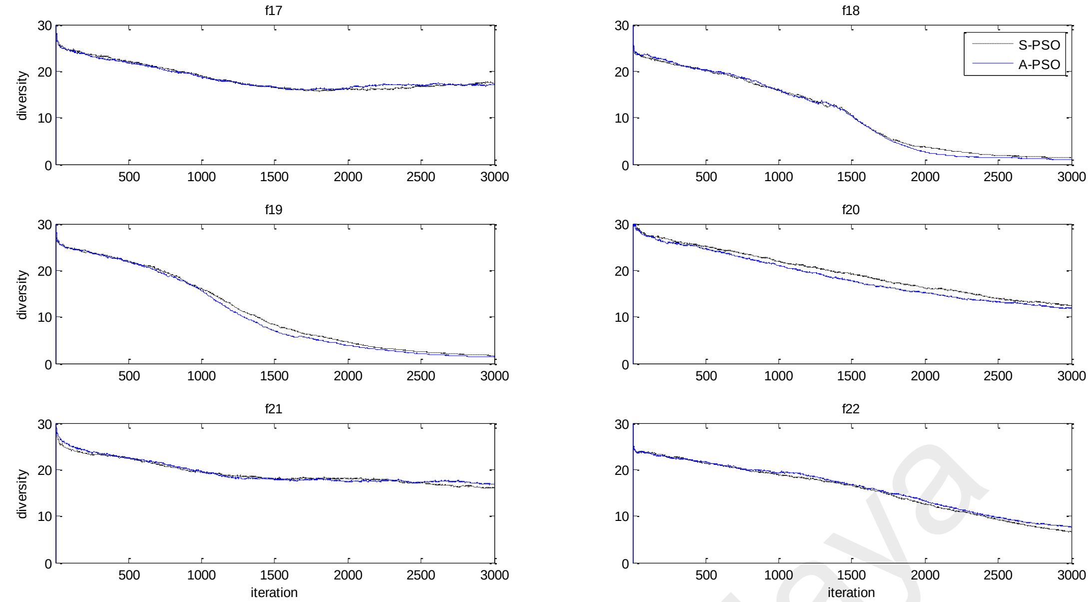

Figure 4.30: Rate of Position Diversity of Hybrid Functions for S-PSO and A-PSO

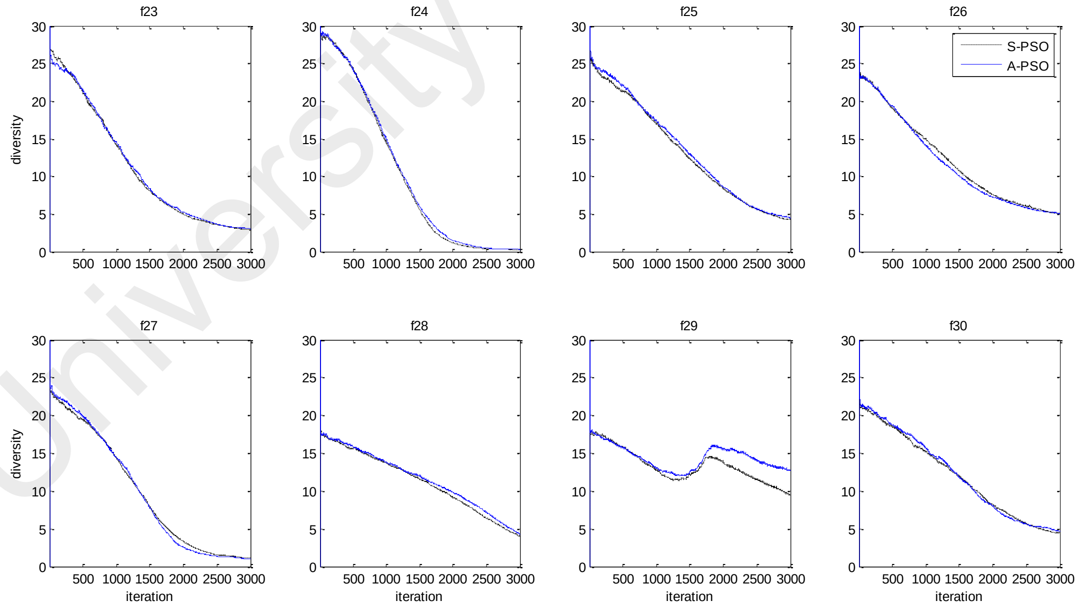

Figure 4.31: Rate of Position Diversity of Composite Functions for S-PSO and A-PSO

GSA - For the 27 test functions where S-GSA outperforms A-GSA the diversity rate of the two variations of GSA exhibits the same behaviour. The diversity of S-GSA decreases rapidly while diversity of A-GSA grows and stagnate. Due to the rapidness of the loss of diversity, the graphs of the position diversity rate shown in Figure 4.32 to Figure 4.35 are plotted in semilog for clearer observation.

It can be observed that during the first five iterations, both S-GSA's and A-GSA's diversity decreased at the same rate before the agents of A-GSA start to diversify. After the tenth iteration, the agents' diversity of A-GSA oscillated at a positive value until the final iteration. On the other hand, the diversity of S-GSA's agents continues reducing rapidly to a value close to zero.

Although diversity is desired, nonconvergence is undesired. Lack of memory usage in GSA reduce the ability of the agents of A-GSA to focus and direct their search towards a point within the search space. Thus, resulting nonconvergence. Nonconvergence causes the agents of A-GSA to overlook area with good performance.

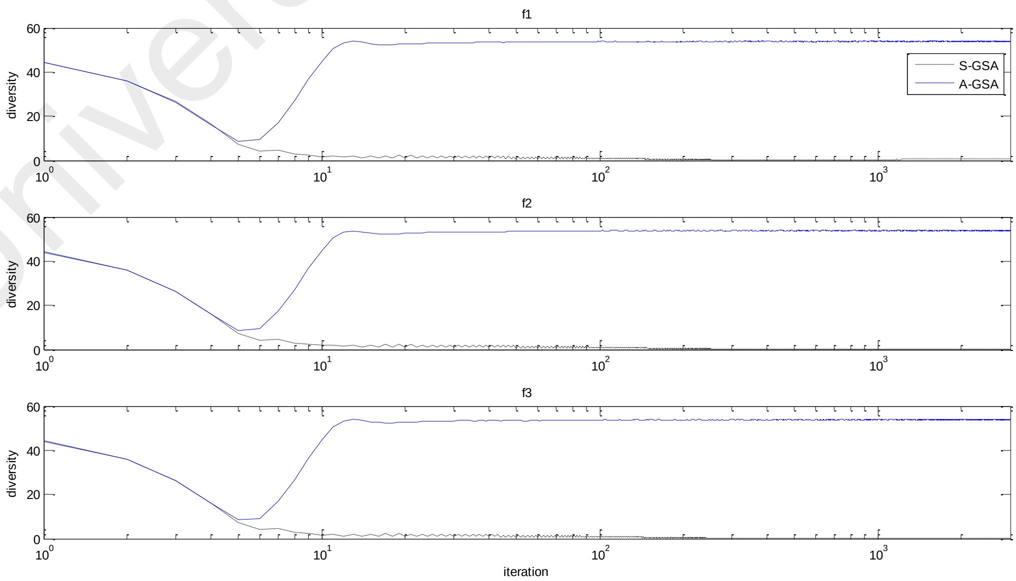

Figure 4.32: Rate of Position Diversity of Unimodal Functions for S-GSA and A-GSA

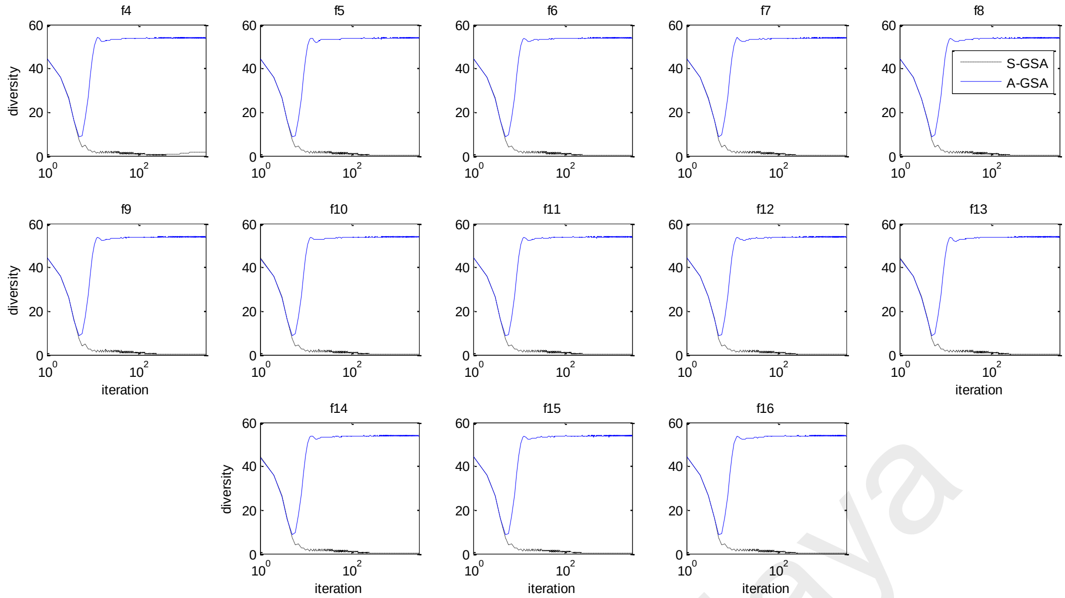

Figure 4.33: Rate of Position Diversity of Simple Multimodal Functions for S-GSA and A-GSA

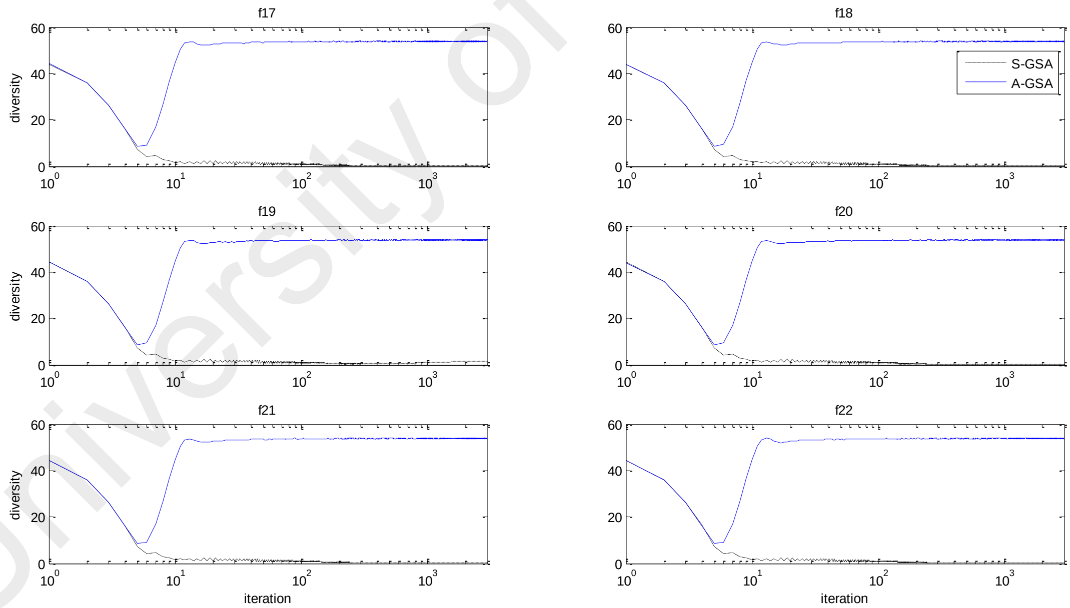

Figure 4.34: Rate of Position Diversity of Hybrid Functions for S-GSA and A-GSA

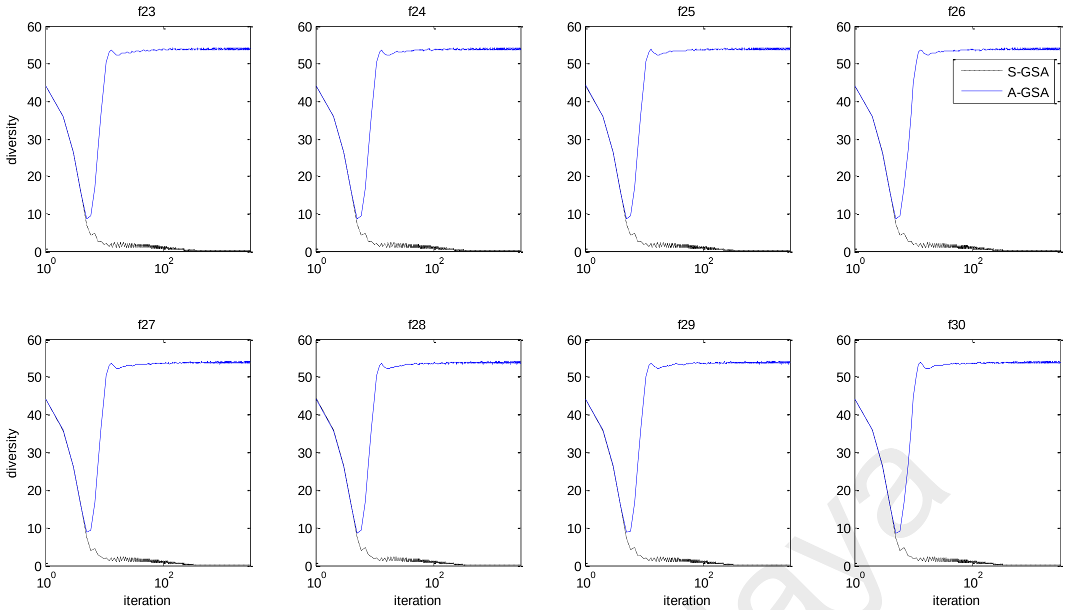

Figure 4.35: Rate of Position Diversity of Composite Functions for S-GSA and A-GSA

SKF **–** SKF's population's diversity is small compared to PSO and GSA. The position diversity of both S-SKF and A-SKF reduced with the iteration. However, unlike S-PSO and A-PSO where the diversity decreases gradually, the decrement rate of S-SKF and A-SKF is exponential. Thus, similar to GSA, the position diversity rate of S-SKF and A-SKF are plotted using semilog.

The graphs of diversity rate of S-SKF and A-SKF are shown in Figure 4.36 to Figure 4.39. A-SKF's diversity rate is observed to decrease at slower rate than S-SKF. Distinct difference between A-SKF's diversity and S-SKF's diversity can be seen especially for hybrid and composite functions. The diversity of A-SKF does not decrease as smoothly as S-SKF. Memory is used to direct the search by the agents in SKF but the effect is not as strong as PSO. Thus, the influence of asynchronous update towards the agents of SKF is stronger. This contributes to disturbance towards the diversity of the agents and the better performance by A-SKF.

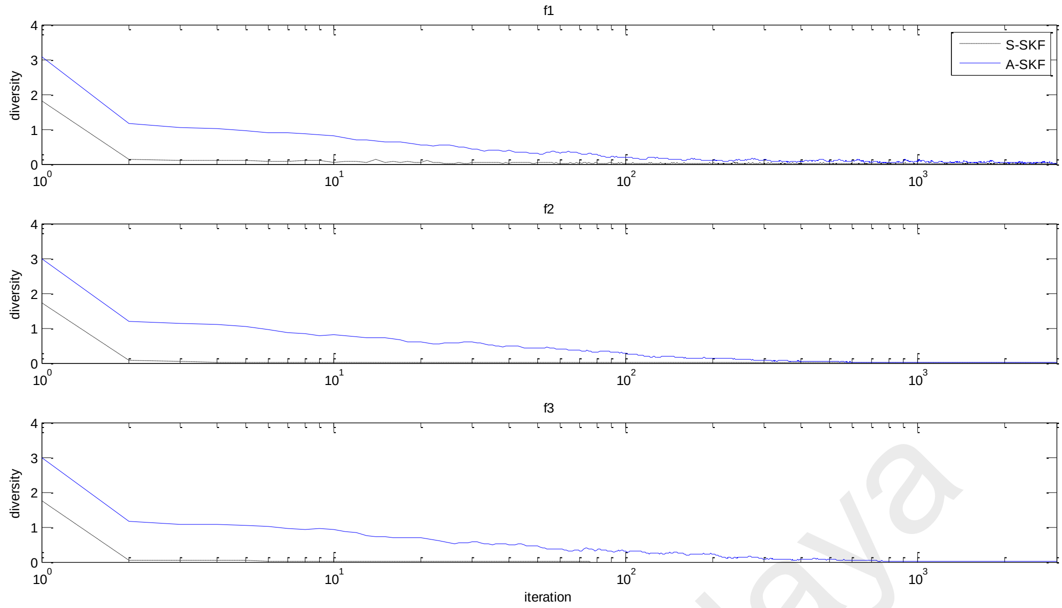

Figure 4.36: Rate of Position Diversity of Unimodal Functions for S-SKF and A-SKF

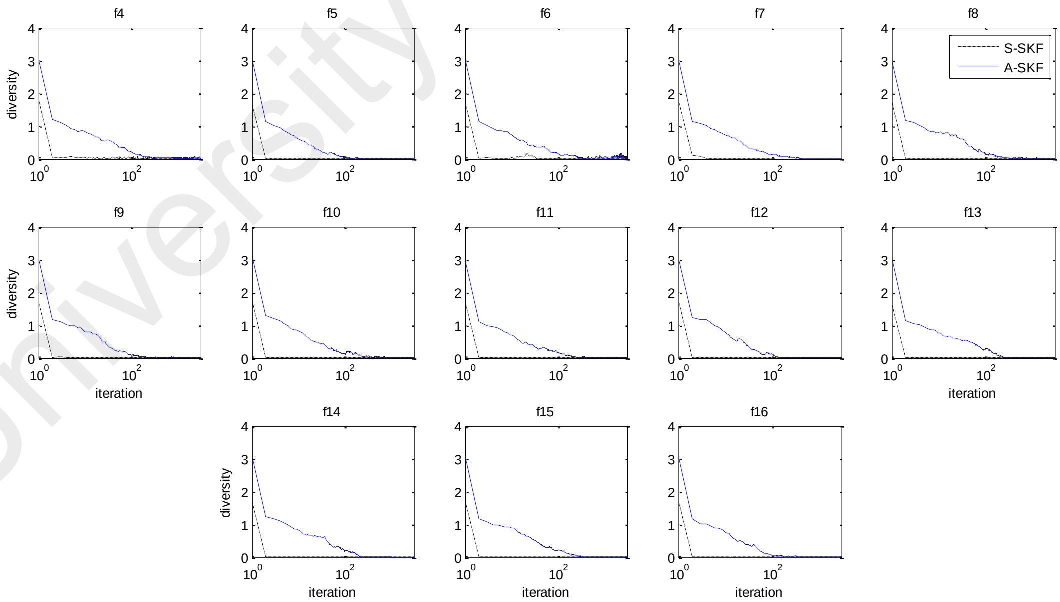

Figure 4.37: Rate of Position Diversity of Simple Multimodal Functions for S-SKF and A-SKF

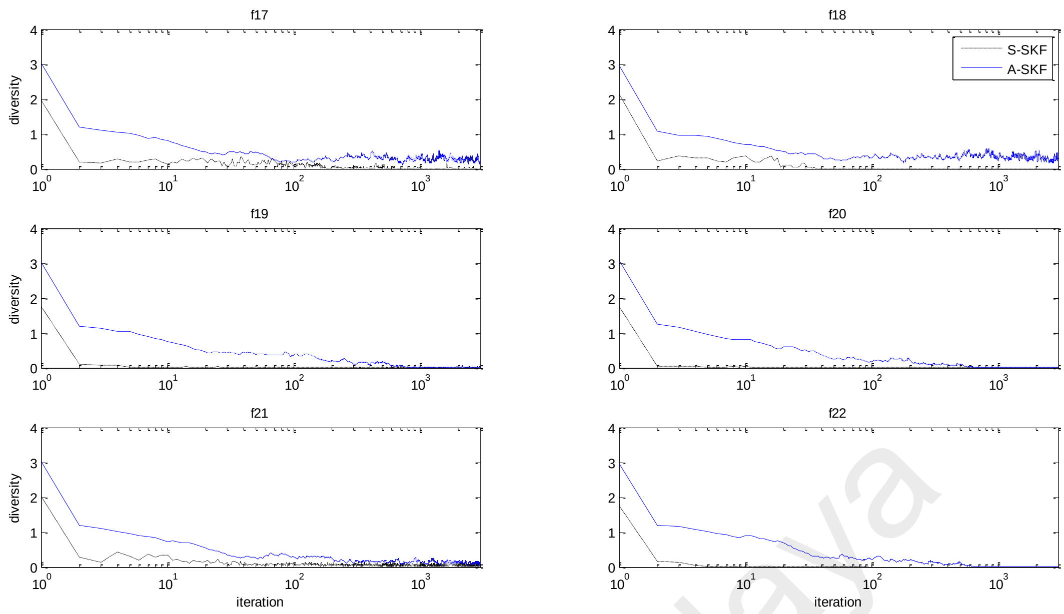

Figure 4.38: Rate of Position Diversity of Hybrid Functions for S-SKF and A-SKF

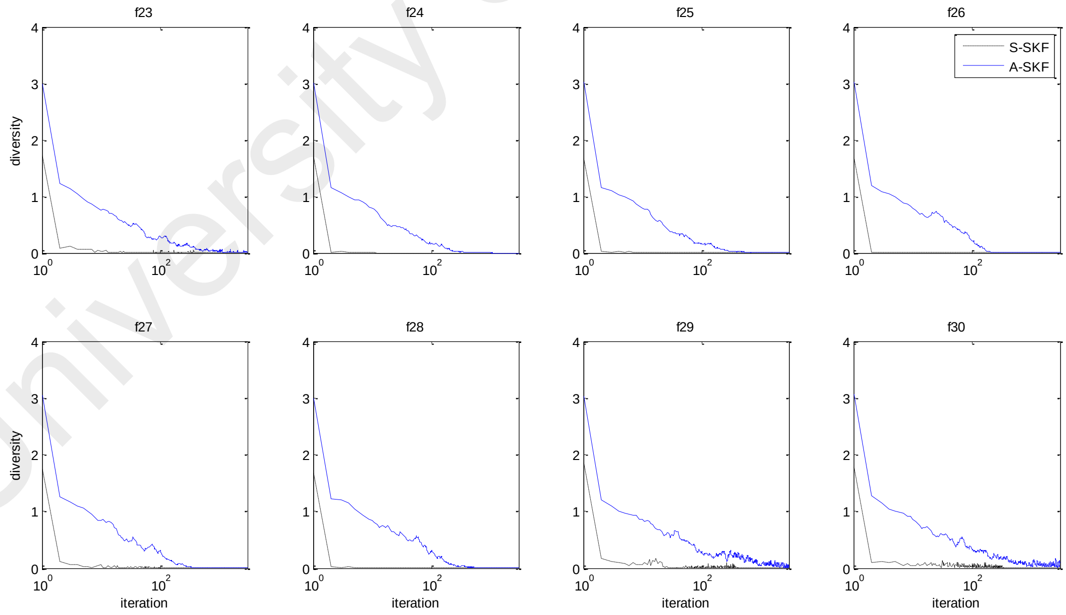

Figure 4.39: Rate of Position Diversity of Composite Functions for S-SKF and A-SKF

4.5 Conclusion

From the tests conducted on the three parent algorithms implemented using the traditional iteration strategies, it is seen that iteration strategy is able to influence performance of population-based algorithms. However, the best iteration strategy for every population-based metaheuristic can't be identified. It is an algorithm and also problem dependent parameter.

Synchronous update is found to be better for GSA while asynchronous is better for SKF. Meanwhile it is found that iteration strategy is a problem dependent parameter for PSO, where S-PSO performs better in some functions while A-PSO has a better performance for the other functions. Even though, S-PSO has more number of success in simple multimodal and composite functions, there are several problems where A-PSO is better at. The same is observed for other type of functions. However, the difference between the performance of the two iteration strategies is small. Asynchronistic has poor result in GSA. This might be contributed due to the lack of memory in GSA. The memoryless population causes frequent change of $best(t)$ and $worst(t)$ in A-GSA which consequently lead to nonconvergence by A-GSA.

The response of the population's diversity towards iteration strategy varies from one algorithm to another. In GSA and SKF, the difference of the two iteration strategies is significant. The asynchronous update is seen to be able to preserve diversity longer than the synchronous update. In A-GSA, asynchronous update prevents the agents from converging for the entire search process, while the effect of asynchronous update in A-SKF is not as extreme. The asynchronous update in A-SKF the population diversity is preserved longer but as the search progresses the population slowly converges. This contributes to better performance of A-SKF. In PSO the effect of asynchronous update towards population's diversity is not obvious.

Usage of memory by a population-based algorithm also influence the population's response towards the iteration strategy. As observed in PSO stronger usage of memory provides more stable performance across different strategy. The asynchronous iteration strategy causes prolonged divergence in the memoryless GSA which affected its performance badly.

CHAPTER 5: RANDOM SWITCHING ITERATION STRATEGY

5.1 Introduction

In this chapter, a new class of iteration strategies is proposed. The strategies within this class are hybrid of the two traditional strategies, where the algorithms that implement this new iteration strategy switch between the synchronous and asynchronous iteration strategies. The number of fitness evaluation of the strategies from the new class is equal to a purely synchronous update algorithm or purely asynchronous update algorithm. The switching does not introduce significant increment of the computational cost. The switching iteration strategy is implemented by the three parent algorithms and the findings are discussed. Before that, a brief review on switching in optimization is presented.

5.2 Literature Review

Switching had been used in many works on optimization algorithms. For example. in (Dulikravich, Martin, Colaco, & Inclan, 2013), various works that focus on achieving optimum solution by switching between optimization algorithms are reviewed. As per the no free lunch theorem, the ultimate optimization algorithm that performs better than any other algorithms for all optimization problems does not exist. Inspired by this, the works reviewed in the work switch between optimization algorithms depending on the progress of the iterative process.

An objective switching genetic algorithm for design optimization (OSGADO) is proposed for multi objective optimization (Chafekar, Xuan, & Rasheed, 2003). In OSGADO a single population is used to optimize problem of multiple objectives sequentially. The population optimize an objective for a certain number of evaluation before switching to another objective, once every objective had been addressed the population switch back to the first objective.

A PSO that balances global and local search by switching from one mode of velocity update to another mode according to the swarm's evolutionary factor is introduced in (Tang, Wang, & Fang, 2011). The algorithm is used for quantification analysis of lateral flow immunoassay test strip for medical diagnostic (Zeng, Hung, Li, & Du, 2014; Zeng, Wang, Li, Du, & Liu, 2012; Zeng, Wang, Zhang, & Alsaadi, 2016), AC servo system disturbance control (Hou, Hou, Wang, Gao, & Sun, 2016), and bankruptcy prediction (Lu, Zeng, Liu, & Yi, 2015; Lu, Zhu, Zhang, & Shao, 2014).

The attractive repulsive PSO (Riget & Vesterstrøm, 2002) also uses switching concept. The swarm switches between attraction and repulsion in order to escape from premature convergence in multimodal optimization problem. The switches are conducted according to the diversity of the swarm.

In (Balsa-canto, Peifer, Banga, Timmer, & Fleck, 2008), parameter optimization for biological systems is optimized by switching between global search and local search method using a unique strategy that determines the most appropriate switching point. The stochastic ranking evolutionary search and differential evolution are used for global optimization while multiple shooting algorithm is used for local optimization. The proposed method is able to efficiently tackle the multimodality of biological system parameter optimization problem.

From the works reviewed above, it can be seen that switching allows two or more good optimization strategies or methods to be combined so that a better optimizer is achieved. It is a simple idea, but able to provide better solution, balances between local and global search and optimizes multimodal and multi objective problems more efficiently. This motivates the work in this chapter.

5.3 Random Switching Iteration Strategy

Random switching iteration strategy randomly alternates the iteration strategy of a population-based metaheuristics algorithm between the synchronous update and asynchronous update throughout the search. Specifically, the population switches its iteration strategy after Δ number of fitness evaluation. The value of Δ is randomly chosen every time a switching occurs. The range of Δ is drawn from uniform random distribution between zero to the maximum number of fitness evaluation. No information of the population's condition is used in selecting the value of Δ. No maximum number of switching is set. This provides a simple switching strategy.

The random switching iteration strategy can be defined as in Definition 5.1.

Definition 5.1: (Random switching iteration strategy)

If $\delta > \Delta$ *then*

If asynchronous update, then

Switch to synchronous update

$\Delta \sim U([0, FES])$

Else

Switch to asynchronous update

$\Delta \sim U([0, FES])$

The general flowchart of the random switching algorithm is shown in Figure 5.1.

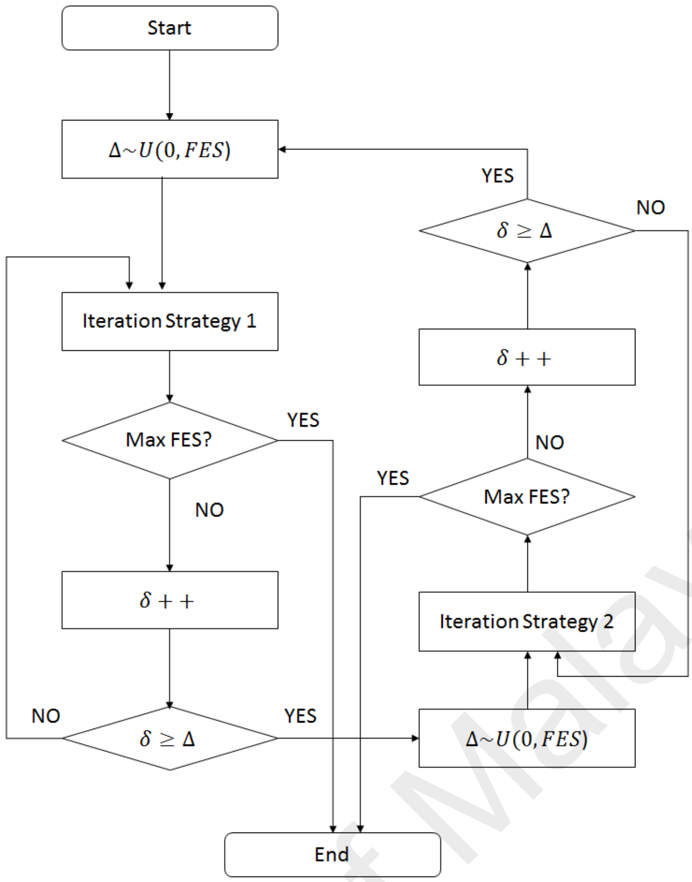

Figure 5.1: General Flowchart of Random Switching

5.3.1 The Proposed Randomly Switching PSO

This section discusses PSO with randomly switching iteration strategy. The PSO used is based on inertia weight PSO with global neighborhood. Therefore, the velocity update equation used is similar to equation 2.8, while the position update equation is the same as equation 2.7. There are two variants of the randomly switching PSO, RSw-PSO$_a$ and RSw-PSO$_s$. The difference between the two variants is the starting iteration strategy. In RSw-PSO$_a$, the swarm initially adopts the asynchronous update, while in RSw-PSO$_s$, the swarm starts with synchronous update. The flowcharts of the PSO with random switching iteration strategy are shown in Figure 5.2 and Figure 5.3.

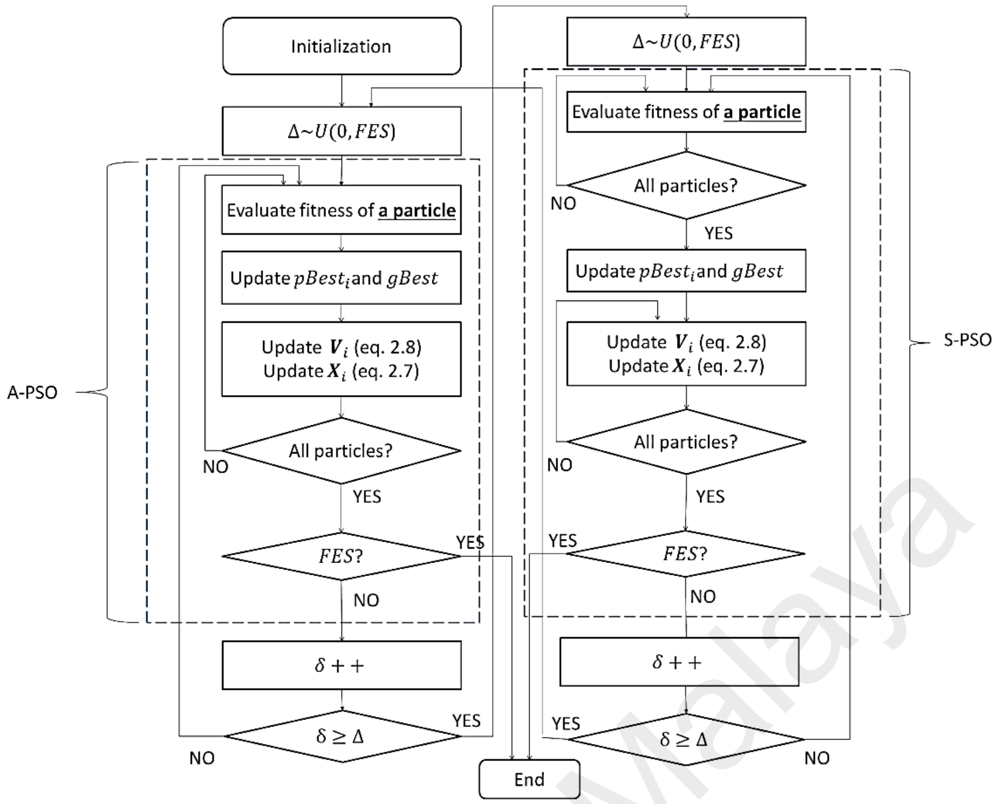

Figure 5.2: Flowchart of RSw-PSO$_a$

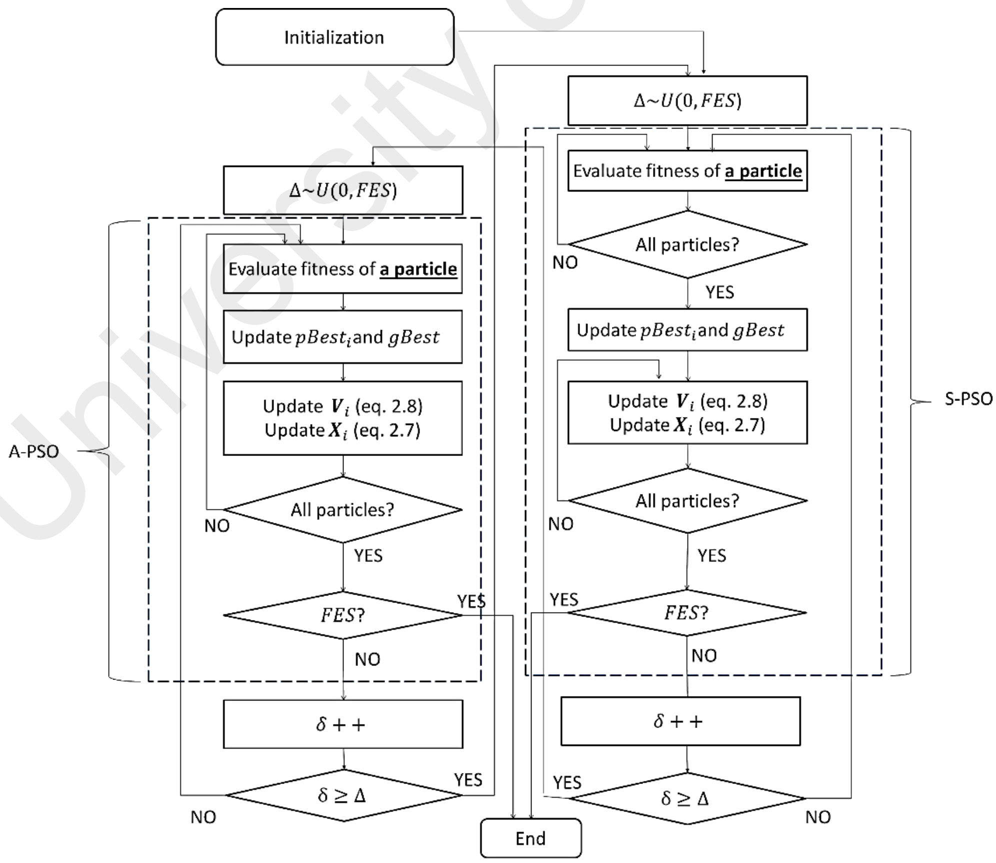

Figure 5.3: Flowchart of RSw-PSO$_s$

5.3.1.1 The Initialization

The algorithm starts with initialization of the particles. Similar to both S-PSO and A-PSO, the swarm's positions and velocities are randomly initialized according to the search space of the problem faced. The initial iteration strategy is either one of the two traditional strategies.

5.3.1.2 The Switching

The population switches between the two iteration strategies based on the switching counter, δ. The switching counter counts the number of fitness evaluation conducted while the switching condition remains unchanged.

During execution of synchronous update, the fitness of the whole population is measured before the best values are selected. After that, the swarm's velocities and positions are updated. In the asynchronous update, the particles go through the steps one by one according to their particle number. Hence, in an iteration, particle number 1 leads the optimization process. It starts with fitness evaluation. If the newly evaluated fitness is found to be better than its own $\boldsymbol{pBest}$ and the population's $\boldsymbol{gBest}$, then the two values are updated. Next, the particle's new velocity and position are computed. After the optimization tasks of particle 1 are completed, the next particle begins its evaluation and update processes.

Before the population moves to next iteration, its switching counter, δ, is incremented and if $\delta \geq \Delta$ then the population switches its iteration strategy. During the switch, the swarm's positions, velocities, and information of the $\boldsymbol{pBest}$ and $\boldsymbol{gBest}$ are preserved, the δ is reset and new Δ is randomly set.

5.3.1.3 The Stopping Condition

The algorithms stop when the stopping condition is met. The stopping condition is evaluated after the velocity and position update phase, before δ is incremented and the switching condition is checked.

Here, the maximum number of fitness evaluation is adopted as the stopping condition. If maximum number of fitness evaluation has been achieved, then the algorithm is stopped and the best-found solution is reported as the optimal solution. The algorithm stops regardless of the iteration strategy it is executing.

5.3.2 The Proposed Randomly Switching GSA

Application of randomly switching iteration strategy on GSA is proposed in this section. Like PSO, two variants of randomly switching GSA are available, RSw-GSA_a and RSw-GSA_s. RSw-GSA_a starts with asynchronous update, while RSw-GSA_s starts with synchronous iteration strategy. The algorithms are based on the original GSA with embedded $Kbest$ elitism. The update equations are similar to the equations in section 2.3.2.1. The flowcharts of the random switching GSAs are presented in Figure 5.4 and Figure 5.5.

5.3.2.1 The Initialization

The algorithms start with random initialization of the agents' positions and velocities. These values are determined according to the size of the search space.

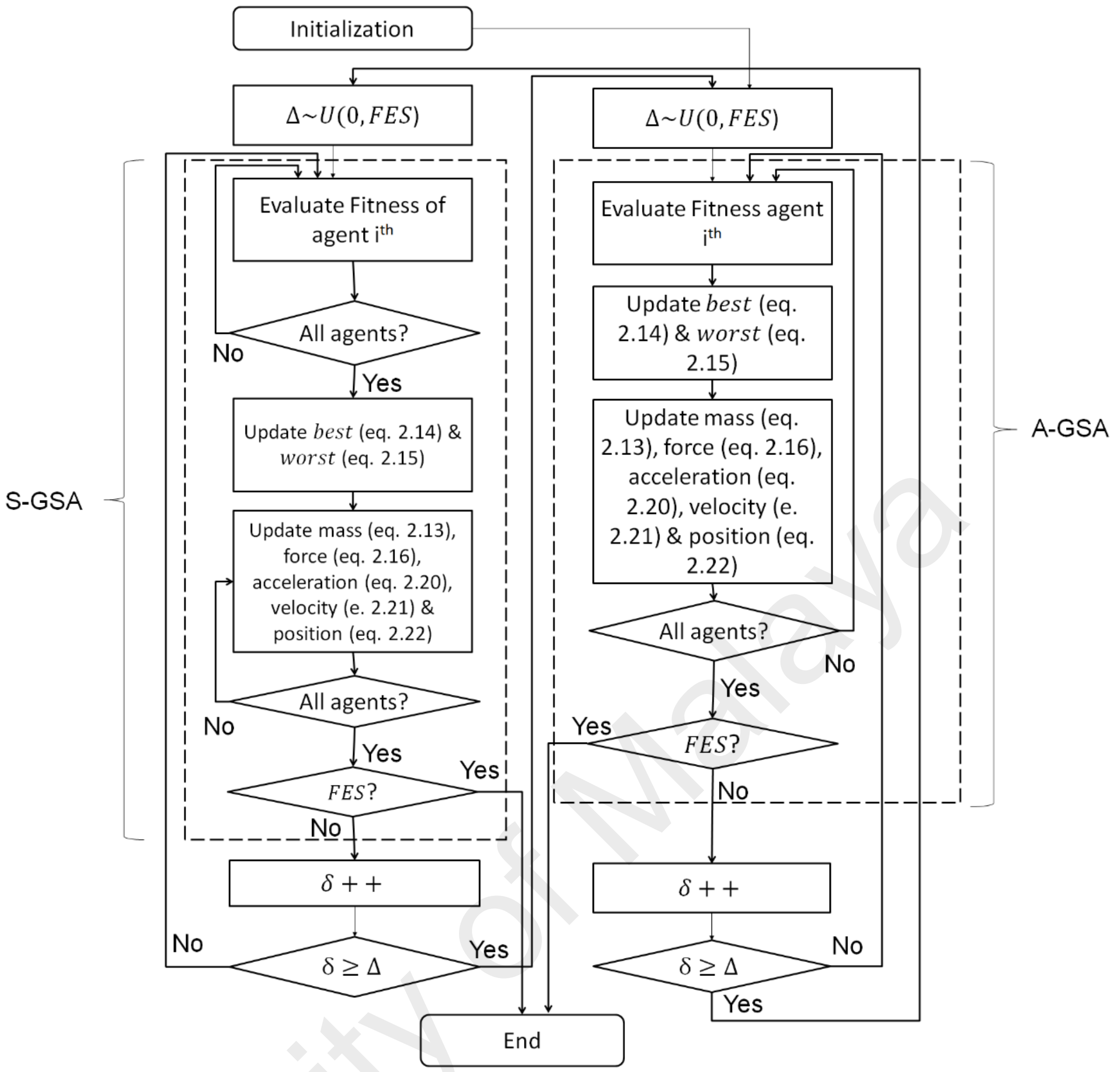

Figure 5.4: Flowchart of RSw-GSA$_a$

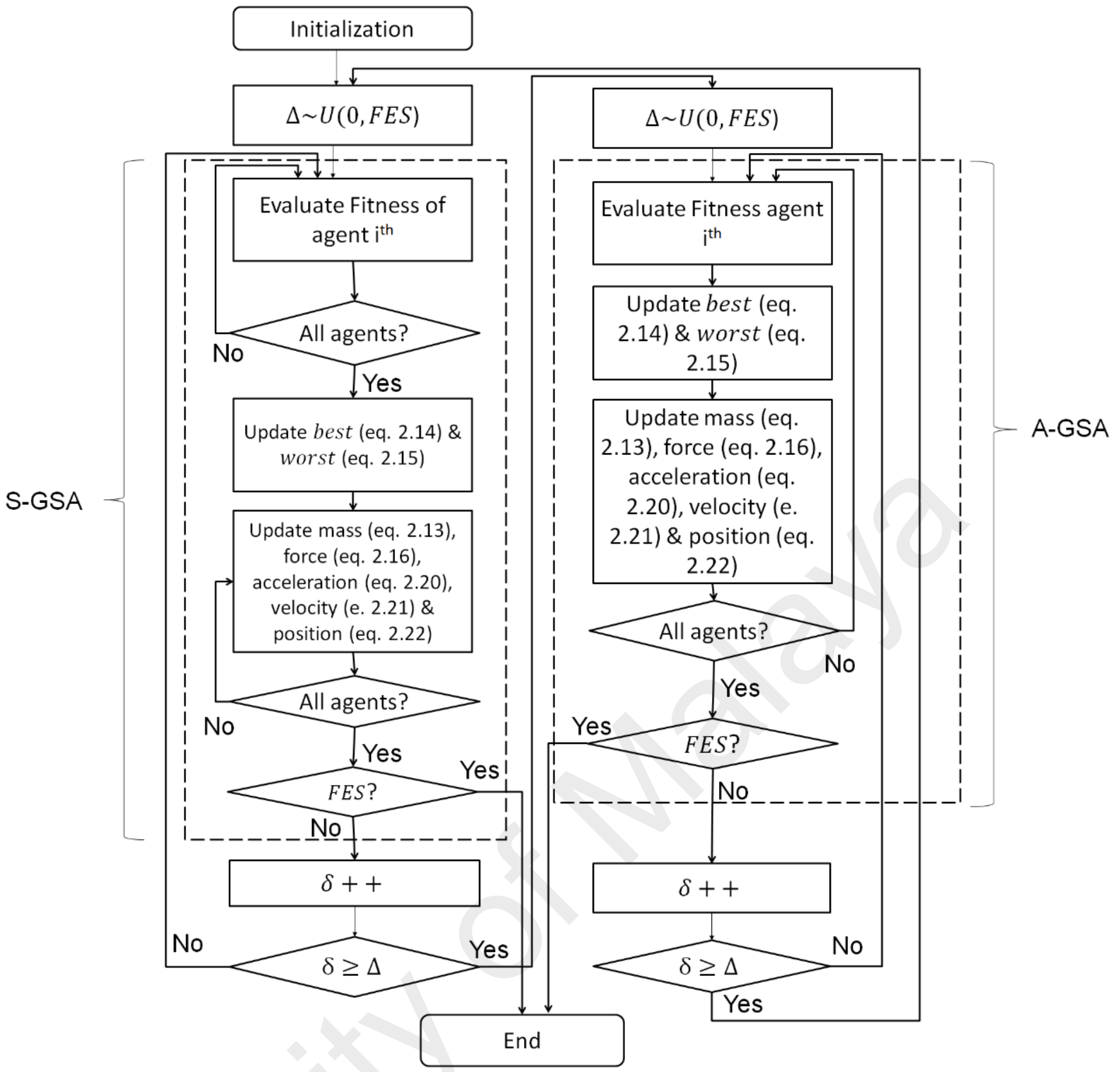

Figure 5.5: Flowchart of RSw-GSA$_S$

5.3.2.2 The Switching

Every time the population switches its iteration strategy, Δ is randomly chosen. The random value ranges from zero to the maximum number of fitness evaluation, FES. The switching counter, δ, is incremented when iteration is increased and δ's value is reset when the iteration strategy is switched. The strategy is switched when $\delta \geq \Delta$.

The switching GSA can start either with asynchronous update or with synchronous update. During execution of asynchronous iteration strategy, the population works similar to A-GSA. On the other hand, during the execution of synchronous update, the population works like S-GSA. The switching GSA preserves the population's positions and velocities as the switching happens.

5.3.2.3 The Stopping Condition

After the positions are updated, the stopping condition is checked. If maximum number of fitness evaluation had been reached, then the algorithm is stopped. Otherwise, the algorithm proceeds to compare δ with the threshold value, Δ.

5.3.3 The Proposed Randomly Switching SKF

This section proposed the usage of randomly switching iteration strategy on SKF. Randomly switching SKF that starts with asynchronous update is noted as, RSw-SKF$_{\mathrm{a}}$, while RSw-SKF$_{s}$, represents randomly switching SKF that starts with synchronous update. The proposed algorithms used the same update equations and parameter setting as the original SKF described in chapter 2. The flowchart of RSw-SKF$_{\mathrm{a}}$ is shown in Figure 5.6, while the flowchart in Figure 5.7 presents the RSw-SKF$_{s}$.

5.3.3.1 The Initialization

In the initialization phase of the algorithms, the filters' estimated values are randomly initialized according to the search space.

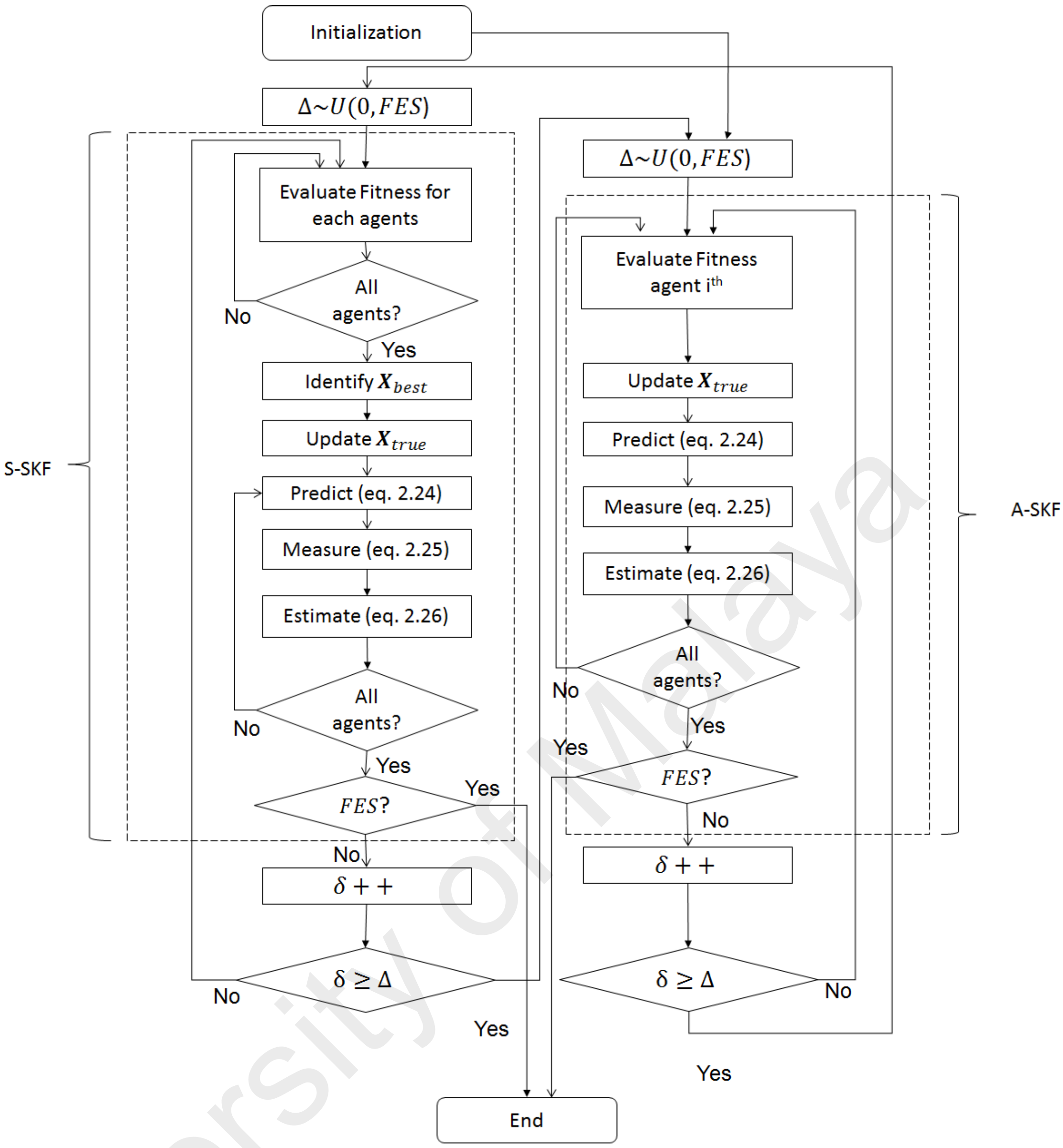

Figure 5.6: Flowchart of RSw-SKF$_a$

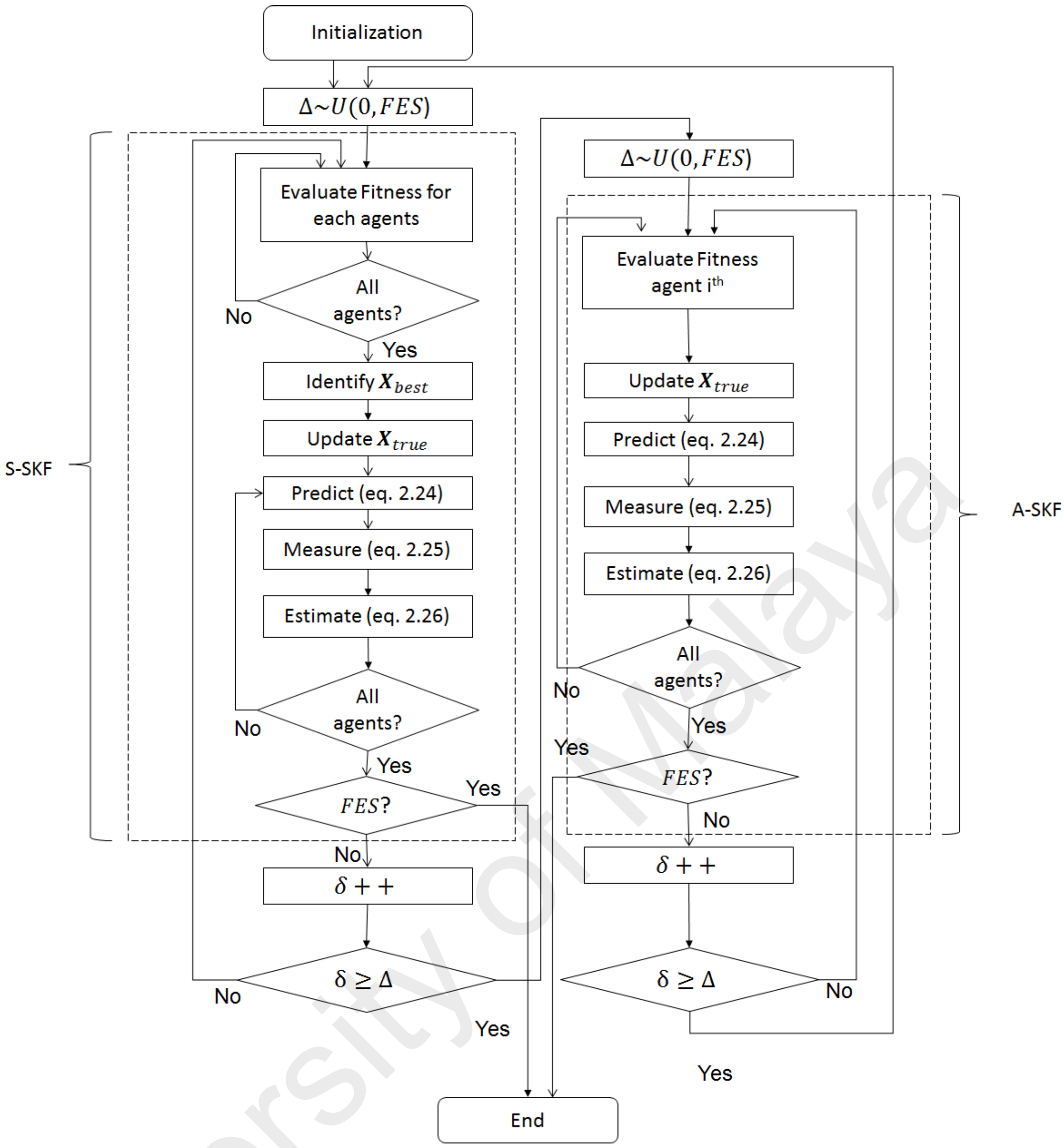

Figure 5.7: Flowchart of RSw-SKF$_s$

5.3.3.2 The Switching

A counter, δ, counts the number of fitness evaluation an SKF population is executing for a particular iteration strategy. If the population had performed Δ number of fitness evaluation using a particular iteration strategy, then its iteration strategy is switched. From S-SKF to A-SKF and vice versa.

If the population is executing a synchronous update population, then SKF migrates and restarts its search as an asynchronous update population. New value of Δ is randomly drawn from zero to FES and δ is reset when switching occurs.

Information on $\boldsymbol{X}_{true}$ is preserved across the switches. The population keep moving towards the previous $\boldsymbol{X}_{true}$ until a better solution or new $\boldsymbol{X}_{true}$ is found.

5.3.3.3 The Stopping Condition

Maximum number of fitness evaluation, FES, is used as the stopping condition. After the predicted, measure, and estimated steps are executed by every filter within the population, the stopping condition is checked. If the maximum number of fitness evaluation has been executed, then the algorithm is terminated.

5.4 Experiments, Results and Discussion

5.4.1 Experimental Parameter Settings

The experiments conducted here use the same parameter settings as the experiments conducted in chapter 4. The performance of the algorithms is measured using the fitness error value and Wilcoxon signed rank test is used for pairwise non-parametric statistical analysis while Friedman and Holm tests are used for multiple algorithms comparison. The change of populations' behaviour towards the iteration strategy is observed using position diversity (equation 2.4).

5.4.2 Fitness Error Value

PSO - RSw-PSO$_a$ and RSw-PSO$_s$ are studied here. The two variants of random switching PSO differ from each other with their starting iteration strategy. The RSw-PSO$_a$ and RSw-PSO$_s$ are compared with S-PSO and A-PSO. Figure 5.8 show the rate of fitness error value over iteration. In chapter 4 it is observed that the graphs of fitness error over iteration for the functions are showing almost the same behavior, thus only four functions, f2, f16, f19 and f26, one from each type of functions, are shown here. The graphs show fitness error of RSw-PSO$_a$ and RSw-PSO$_s$ decrease at similar rate to S-PSO and A-PSO and the differences of the fitness errors for the four PSO variant are small.

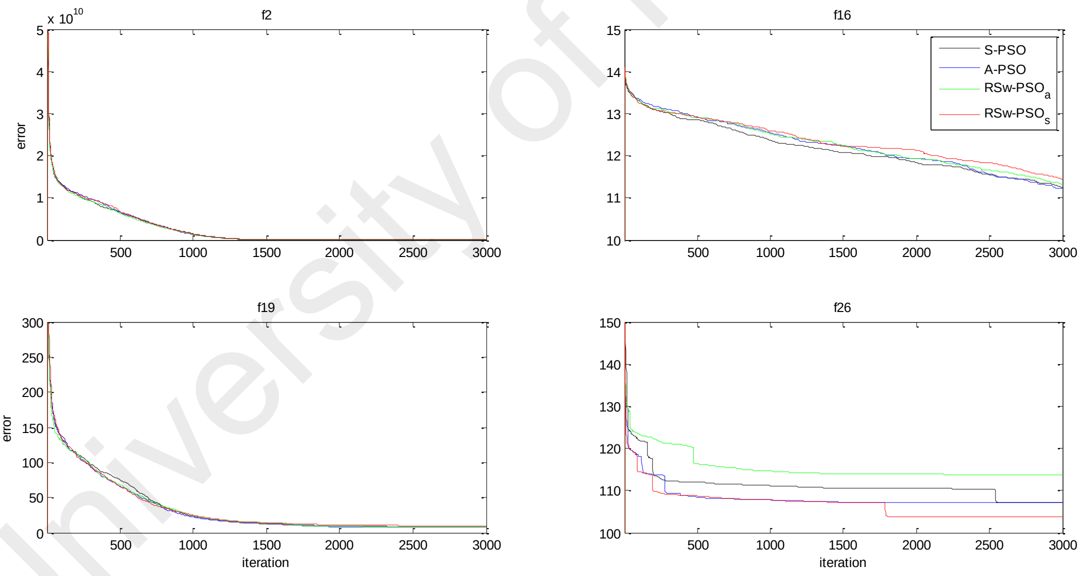

Figure 5.8: Fitness Error Rate of RSw-PSO

The algorithms' fitness error value distributions are shown in Figure 5.9 to Figure 5.12. The boxplots for the four algorithms are located at almost similar level. Some small differences are observed in the width of the boxes and whiskers, but, no uniform trend is observed. For example, in unimodal function, RSw-PSO$_s$ has the widest box for f1 indicating its bad performance. However, RSw-PSO$_s$ has the smallest box for f2, which is under the same category as f1. In the experiment involving hybrid functions, RSw-PSO$_a$ is seen to have large number of extreme outliers for f18. On the other hand, for f21, RSw-PSO$_a$ has the smallest boxplot with the least outliers.

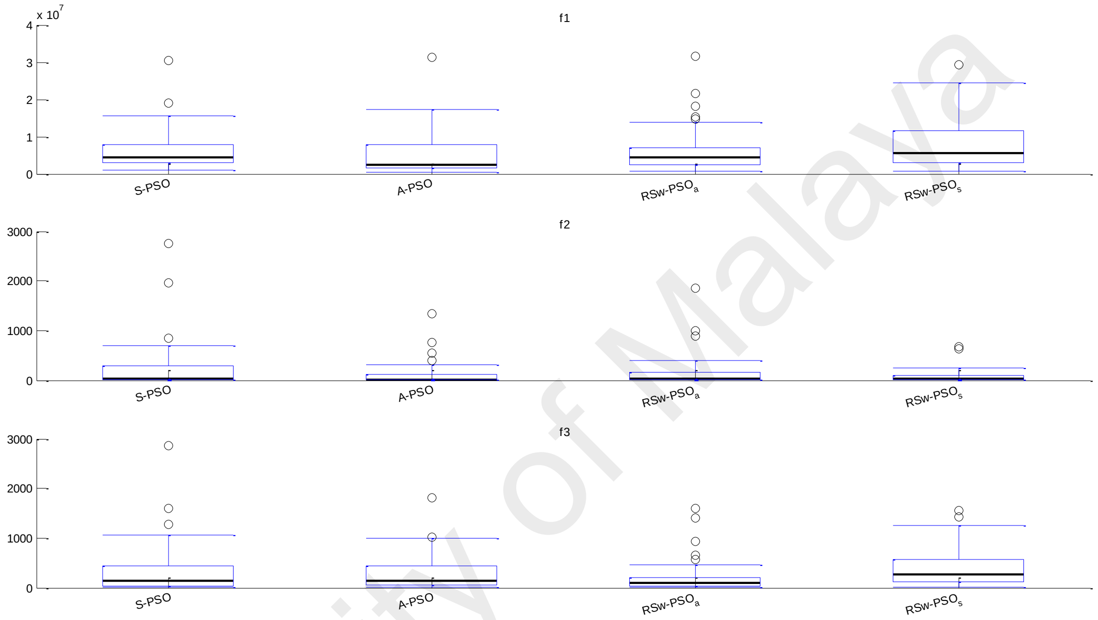

Figure 5.9: Fitness Error Distribution of Unimodal Functions for RSw-PSO

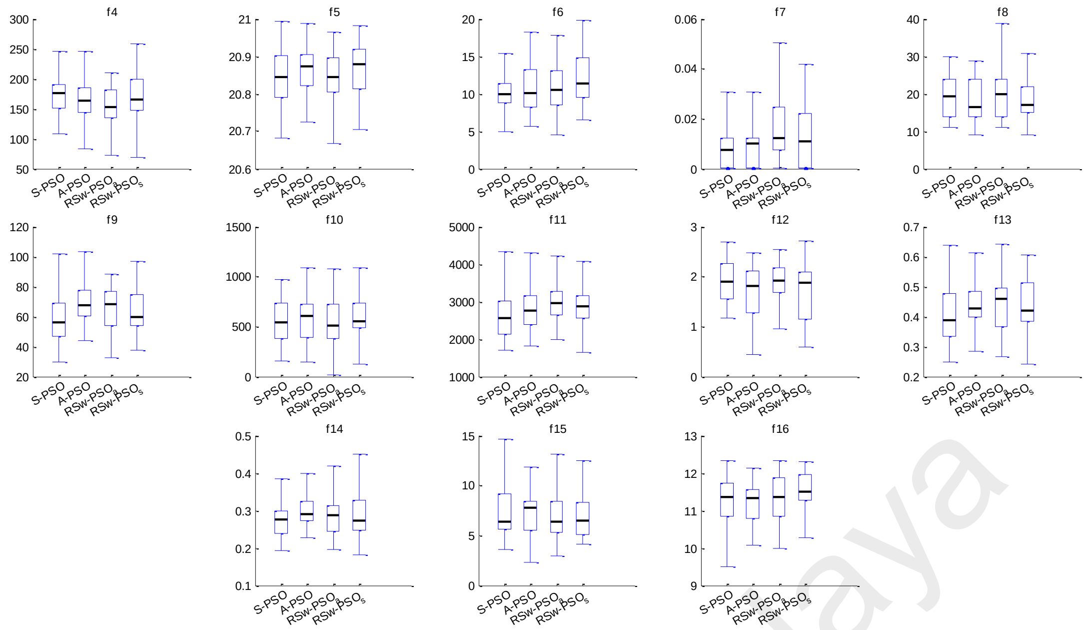

Figure 5.10: Fitness Error Distribution of Simple Multimodal Functions for RSw-PSO

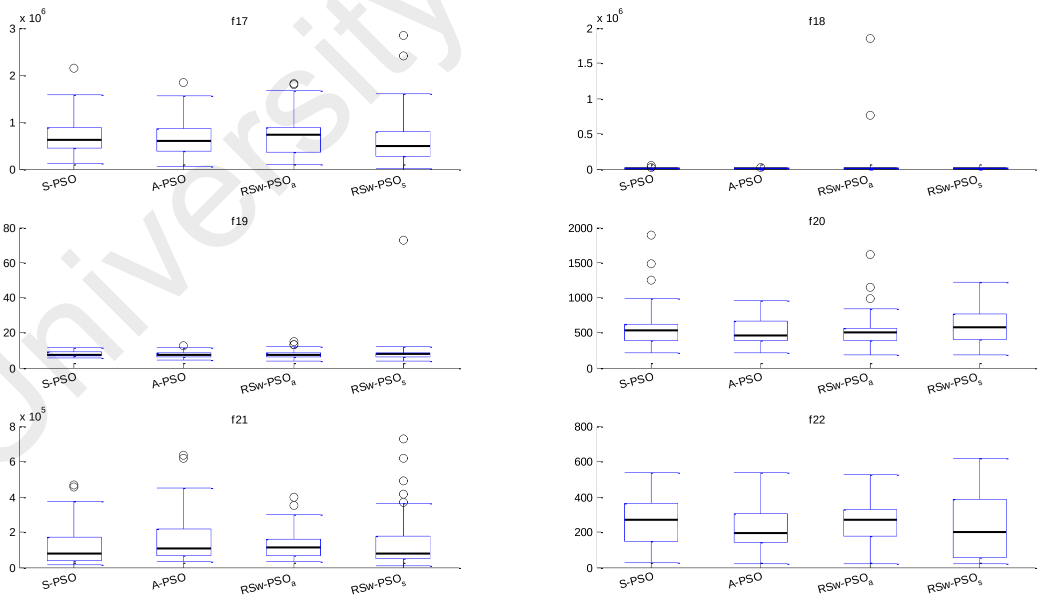

Figure 5.11: Fitness Error Distribution of Hybrid Functions for RSw-PSO

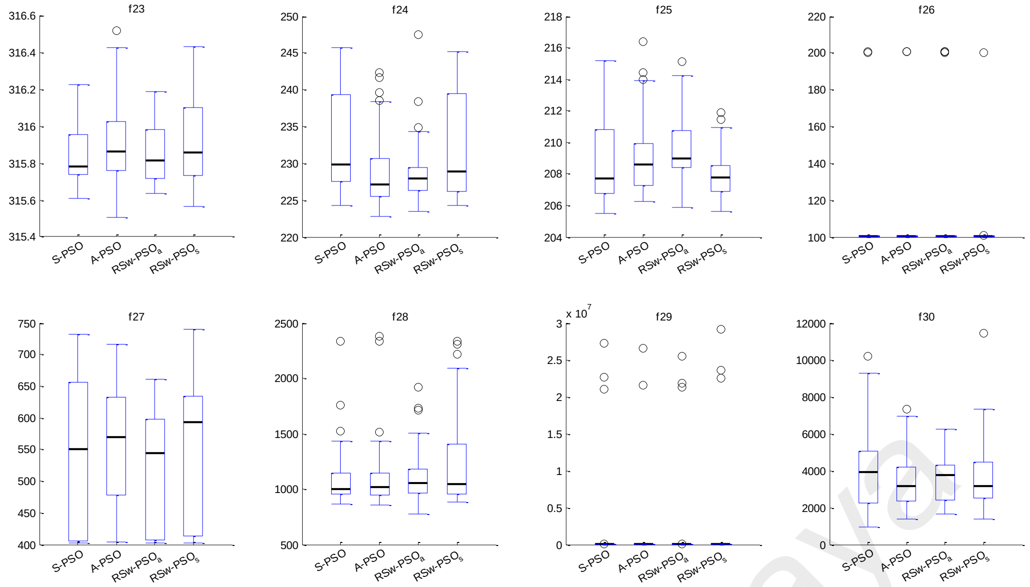

Figure 5.12: Fitness Error Distribution of Composite Functions for RSw-PSO

GSA- The rate of fitness error value over iteration for RSw-GSA$_a$, RSw-GSA$_s$ S-GSA and A-GSA are shown in Figure 5.13. RSw-GSA$_a$ and RSw-GSA$_s$ showed similar trend where the curves of RSw-GSA$_a$'s and RSw-GSA$_s$'s fitness error rate are between S-GSA and A-GSA. For f16 and f26, S-GSA was outperformed by A-GSA, both random GSA are able to match the performance of A-GSA. This shows how randomness is able to drive the parent algorithm towards the best performer between the two traditional iteration strategies.

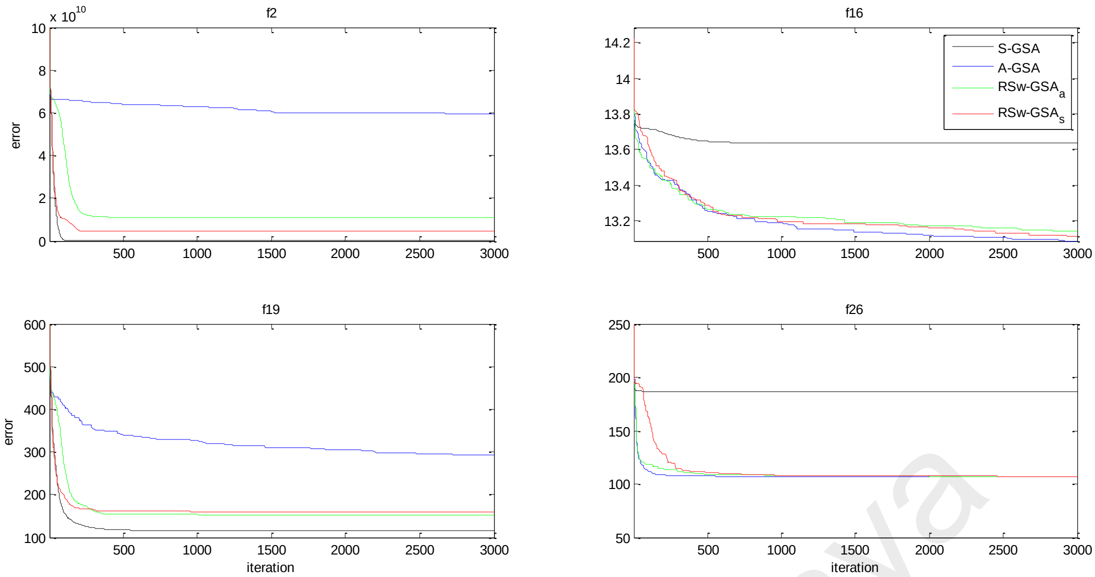

Figure 5.13: Fitness Error Rate of RSw-GSA

The fitness error distributions are presented using the boxplots in Figure 5.14 to Figure 5.17. In most functions, the fitness error distribution of RSw-GSA$_a$ and RSw-GSA$_s$ is located between S-GSA and A-GSA. The location of the boxplots for RSw-GSA$_a$ and RSw-GSA$_s$ is close to each other. However, the location is higher than S-GSA. This shows the inability of RSw-GSA$_a$ and RSw-GSA$_s$ to outperform S-GSA.

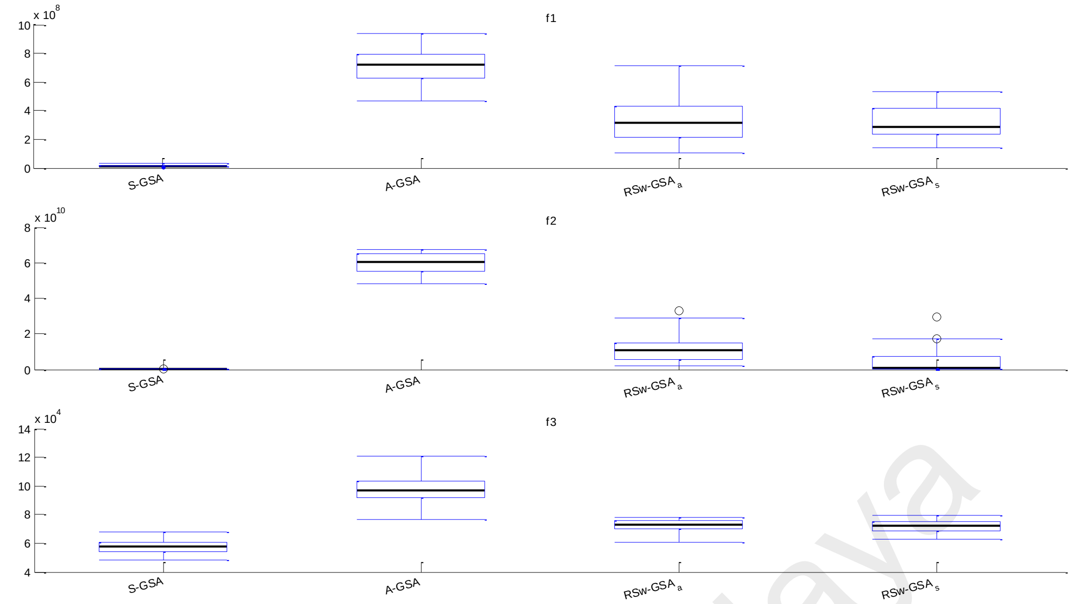

Figure 5.14: Fitness Error Distribution of Unimodal Functions for RSw-GSA

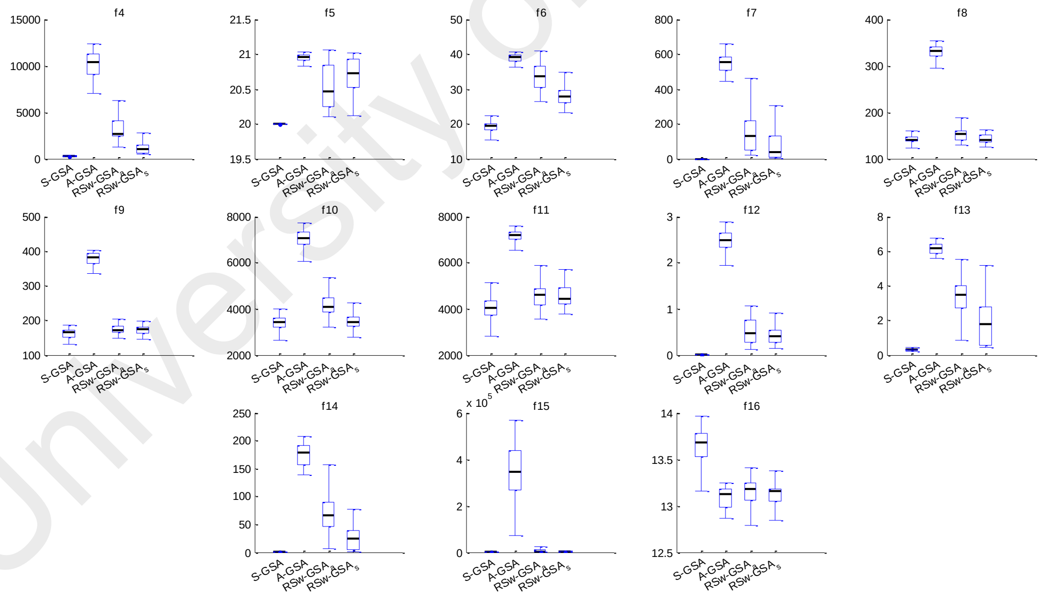

Figure 5.15: Fitness Error Distribution of Simple Multimodal Functions for RSw-GSA

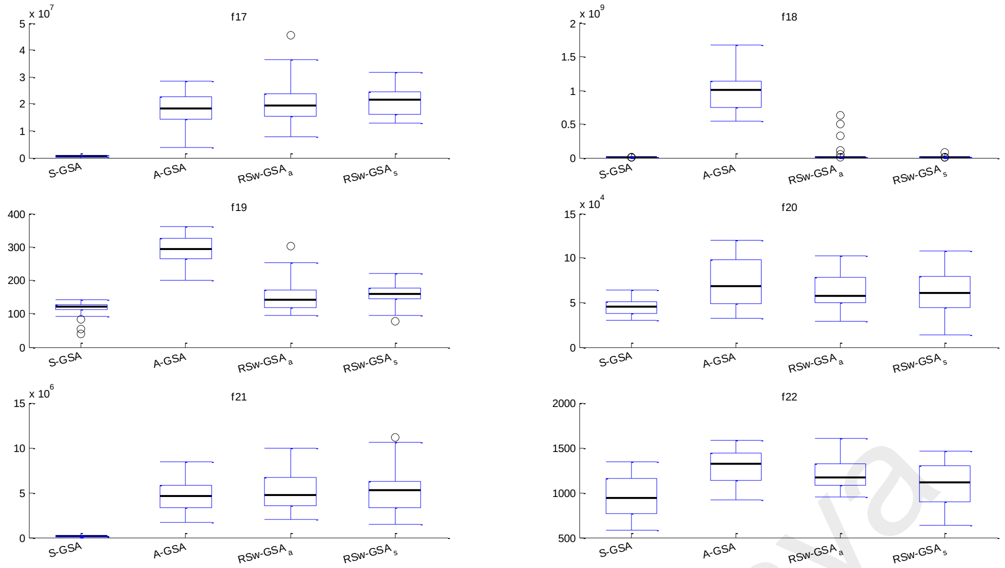

Figure 5.16: Fitness Error Distribution of Hybrid Functions for RSw-GSA

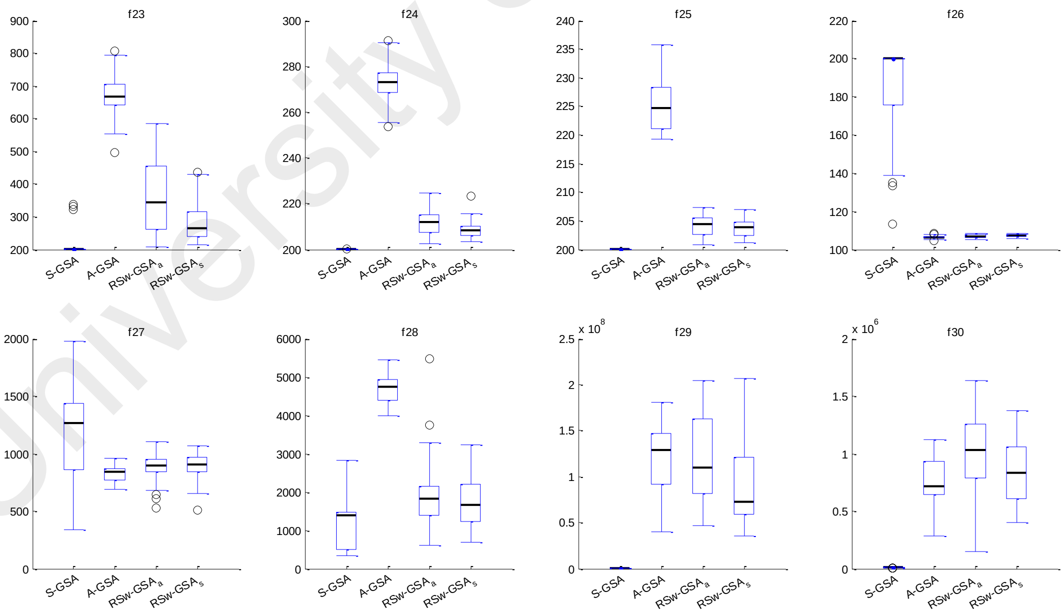

Figure 5.17: Fitness Error Distribution of Composite Functions for RSw-GSA

SKF – The fitness error rate of RSw-SKF$_a$ and RSw-SKF$_s$ are shown in Figure 5.18, The error rate of RSw-SKF$_a$ and RSw-SKF$_s$ decrease as rapid as S-SKF however the populations of the random switching are able to settle at a smaller error rate.

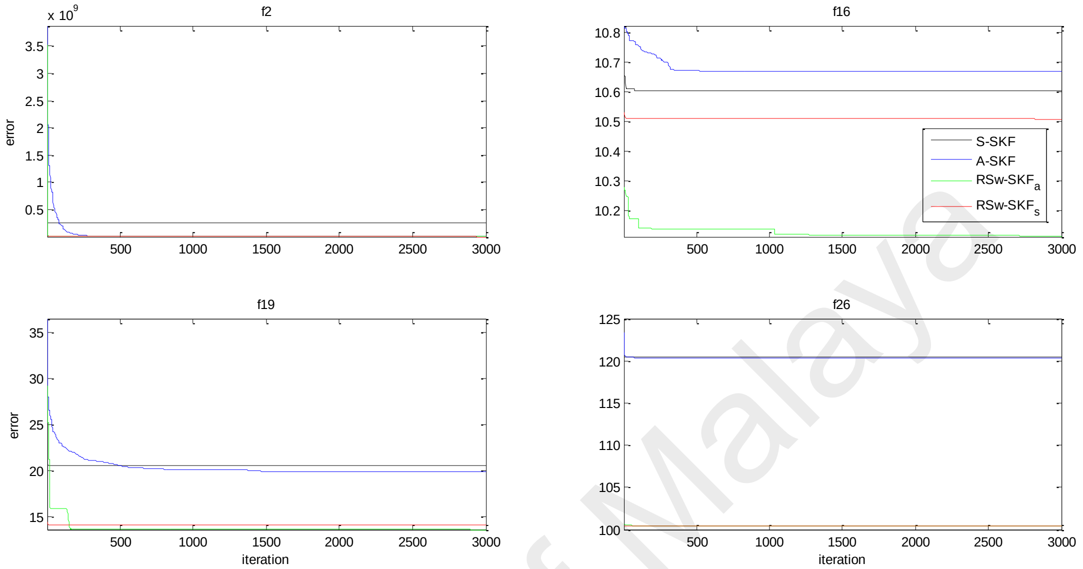

Figure 5.18: Fitness Error Rate of RSw-SKF

The boxplots in Figure 5.28 to Figure 5.31 show the distribution of the fitness error value for RSw-SKF$_a$, RSw-SKF$_s$, S-SKF and A-SKF. RSw-SKF$_a$ and RSw-SKF$_s$ are able to achieve significantly lower and smaller boxplot in a number of functions such as f1, f2, f3, f4, f5, f8, f10, f17, f18, f19, f20, f21, f23 and f30.

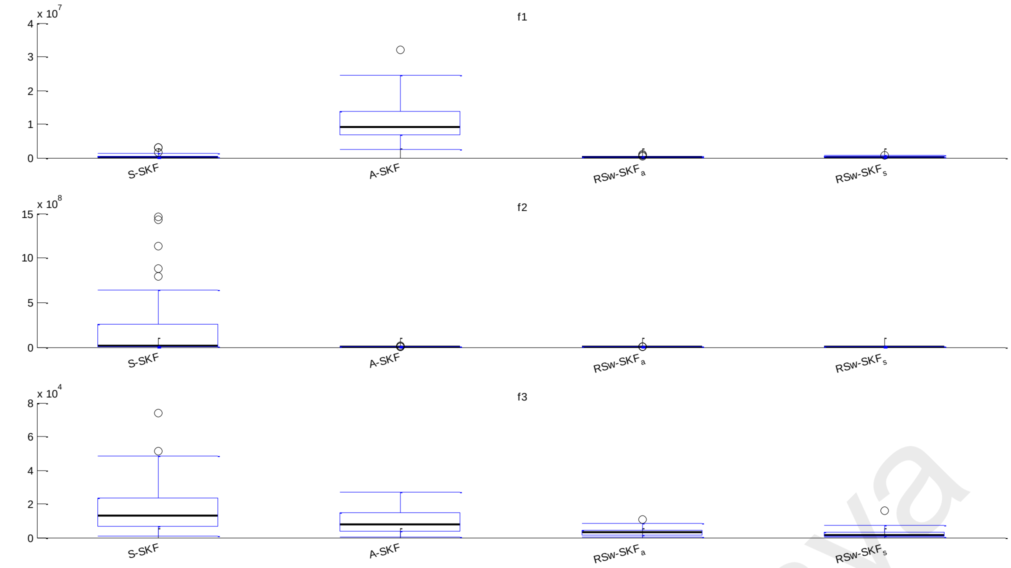

Figure 5.19: Fitness Error Distribution of Unimodal Functions for RSw-SKF

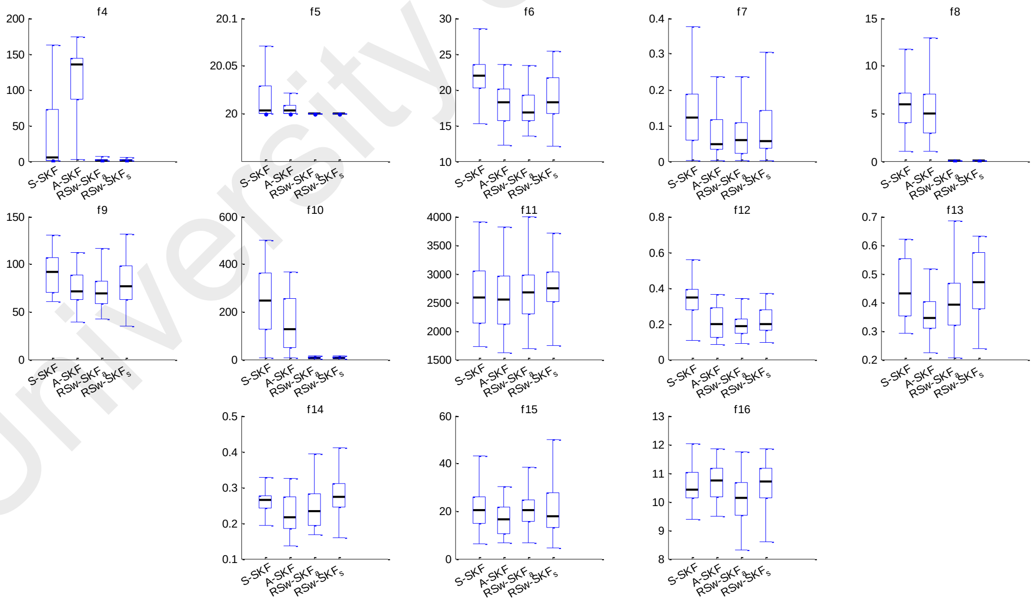

Figure 5.20: Fitness Error Distribution of Simple Multimodal Functions for RSw-SKF

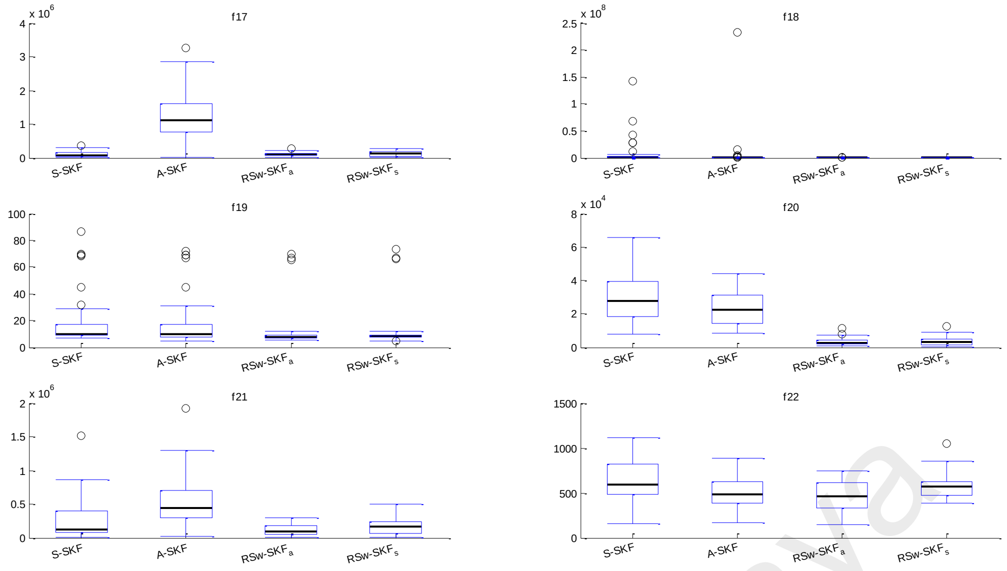

Figure 5.21: Fitness Error Distribution of Hybrid Functions for RSw-SKF

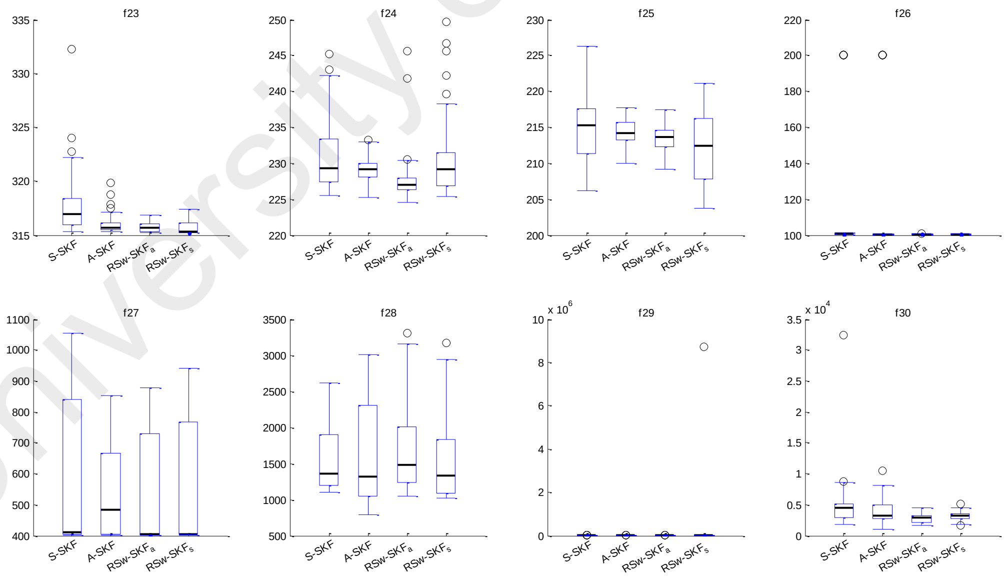

Figure 5.22: Fitness Error Distribution of Composite Functions for RSw-SKF

5.4.3 Statistical Analysis

PSO- The average fitness errors by RSw-PSO$_a$ and RSw-PSO$_s$ are compared with S-PSO and A-PSO in Table 5.1. It is observed that A-PSO has the smallest average error in most of the functions (11 out of 30) this is followed by S-PSO (8 out of 30), RSw-PSO$_a$ (6 out of 30), and RSw-PSO$_s$ (5 out of 30).

Table 5.1: Average Error of RSw-PSO

Function ID	S-PSO	A-PSO	RSw-PSO$_a$	RSw-PSO$_s$
f1	6.670E+06	**5.200E+06**	6.700E+06	8.480E+06
f2	2.879E+02	1.389E+02	1.807E+02	**9.181E+01**
f3	3.663E+02	2.945E+02	**2.534E+02**	3.997E+02
f4	1.746E+02	1.608E+02	**1.516E+02**	1.723E+02
f5	2.085E+01	2.086E+01	**2.084E+01**	2.087E+01
f6	**1.033E+01**	1.071E+01	1.062E+01	1.200E+01
f7	1.058E-02	**9.766E-03**	2.039E-02	1.288E-02
f8	1.917E+01	1.857E+01	2.034E+01	**1.798E+01**
f9	**5.871E+01**	6.879E+01	6.525E+01	6.414E+01
f10	**5.584E+02**	6.090E+02	5.703E+02	6.036E+02
f11	**2.639E+03**	2.839E+03	3.006E+03	2.902E+03
f12	1.893E+00	**1.658E+00**	1.840E+00	1.693E+00
f13	**4.086E-01**	4.446E-01	4.408E-01	4.377E-01
f14	**2.850E-01**	3.454E-01	3.285E-01	3.091E-01
f15	7.404E+00	7.254E+00	6.877E+00	**6.848E+00**
f16	1.126E+01	**1.122E+01**	1.132E+01	1.145E+01
f17	6.780E+05	**6.340E+05**	7.260E+05	6.660E+05
f18	7.474E+03	**4.828E+03**	9.331E+04	8.305E+03
f19	8.054E+00	**7.416E+00**	7.731E+00	9.508E+00
f20	6.018E+02	**5.209E+02**	5.420E+02	6.005E+02
f21	1.360E+05	1.660E+05	**1.270E+05**	1.590E+05
f22	2.559E+02	**2.294E+02**	2.549E+02	2.354E+02
f23	**3.158E+02**	3.159E+02	3.159E+02	3.159E+02
f24	2.329E+02	2.293E+02	**2.288E+02**	2.322E+02
f25	2.087E+02	2.091E+02	2.094E+02	**2.080E+02**
f26	1.071E+02	1.071E+02	1.138E+02	**1.037E+02**
f27	5.512E+02	5.556E+02	**5.170E+02**	5.599E+02
f28	**1.103E+03**	1.142E+03	1.132E+03	1.245E+03
f29	2.370E+06	**1.600E+06**	2.290E+06	2.510E+06
f30	3.970E+03	**3.391E+03**	3.658E+03	3.643E+03

The Wilcoxon signed rank test is conducted on RSw-PSO_a and RSw-PSO_s against S-PSO and A-PSO. The statistical values of the test are shown in Table 5.2. With statistical value of 230 and 182 which are bigger than 152, RSw-PSO_a is statistically on par with both S-PSO and A-PSO. While with statistical value of 178, RSw-PSO_s is on par with S-PSO. However, comparison of RSw-PSO_s and A-PSO shows a statistical value of 129 (<137) indicating A-PSO is significantly better with significance level of 5%. Both RSw-PSO_a and RSw-PSO_s statistically are on par with each other (200>152).

Table 5.2: Wilcoxon Signed Rank Test Statistical Values for RSw-PSO

	$R+$	$R-$
S-PSO vs RSw-PSO_a	230	235
S-PSO vs RSw-PSO_s	283	182
A-PSO vs RSw-PSO_a	287	178
A-PSO vs RSw-PSO_s	336	<u>129</u>
RSw-PSO_a vs RSw-PSO_s	265	200

GSA **-** The average fitness error value of RSw-GSA_a and RSw-GSA_s for each test functions are compared with S-GSA and A-GSA and tabulated in Table 5.3. Synchronous update is the best iteration strategy for GSA. S-GSA found the most number of smallest average error.

Table 5.3: Average Error of RSw-GSA

Function ID	S-GSA	A-GSA	RSw-GSA$_a$	RSw-GSA$_s$
f1	**1.300E+07**	7.110E+08	3.300E+08	3.210E+08
f2	**8.603E+03**	5.940E+10	1.110E+10	4.530E+09
f3	**5.784E+04**	9.770E+04	7.215E+04	7.149E+04
f4	**3.017E+02**	1.013E+04	3.203E+03	1.123E+03
f5	**2.000E+01**	2.095E+01	2.053E+01	2.071E+01
f6	**1.907E+01**	3.895E+01	3.366E+01	2.793E+01
f7	**0.000E+00**	5.439E+02	1.485E+02	7.061E+01
f8	**1.405E+02**	3.285E+02	1.531E+02	1.430E+02
f9	**1.624E+02**	3.781E+02	1.741E+02	1.728E+02
f10	**3.370E+03**	7.018E+03	4.159E+03	3.543E+03
f11	**4.058E+03**	7.155E+03	4.553E+03	4.541E+03
f12	**4.870E-04**	2.450E+00	5.182E-01	4.163E-01
f13	**3.017E-01**	6.146E+00	3.274E+00	1.737E+00
f14	**2.433E-01**	1.751E+02	6.920E+01	2.645E+01
f15	**3.659E+00**	3.470E+05	6.759E+03	2.338E+03
f16	1.363E+01	**1.309E+01**	1.314E+01	1.311E+01
f17	**5.310E+05**	1.840E+07	2.060E+07	2.110E+07
f18	**3.817E+02**	9.810E+08	5.430E+07	3.580E+06
f19	**1.153E+02**	2.924E+02	1.511E+02	1.603E+02
f20	**4.521E+04**	7.100E+04	6.270E+04	6.030E+04
f21	**1.550E+05**	4.760E+06	5.250E+06	5.060E+06
f22	**9.562E+02**	1.300E+03	1.224E+03	1.100E+03
f23	**2.130E+02**	6.697E+02	3.628E+02	2.847E+02
f24	**2.000E+02**	2.726E+02	2.118E+02	2.085E+02
f25	**2.000E+02**	2.249E+02	2.042E+02	2.036E+02
f26	1.868E+02	**1.064E+02**	1.069E+02	1.072E+02
f27	1.179E+03	**8.293E+02**	8.819E+02	8.981E+02
f28	**1.257E+03**	4.703E+03	1.882E+03	1.724E+03
f29	**2.001E+02**	1.170E+08	1.220E+08	8.930E+07
f30	**1.096E+04**	7.470E+05	1.030E+06	8.430E+05

The statistical values of Wilcoxon signed rank test are shown in Table 5.4. These values show that S-GSA is statistically better than RSw-GSA$_a$ and RSw-GSA$_s$ with statistical value lesser than 109, thus, the significance level is 1%. RSw-GSA$_a$ and RSw-GSA$_s$ are significantly better than A-GSA with significance level of 2% (113<120) and 1% (86<109) respectively. Comparison between RSw-GSA$_a$ and RSw-GSA$_s$ shows that using the best of the traditional strategies as the initial strategy is better. RSw-GSA$_s$ is found to be better than RSw-GSA$_a$ with statistical value of 56 which is lesser than critical value of 109, giving 1% significance level.

Table 5.4: Wilcoxon Signed Rank Test Statistical Values for RSw-GSA

	R+	*R*−
S-GSA vs RSw-GSA$_a$	436	<u>39</u>
S-GSA vs RSw-GSA$_s$	432	<u>33</u>
A-GSA vs RSw-GSA$_a$	<u>113</u>	352
A-GSA vs RSw-GSA$_s$	<u>86</u>	379
RSw-GSA$_a$ vs RSw-GSA$_s$	<u>56</u>	409

SKF **-** Table 5.5 listed the average fitness error values of RSw-SKF$_a$, RSw-SKF$_s$, S-SKF, and A-SKF according to the test functions. RSw-SKF$_a$ found the most number of the smallest average error (20 out of 30). This is followed by RSw-SKF$_s$ (8 out of 30) and A-SKF (4 out of 30). Both RSw-SKF$_a$ and RSw-SKF$_s$ found the smallest average fitness error for function f5 and f26.

According to the Wilcoxon signed rank test conducted, RSw-SKF$_a$ and RSw-SKF$_s$ are found to be significantly better than S-SKF and A-SKF. RSw-SKF$_a$ is significantly better than S-SKF and A-SKF with statistic value of 36 and 57 respectively (<109). These values give significance level of 1%. RSw-SKF$_s$ is better than S-SKF with significance level of 1% (84<109). RSw-SKF$_s$ is also better than A-SKF, but with a higher significance level of 5% (132<137). Similar as randomly switching GSA, randomly switching SKF that starts with the best traditional iteration strategy has a better performance. RSw-SKF$_a$ is found to be significantly better than RSw-SKF$_s$ with 2% significant level (112.5<120). The statistical value of the test is shown in Table 5.6.

Table 5.5: Average Error of RSw-SKF

Function ID	S-SKF	A-SKF	RSw-SKF$_a$	RSw-SKF$_s$
f1	4.860E+05	1.100E+07	**1.980E+05**	3.330E+05
f2	2.450E+08	1.290E+06	1.095E+04	**1.085E+04**
f3	1.841E+04	9.901E+03	3.212E+03	**2.714E+03**
f4	3.646E+01	1.177E+02	9.487E+00	**6.991E+00**
f5	2.002E+01	2.001E+01	**2.000E+01**	**2.000E+01**
f6	2.195E+01	1.817E+01	**1.738E+01**	1.879E+01
f7	1.635E-01	8.444E-02	**8.260E-02**	9.861E-02
f8	5.878E+00	5.473E+00	2.322E-01	**2.012E-01**
f9	9.087E+01	7.526E+01	**7.204E+01**	7.930E+01
f10	2.263E+02	1.620E+02	**6.586E+00**	1.452E+01
f11	2.640E+03	**2.585E+03**	2.686E+03	2.739E+03
f12	3.592E-01	2.099E-01	**1.944E-01**	2.119E-01
f13	4.443E-01	**3.567E-01**	4.034E-01	4.673E-01
f14	2.593E-01	**2.273E-01**	2.426E-01	2.850E-01
f15	2.192E+01	**1.640E+01**	2.150E+01	2.097E+01
f16	1.060E+01	1.067E+01	**1.011E+01**	1.051E+01
f17	1.050E+05	1.170E+06	**9.714E+04**	1.220E+05
f18	1.150E+07	8.560E+06	**1.861E+03**	4.327E+03
f19	2.050E+01	1.985E+01	**1.355E+01**	1.404E+01
f20	2.984E+04	2.415E+04	**3.443E+03**	3.736E+03
f21	2.610E+05	5.550E+05	**1.120E+05**	1.660E+05
f22	6.217E+02	4.973E+02	**4.636E+02**	5.623E+02
f23	3.181E+02	3.161E+02	**3.157E+02**	3.158E+02
f24	2.310E+02	2.292E+02	**2.282E+02**	2.312E+02
f25	2.151E+02	2.143E+02	2.136E+02	**2.120E+02**
f26	1.204E+02	1.204E+02	**1.005E+02**	**1.005E+02**
f27	5.985E+02	5.476E+02	**5.348E+02**	5.828E+02
f28	1.574E+03	1.610E+03	1.684E+03	**1.518E+03**
f29	2.477E+03	1.189E+03	**1.046E+03**	2.910E+05
f30	5.438E+03	3.848E+03	**2.805E+03**	3.163E+03

Table 5.6: Wilcoxon Signed Rank Test Statistical Values for RSw-SKF

	$R+$	$R-$
S-SKFvs RSw-SKF$_a$	<u>36</u>	429
S-SKF vs RSw-SKF$_s$	<u>84</u>	381
A-SKF vs RSw-SKF$_a$	<u>57</u>	408
A-SKF vs RSw-SKF$_s$	<u>132</u>	333
RSw-SKF$_a$ vs RSw-SKF$_s$	<u>112.5</u>	352.5

Multiple Comparisons Among Algorithms– The Friedman ranks of the random switching algorithms and the parent algorithms in synchronous and asynchronous update are tabulated in Table 5.7. Random switching can be seen to benefit SKF the most. RSw-$\mathrm{SKF_a}$ is now ranked the best among all algorithms even higher than A-PSO which is ranked the best in chapter 4. However, the statistical values in Table 5.8 which are from Holm procedure with significance level of 5% show that statistically RSw-$\mathrm{SKF_a}$ and A-PSO are on par. The statistical values also show that random switching does not benefit GSA.

Table 5.7: Average Rankings of Friedman Test for Random Switching

Algorithm	Ranking
RSw-SKF_a	3.75
A-PSO	4.9
RSw-SKF_s	5
RSw-PSO_a	5.15
S-PSO	5.2333
RSw-PSO_s	5.3667
A-SKF	5.7
S-GSA	5.85
S-SKF	6.7167
RSw-GSA_s	9
RSw-GSA_a	10
A-GSA	11.3333
p-value: 7.04×10^{-11}	

Table 5.8: Statistics of Holm Test for Random Switching

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
66	A-GSA vs. RSw-SKF$_a$	8.145807	0	0.000758
65	A-PSO vs. A-GSA	6.910509	0	0.000769
64	A-GSA vs. RSw-SKF$_s$	6.803091	0	0.000781
63	RSw-GSA$_a$ vs. RSw-SKF$_a$	6.713577	0	0.000794
62	RSw-PSO$_a$ vs. A-GSA	6.641965	0	0.000806
61	S-PSO vs. A-GSA	6.552451	0	0.00082
60	RSw-PSO$_s$ vs. A-GSA	6.409228	0	0.000833
59	A-GSA vs. A-SKF	6.051171	0	0.000847
58	S-GSA vs. A-GSA	5.890045	0	0.000862
57	RSw-GSA$_s$ vs. RSw-SKF$_a$	5.639405	0	0.000877
56	A-PSO vs. RSw-GSA$_a$	5.478279	0	0.000893
55	RSw-GSA$_a$ vs. RSw-SKF$_s$	5.370862	0	0.000909
54	RSw-PSO$_a$ vs. RSw-GSA$_a$	5.209736	0	0.000926
53	S-PSO vs. RSw-GSA$_a$	5.120221	0	0.000943
52	RSw-PSO$_s$ vs. RSw-GSA$_a$	4.976998	0.000001	0.000962
51	A-GSA vs. S-SKF	4.959096	0.000001	0.00098
50	RSw-GSA$_a$ vs. A-SKF	4.618941	0.000004	0.001
49	S-GSA vs. RSw-GSA$_a$	4.457815	0.000008	0.00102
48	A-PSO vs. RSw-GSA$_s$	4.404106	0.000011	0.001042
47	RSw-GSA$_s$ vs. RSw-SKF$_s$	4.296689	0.000017	0.001064
46	RSw-PSO$_a$ vs. RSw-GSA$_s$	4.135563	0.000035	0.001087
45	S-PSO vs. RSw-GSA$_s$	4.046049	0.000052	0.001111
44	RSw-PSO$_s$ vs. RSw-GSA$_s$	3.902826	0.000095	0.001136
43	RSw-GSA$_s$ vs. A-SKF	3.544769	0.000393	0.001163
42	RSw-GSA$_a$ vs. S-SKF	3.526866	0.000421	0.00119
41	S-GSA vs. RSw-GSA$_s$	3.383643	0.000715	0.00122
40	S-SKF vs. RSw-SKF$_a$	3.186711	0.001439	0.00125
39	A-GSA vs. RSw-GSA$_s$	2.506402	0.012197	0.001282
38	RSw-GSA$_s$ vs. S-SKF	2.452693	0.014179	0.001316
37	S-GSA vs. RSw-SKF$_a$	2.255762	0.024086	0.001351
36	A-SKF vs. RSw-SKF$_a$	2.094636	0.036203	0.001389
35	A-PSO vs. S-SKF	1.951413	0.051008	0.001429
34	S-SKF vs. RSw-SKF$_s$	1.843996	0.065184	0.001471
33	RSw-PSO$_s$ vs. RSw-SKF$_a$	1.736579	0.082462	0.001515
32	RSw-PSO$_a$ vs. S-SKF	1.68287	0.0924	0.001563
31	S-PSO vs. RSw-SKF$_a$	1.593356	0.11108	0.001613
30	S-PSO vs. S-SKF	1.593356	0.11108	0.001667
29	RSw-PSO$_a$ vs. RSw-SKF$_a$	1.503841	0.132622	0.001724
28	RSw-PSO$_s$ vs. S-SKF	1.450133	0.147022	0.001786
27	A-GSA vs. RSw-GSA$_a$	1.43223	0.152078	0.001852
26	RSw-SKF$_a$ vs. RSw-SKF$_s$	1.342715	0.179364	0.001923
25	A-PSO vs. RSw-SKF$_a$	1.235298	0.21672	0.002
24	S-SKF vs. A-SKF	1.092075	0.2748	0.002083
23	RSw-GSA$_a$ vs. RSw-GSA$_s$	1.074172	0.282745	0.002174
22	A-PSO vs. S-GSA	1.020464	0.307509	0.002273
21	S-GSA vs. S-SKF	0.930949	0.35188	0.002381
20	S-GSA vs. RSw-SKF$_s$	0.913046	0.361218	0.0025
19	A-PSO vs. A-SKF	0.859338	0.390154	0.002632
18	RSw-PSO$_a$ vs. S-GSA	0.751921	0.452099	0.002778
17	A-SKF vs. RSw-SKF$_s$	0.751921	0.452099	0.002941
16	S-PSO vs. S-GSA	0.662406	0.507711	0.003125
15	RSw-PSO$_a$ vs. A-SKF	0.590795	0.554658	0.003333
14	RSw-PSO$_s$ vs. S-GSA	0.519183	0.603633	0.003571
13	A-PSO vs. RSw-PSO$_s$	0.50128	0.616174	0.003846
12	S-PSO vs. A-SKF	0.50128	0.616174	0.004167
11	RSw-PSO$_s$ vs. RSw-SKF$_s$	0.393863	0.693682	0.004545
10	S-PSO vs. A-PSO	0.358057	0.7203	0.005
9	RSw-PSO$_s$ vs. A-SKF	0.358057	0.7203	0.005556
8	A-PSO vs. RSw-PSO$_a$	0.268543	0.788281	0.00625
7	S-PSO vs. RSw-SKF$_s$	0.25064	0.802092	0.007143
6	RSw-PSO$_a$ vs. RSw-PSO$_s$	0.232737	0.815965	0.008333
5	RSw-PSO$_a$ vs. RSw-SKF$_s$	0.161126	0.871994	0.01
4	S-GSA vs. A-SKF	0.161126	0.871994	0.0125
3	S-PSO vs. RSw-PSO$_s$	0.143223	0.886114	0.016667
2	A-PSO vs. RSw-SKF$_s$	0.107417	0.914458	0.025
1	S-PSO vs. RSw-PSO$_a$	0.089514	0.928673	0.05

5.4.4 Population's Diversity

PSO - Figure 5.23 to Figure 5.26 show the behaviour of the populations' position diversity. The RSw-PSO$_a$ and RSw-PSO$_s$ populations exhibit similar behaviour, where in all test functions the particles gradually converge as their search progress. This is due to the fact that both S-PSO and A-PSO have the same behaviour, thus, combining the two iteration strategies does not change the agents' behaviour.

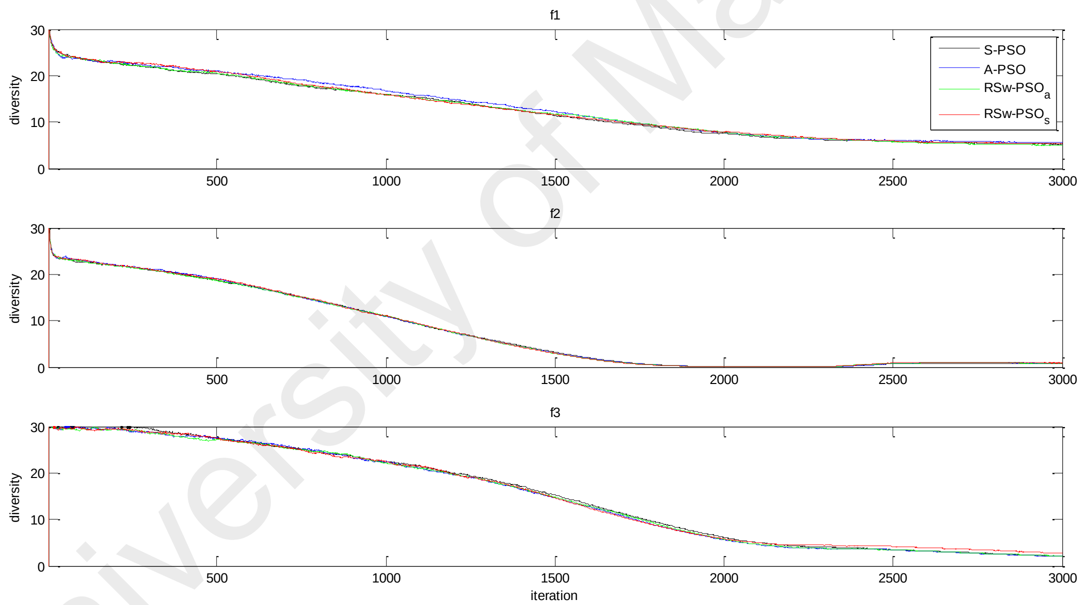

Figure 5.23: Rate of Position Diversity of Unimodal Functions for RSw-PSO

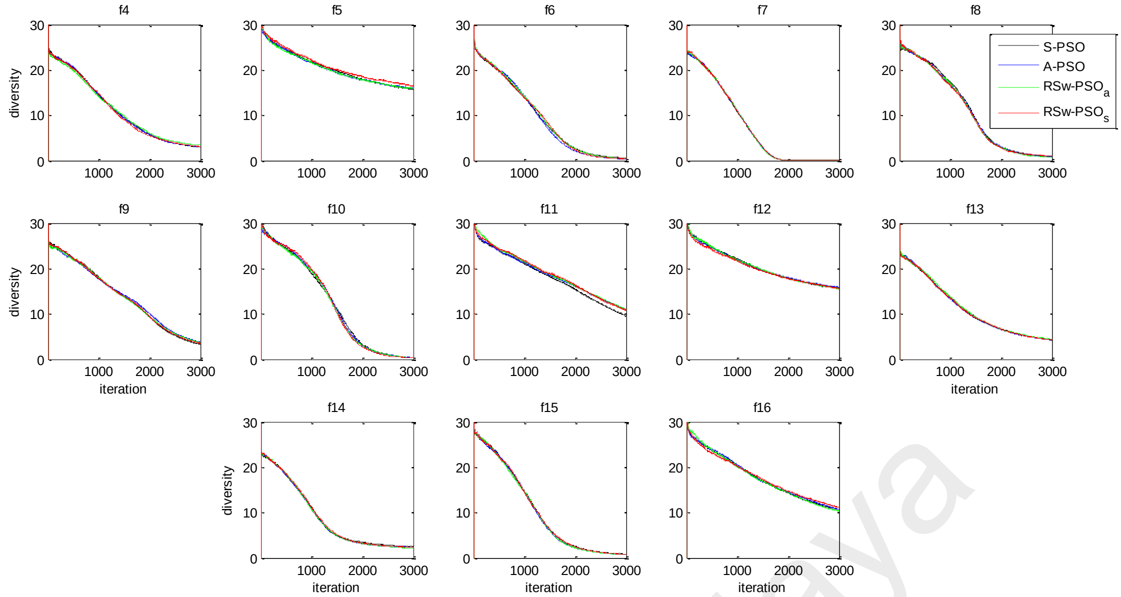

Figure 5.24: Rate of Position Diversity of Simple Multimodal Functions for RSw-PSO

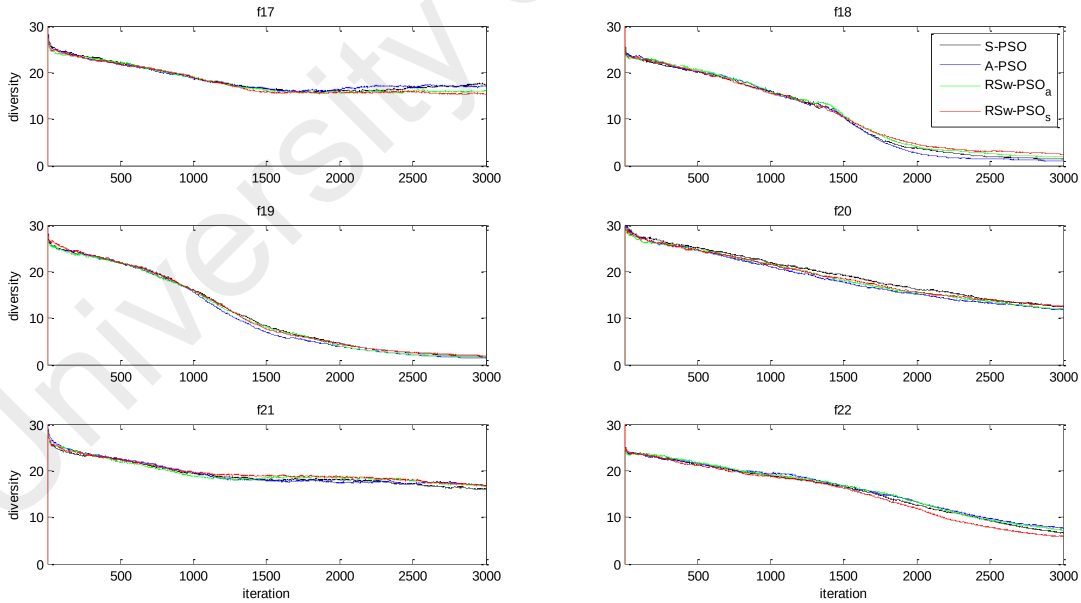

Figure 5.25: Rate of Position Diversity of Hybrid Functions for RSw-PSO

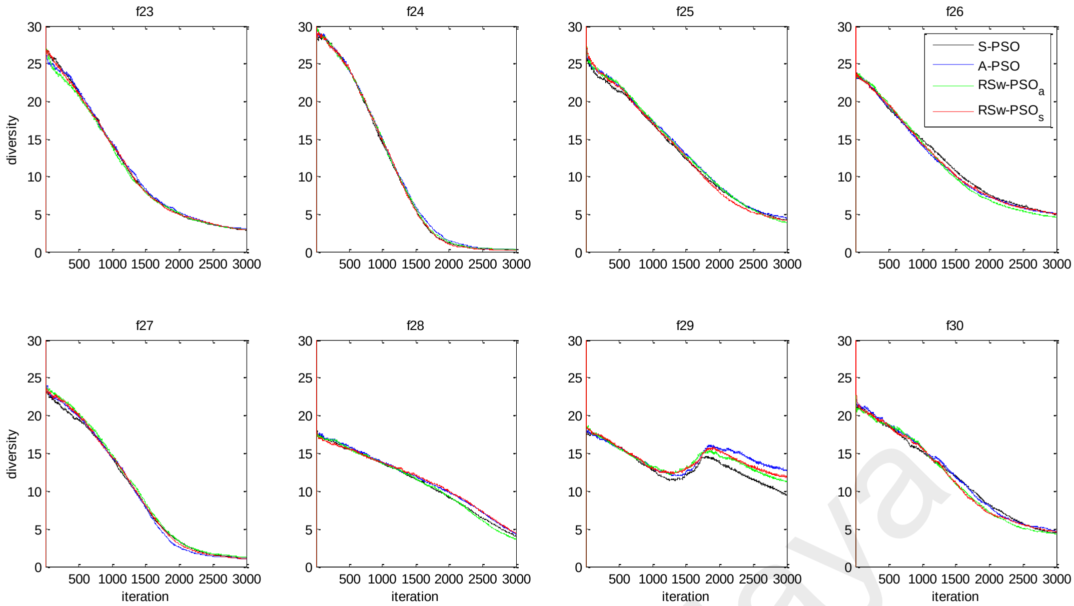

Figure 5.26: Rate of Position Diversity of Composite Functions for RSw-PSO

GSA- The rate of the position diversity for RSw-GSA$_a$, RSw-GSA$_s$, S-GSA, and A-GSA are shown in Figure 5.27 to Figure 5.30. The diversity of the population of RSw-GSA$_a$ decrease and then increase by the tenth iteration. The diversity oscillates at a high value for a period of time and decreased again before the 100th iteration. On the other hand, the population of RSw-GSA$_s$ follows the rapid convergence of S-GSA and then as the population switch its iteration strategy and adopts asynchronous update, the diversity is increased. Both RSw-GSA$_a$'s and RSw-GSA$_s$'s diversity increased after 100th iteration and kept oscillating at a positive value without converging after one third of the total iteration. Overall, the position diversity of RSw-GSA$_a$ and RSw-GSA$_s$ is higher than S-GSA but lower than A-GSA.

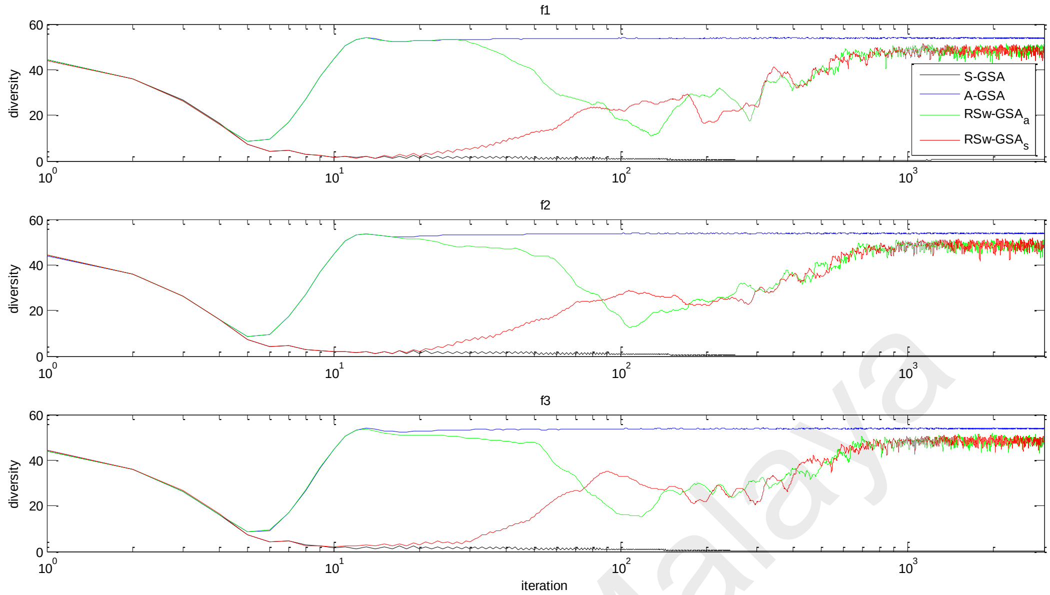

Figure 5.27: Rate of Position Diversity of Unimodal Functions for RSw-GSA

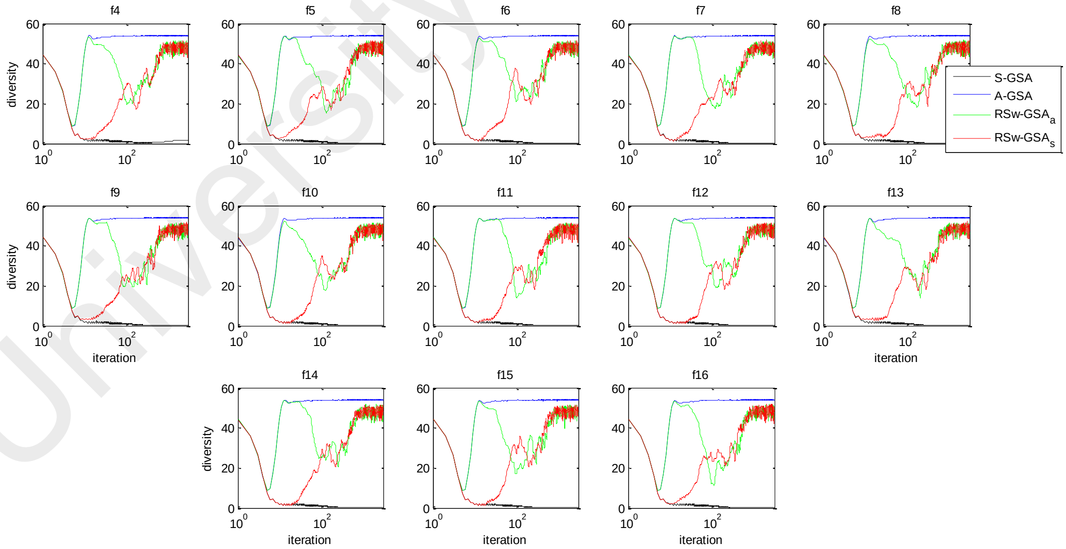

Figure 5.28: Rate of Position Diversity of Simple Multimodal Functions for RSw-GSA

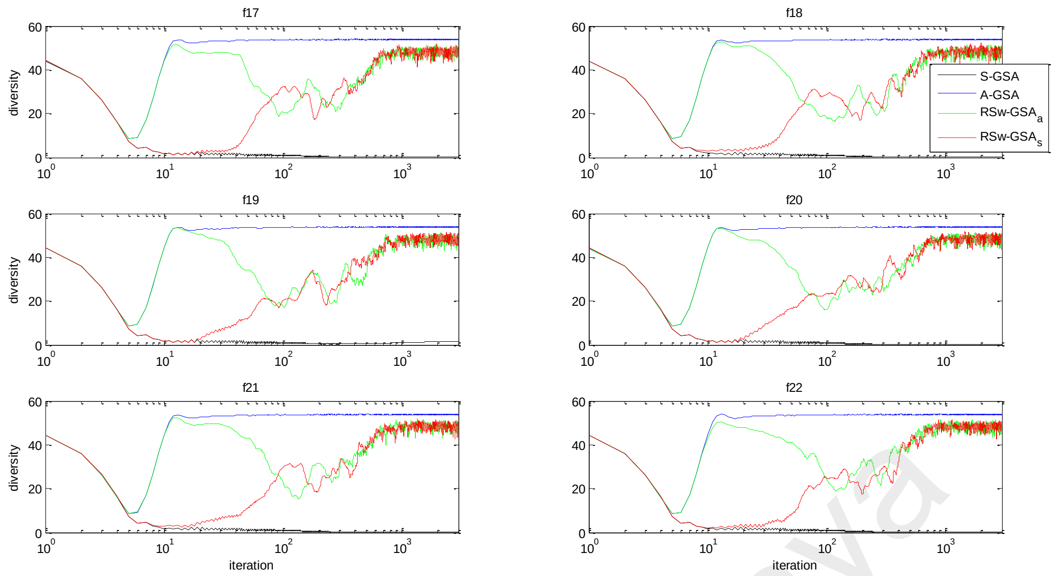

Figure 5.29: Rate of Position Diversity of Hybrid Functions for RSw-GSA

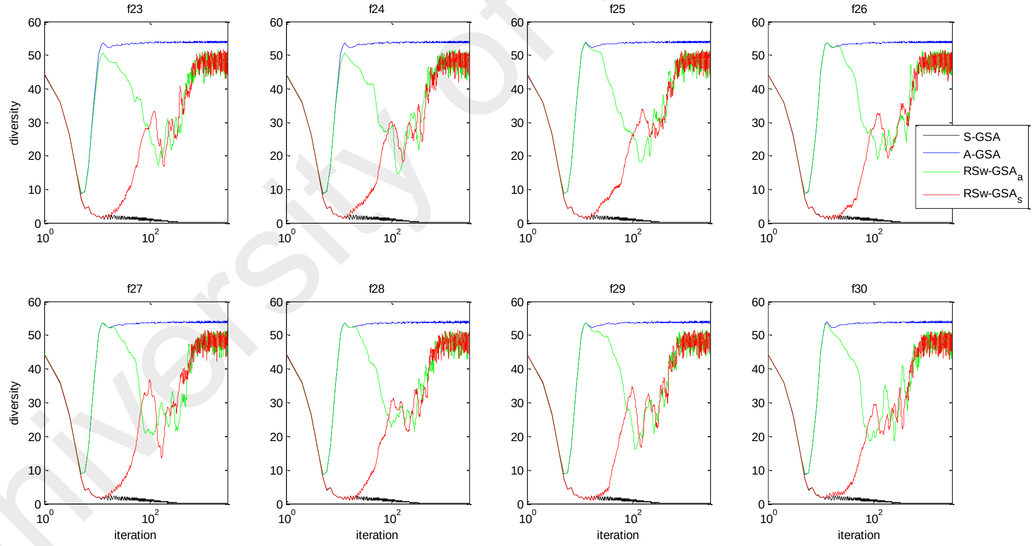

Figure 5.30: Rate of Position Diversity of Composite Functions for RSw-GSA

SKF- Figure 5.31 to Figure 5.34 clearly show the effect of the random switching towards the agents of SKF. Each time a switch occurs it causes small disturbance to the diversity. The agents' convergence is disturbed when the strategy is switched thus the agents are allowed to explore for better solution. This change of behavior helps to improve the performance of SKF.

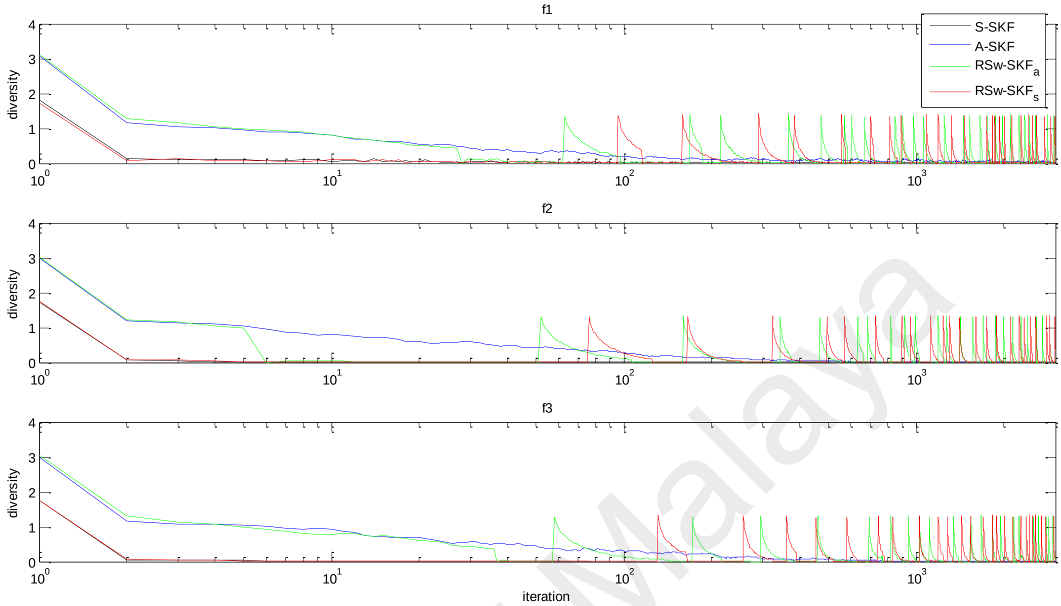

Figure 5.31: Rate of Position Diversity of Unimodal Functions for Random Switching SKF

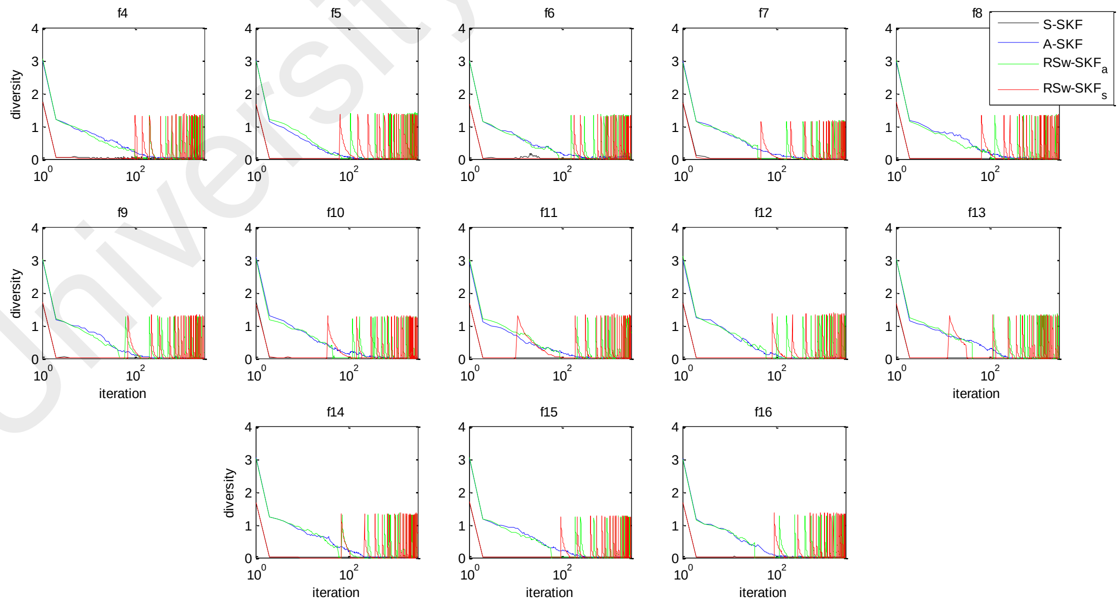

Figure 5.32: Rate of Position Diversity of Simple Multimodal Functions for RSw-SKF

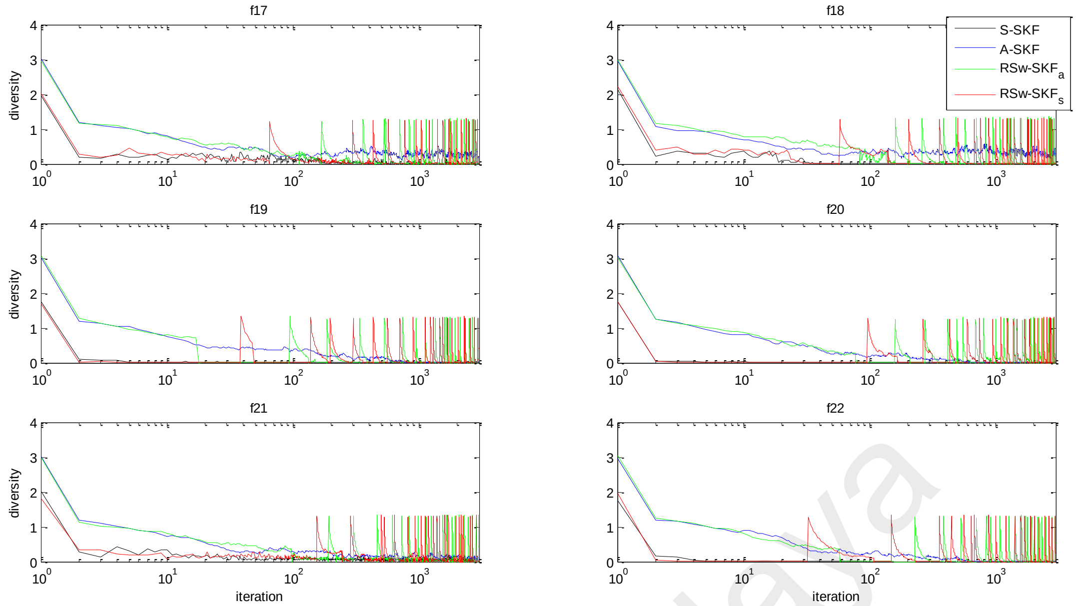

Figure 5.33: Rate of Position Diversity of Hybrid Functions for RSw-SKF

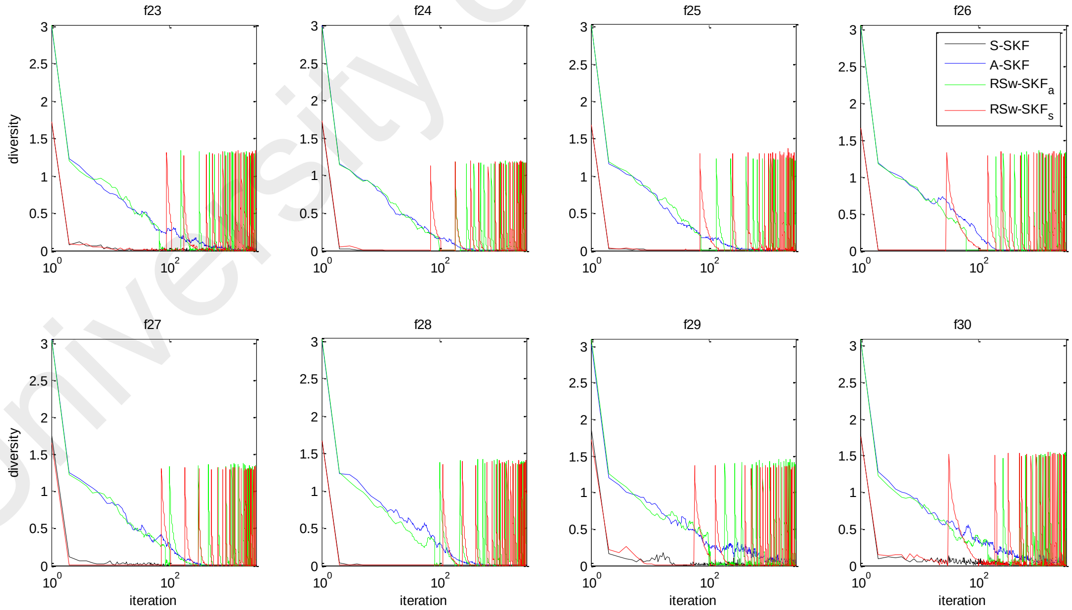

Figure 5.34: Rate of Position Diversity of Composite Functions for RSw-SKF

5.5 Conclusion

The average number of switching for RSw-PSO_a, RSw-PSO_s, RSw-GSA_a, RSw-GSA_s, RSw-SKF_a and RSw-SKF_s are tabulated in Table 5.9. The switching occurs between 42 to 44 time for each of the algorithms.

Table 5.9: Average Number of Switching

	RSw-PSO_a	**RSw-PSO_s**	**RSw-GSA_a**	**RSw-GSA_s**	**RSw-SKF_a**	**RSw-SKF_s**
Average Number of Switch	43.81	43.89	43.02	43.07	42.64	42.55

PSO does not benefit from random switching. This is due to the fact that particles of S-PSO and A-PSO are having similar behavior. As observed in chapter 4, the S-PSO and A-PSO particles' diversity and error rate have identically similar convergence curve in majority of the functions. Thus, merging the two iteration strategies does not alter the search behavior of the particles which result in on par performance.

The RSw-GSA_a and RSw-GSA_s are not able to perform as good as S-GSA. Like A-GSA, the agents of RSw-GSA_a and RSw-GSA_s are not able to converge. Non-convergence causes population-based algorithm to perform badly.

SKF benefit the most from the switching iteration strategy. RSw-SKF_a's and RSw-SKF_s's performances are better than S-SKF and A-SKF. Both S-SKF and A-SKF do not share same diversity behavior. Thus, alternation between synchronous and asynchronous update are able to change the agents' diversity behavior allowing exploration for better solution.

CHAPTER 6: ADAPTIVE SWITCHING ITERATION STRATEGY

6.1 Introduction

Adaptiveness is a common approach in optimization. In this chapter, adaptivity is reviewed and then the second hybrid iteration strategy is proposed, namely, adaptive switching iteration strategy. In the proposed adaptive switching strategy, the decision to switch is made based on the condition of the population. The condition is known as switching indicator. Implementation of the adaptive switching strategy by the three parent algorithms are presented and the results of the experiments conducted are presented in the fourth section of this chapter.

6.2 Literature Review

As discussed in (Peter J Angeline, 1995), adaptive optimization algorithms, change their optimization mechanism (W. N. Chen et al., 2013; Kaucic, 2013; Mirjalili & Lewis, 2014; Ostadmohammadi Arani et al., 2013; Shan, Yasuda, & Ohkura, 2015) or parameters (Kessentini & Barchiesi, 2015; X. Li & Yin, 2015; Precup, David, Petriu, Preitl, & Radac, 2013; Qin et al., 2006; Zhan et al., 2009) or both parameters and the search mechanism (C. Liu & Ouyang, 2010; Wu & Gao, 2013) according to the condition of the search.

Parameter setting greatly affects the performance of an optimizer and this setting can change with time (A. E. Eiben, Hinterding, & Michalewicz, 1999; A. Eiben, Michalewicz, Schoenauer, & Smith, 2007). The usage of adaptive parameters ensures the best parameter setting is used in each situation (Meyer-nieberg & Beyer, 2007). The adaptive mechanism on the other hand allows the agents' search behavior to change according to their current state, for example from exploration to exploitation (Shan et al., 2015).

Among the metric commonly used in adaptive works are, fitness of the search agents (Wu & Gao, 2013), the agents distribution or diversity (Kessentini & Barchiesi, 2015; Qin et al., 2006; Zhan et al., 2009) and the period of the search (W. N. Chen et al., 2013; C. Liu & Ouyang, 2010; W. Liu et al., 2009; Mirjalili & Lewis, 2014; Ostadmohammadi Arani et al., 2013; Precup et al., 2013; Shan et al., 2015).

6.3 Adaptive Switching Iteration Strategy

Like random switching iteration strategy, adaptive switching strategy also alternates between the synchronous update and asynchronous update. However, rather than blindly switching, the decision to switch in adaptive switching strategy is made based on the information of the population.

The information of the population's condition is stored by a switching indicator. Two switching indicators are investigated here; the best found solution, fit^* or the population's diversity, D^p. If the switching indicator is found to be static, $\frac{fit^*(t+1)}{fit^*(t)} = 1$ or $\frac{D^p(t+1)}{D^p(t)} = 1$ then the switching counter, δ is incremented. The counter, δ is initially set to zero. A population's iteration strategy is switched if the indicator is found to be static for Δ number of fitness evaluation, $\delta \geq \Delta$. As the iteration strategy is switched δ is reset to zero.

A stagnant indicator might indicate that the population is trapped within local optima and the agents had prematurely converged. Thus, the iteration strategy is switched to encourage diversity or to focus on fine tuning.

The random switching iteration strategy can be defined as in Definition 6.1.

Definition 6.1: (Adaptive switching iteration strategy)

If $\delta > \Delta$ *then*

If asynchronous update, then

Switch to synchronous update

Else

Switch to asynchronous update

The general flowchart of the adaptive switching algorithm is shown in Figure 6.1.

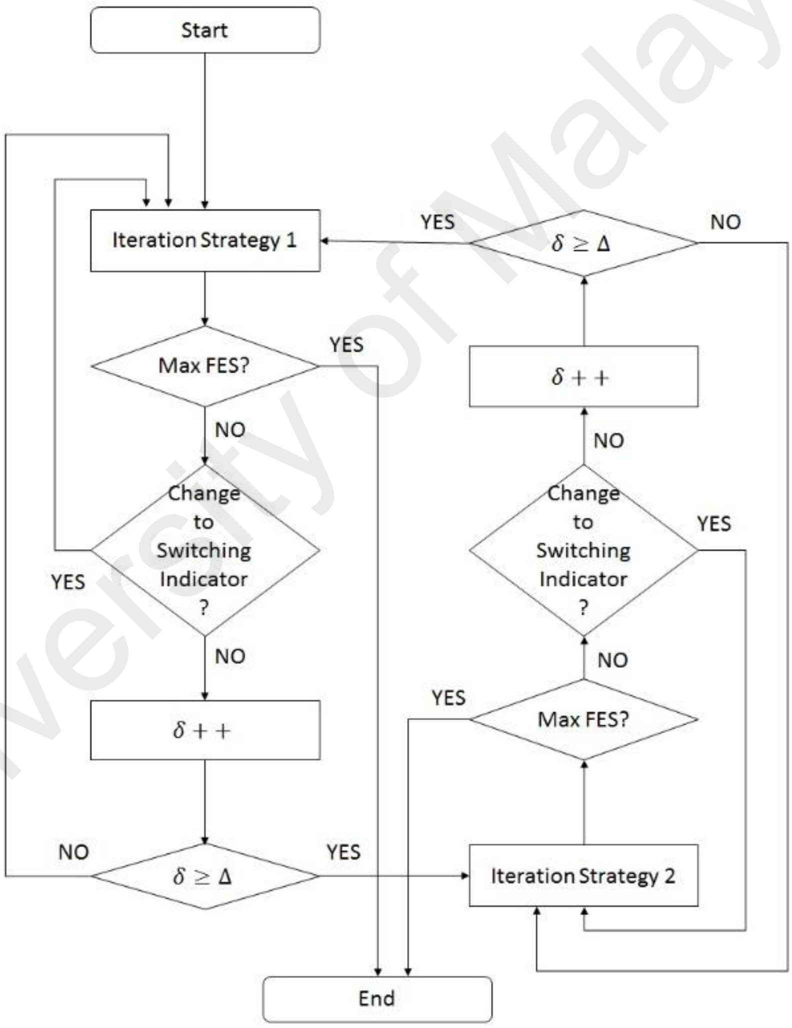

Figure 6.1: General Flowchart of Adaptive Switching

6.3.1 The Proposed Adaptive Switching PSO

PSO with adaptive switching that starts with asynchronous update is represented as, ASw-PSO $_a^b$ while ASw-PSO $_s^b$ represents adaptive switching PSO that starts with synchronous update. The switching indicator used is represented by, b, in the notation. The indicator is either fit^* or D^p. The flowchart in Figure 6.2 shows the flow of ASw-PSO $_a^b$, while Figure 6.3 shows the flow of ASw-PSO $_s^b$.

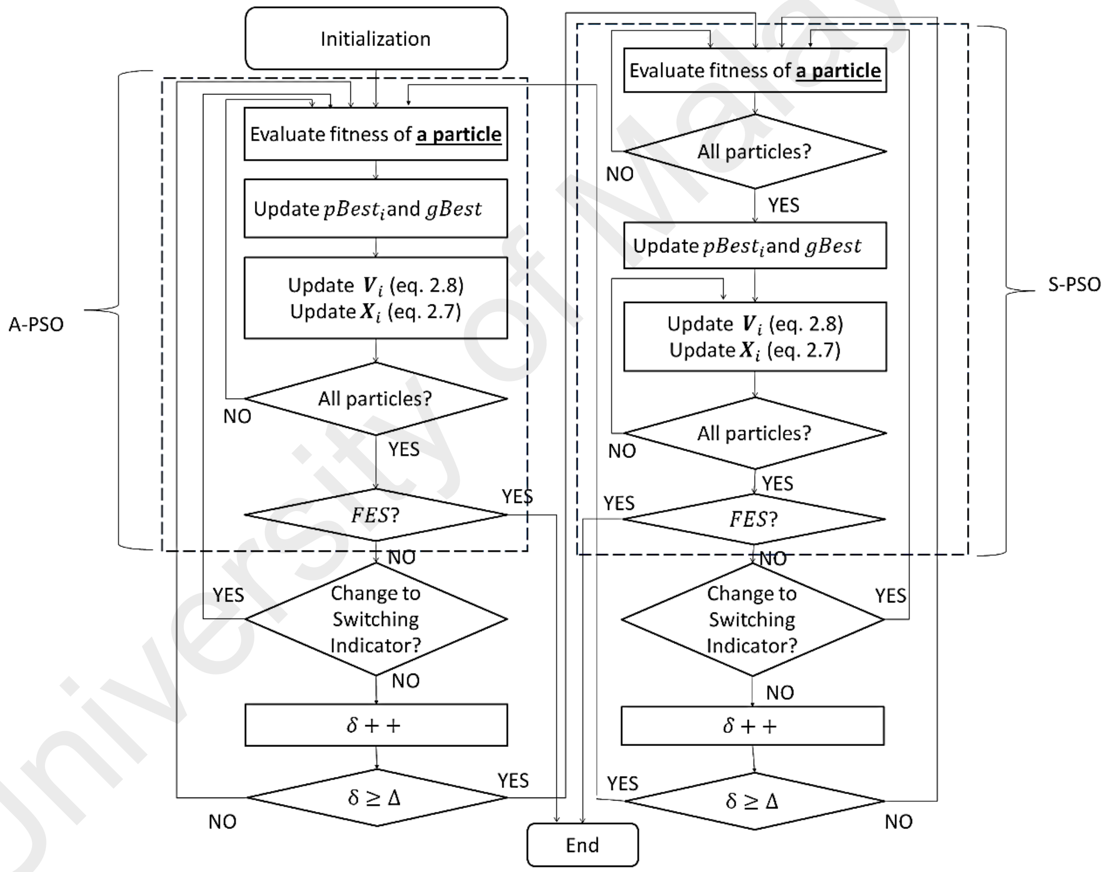

Figure 6.2: Flowchart of ASw-PSO $_a^b$

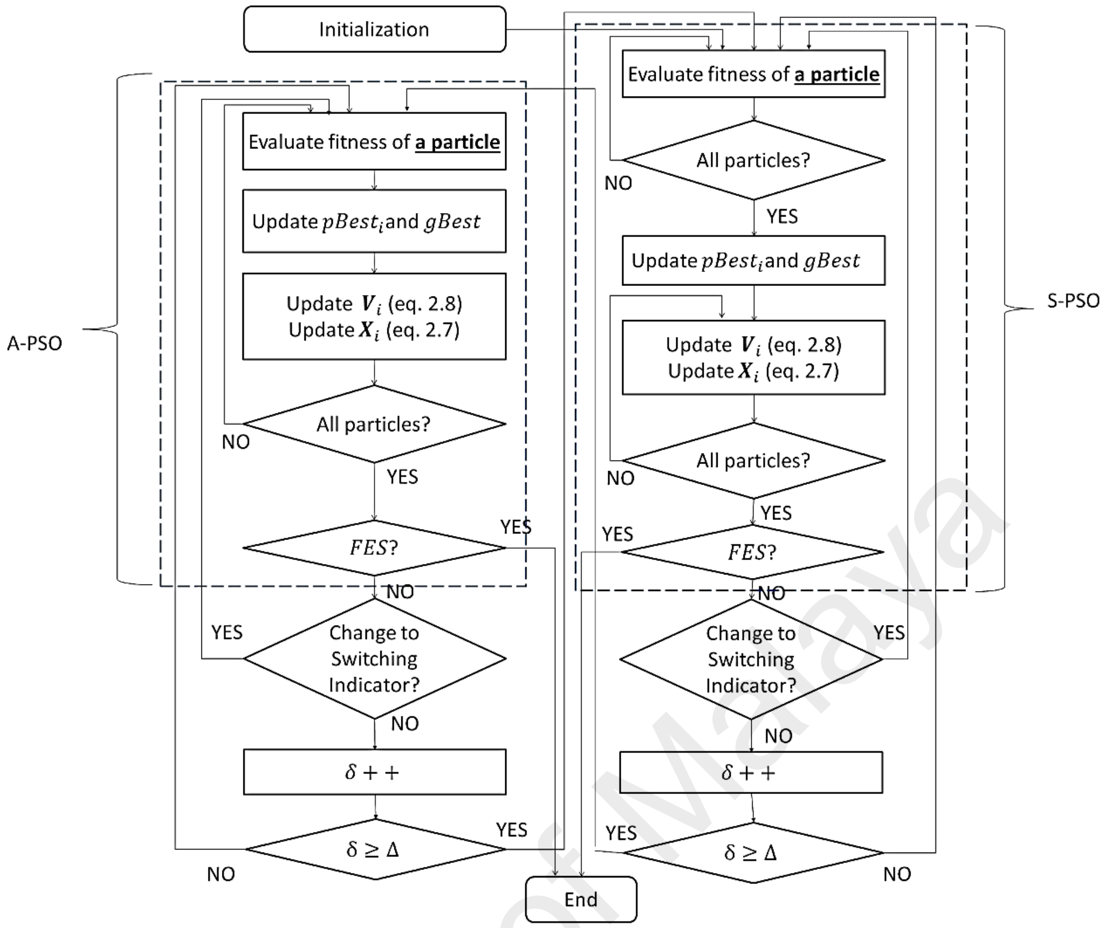

Figure 6.3: Flowchart of ASw-PSO $_s^b$

6.3.1.1 The Initialization

During the initialization phase the swarm's positions and velocities are randomly initialized according to the problem's search space. The population starts with either one of the traditional iteration strategies.

6.3.1.2 The Switching

The switching counter, δ, keeps track the number of fitness evaluation, that the switching indicator remains unchanged. In case where the best fitness of the solution is used as the switching indicator, fit^* is the fitness of, $\boldsymbol{gBest}$. On the other hand, if D^p is chosen as the indicator, the population's diversity need to be computed. This added extra computation to the algorithm.

Similar to the random switching, positions, velocities, and information of the $\boldsymbol{pBest}$ and $\boldsymbol{gBest}$ are preserved from an iteration strategy to the other strategy.

6.3.1.3 The Stopping Condition

Maximum number of fitness evaluation, FES, is used as the stopping condition for adaptive switching PSO. If the stopping condition is not met, then the switching condition and counter are checked before the next iteration is started. The value of Δ is a percentage from the maximum number of fitness evaluation, FES.

6.3.2 The Proposed Adaptive Switching GSA

Adaptive switching GSA that starts its search with asynchronous update, ASw-GSA$_a^b$, and adaptive switching GSA that starts with synchronous update, ASw-GSA$_s^b$. are proposed here. The flowchart of the two GSAs with adaptive switching strategy are shown in Figure 6.4 and Figure 6.5.

6.3.2.1 The Initialization

The adaptive switching GSAs start with random initialization of the agents. The initialization is made according to the problem's search space.

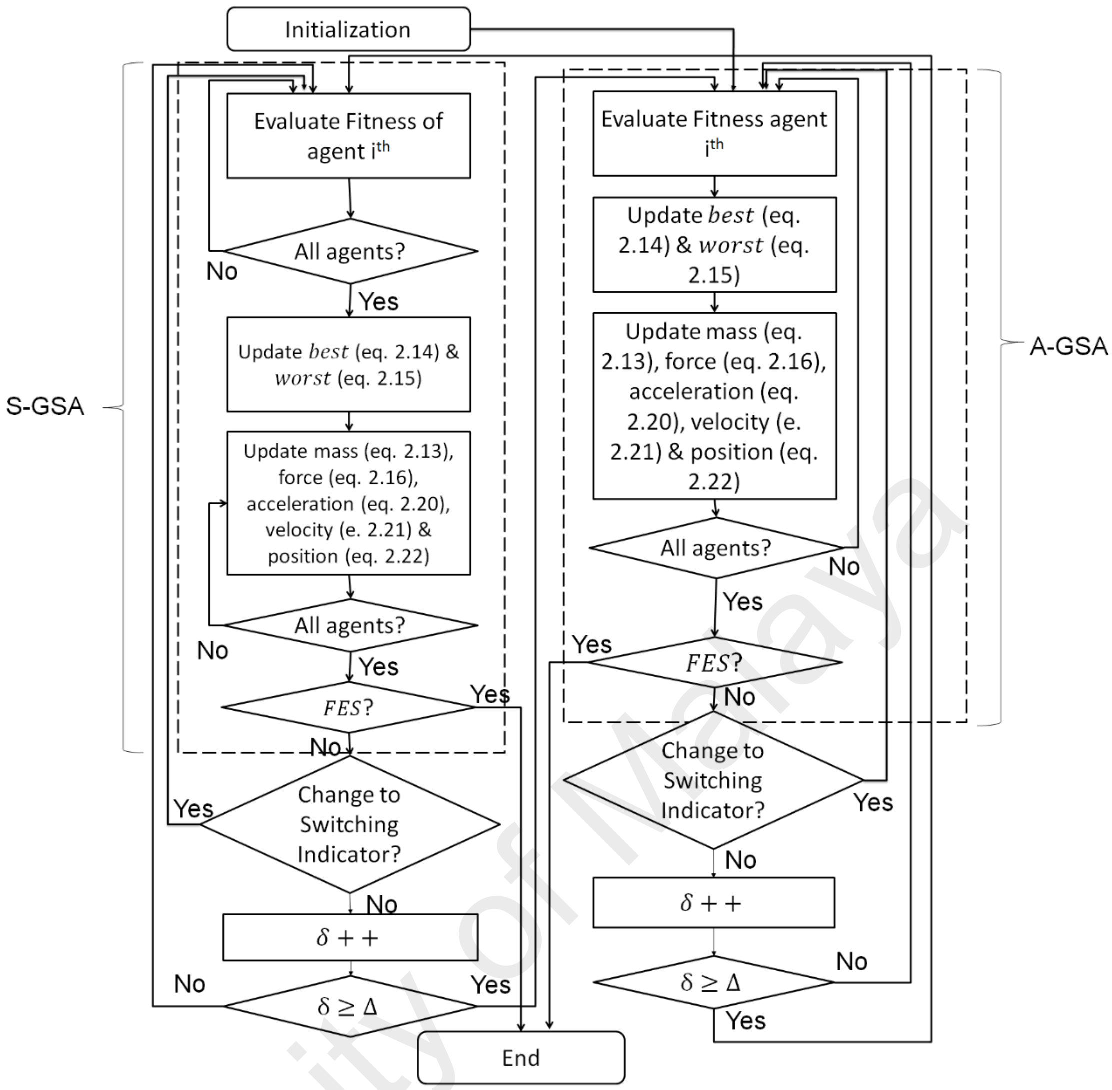

Figure 6.4: Flowchart of ASw-GSA$_a^b$

6.3.2.2 The Switching

Unlike PSO, GSA is a memoryless algorithm, there is no $\boldsymbol{gBest}$ term in GSA. Hence, the concept of memory need to be introduced for adaptive switching GSA. If fit^* is used as the switching indicator, then the population need to remembers the fitness of the best solution ever found, whereas when D^p is used, then the population remembers its position diversity.

Switching frequency is controlled by Δ. The frequency reduces with increase in the value of Δ. The population is preserved during the switch.

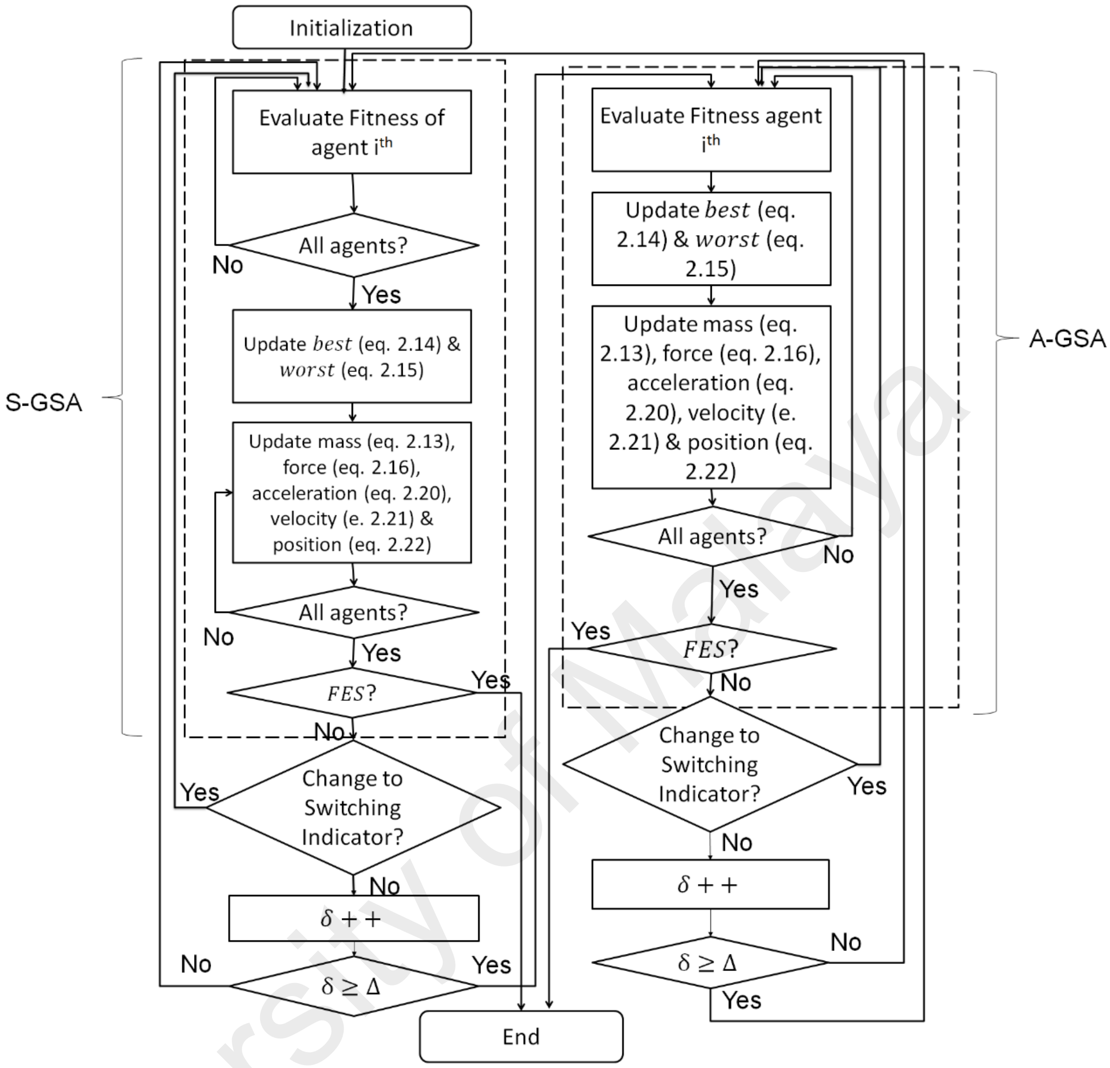

Figure 6.5: Flowchart of ASw-GSA$_s^b$

6.3.2.3 The Stopping Condition

Once again, maximum number of fitness evaluation, FES is used as the stopping condition. Both ASw-GSA$_a^b$ and ASw-GSA$_s^b$ stop after FES fitness evaluation had been done.

6.3.3 The Adaptive Switching SKF

SKF with adaptive switching iteration strategy, ASw-SKF$_a^b$ and ASw-SKF$_s^b$ are proposed in this section. The difference between the two is the former starts with asynchronous update while the later starts with synchronous update. Adaptive switching SKF is similar to random switching SKF, however, rather than making random decision on when to switch, adaptive switching made an educated decision based on information of the population. The flowchart of ASw-SKF$_a^b$ and ASw-SKF$_s^b$ are shown in Figure 6.6 and Figure 6.7 respectively.

6.3.3.1 The Initialization

Adaptive switching SKFs start with random initialization of the filters' estimated values. The random initialization is made according to the problem to be solved.

6.3.3.2 The Switching

Like the adaptive switching PSO and GSA, fitness of the best solution ever found by the population, fit^*, and population's position diversity, D^p can be used to determine when to switch. In SKF, fit^* is the fitness of $\boldsymbol{X_{true}}$.

When the iteration strategy is switched, the information on $\boldsymbol{X_{true}}$ is maintained, thus the agents are steered to find better solution within the area around $\boldsymbol{X_{true}}$.

6.3.3.3 The Stopping Condition

Adaptive switching SKFs stop after maximum number of fitness evaluation, FES, is conducted.

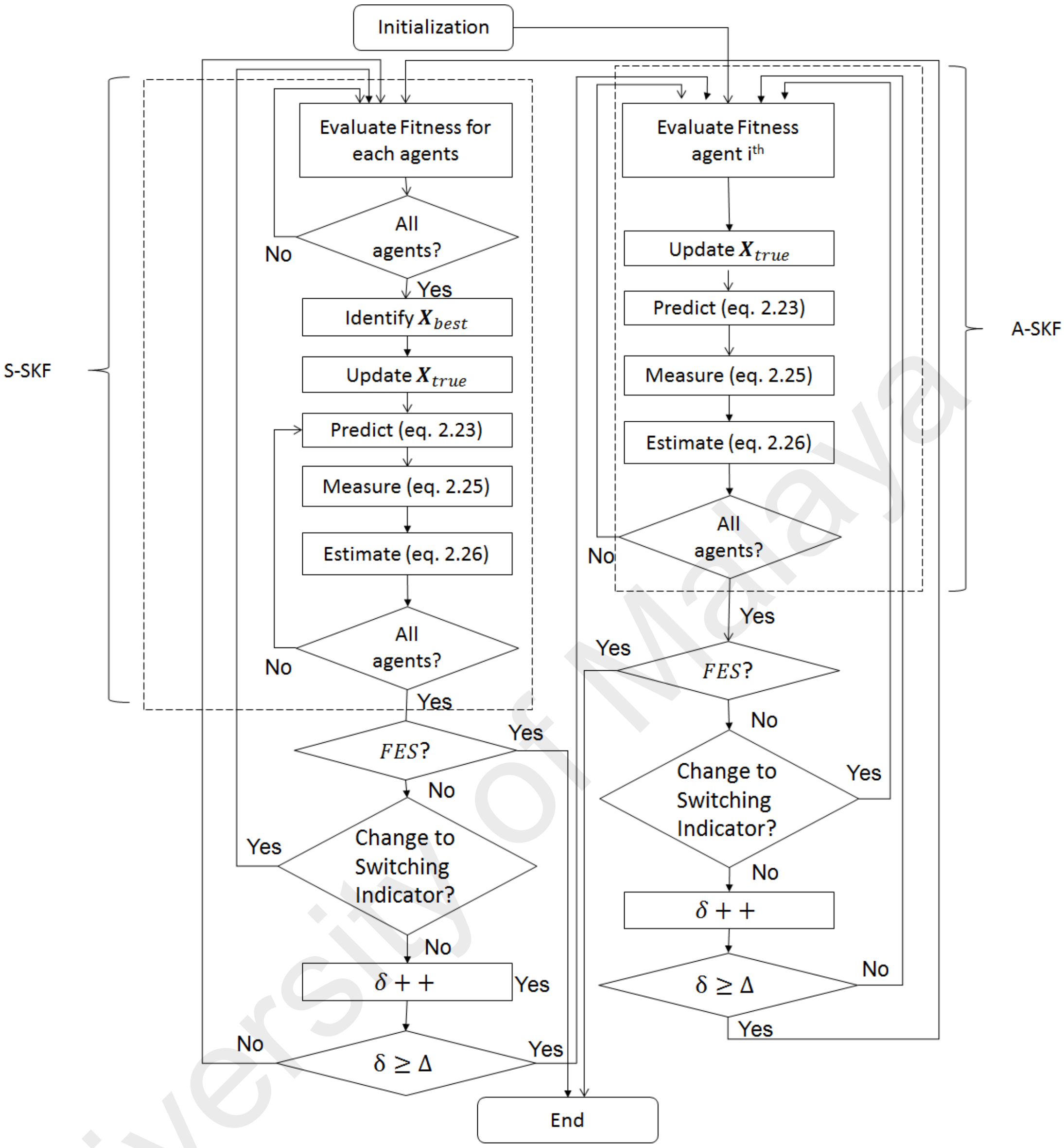

Figure 6.6: Flowchart of ASw-SKF$_a^b$

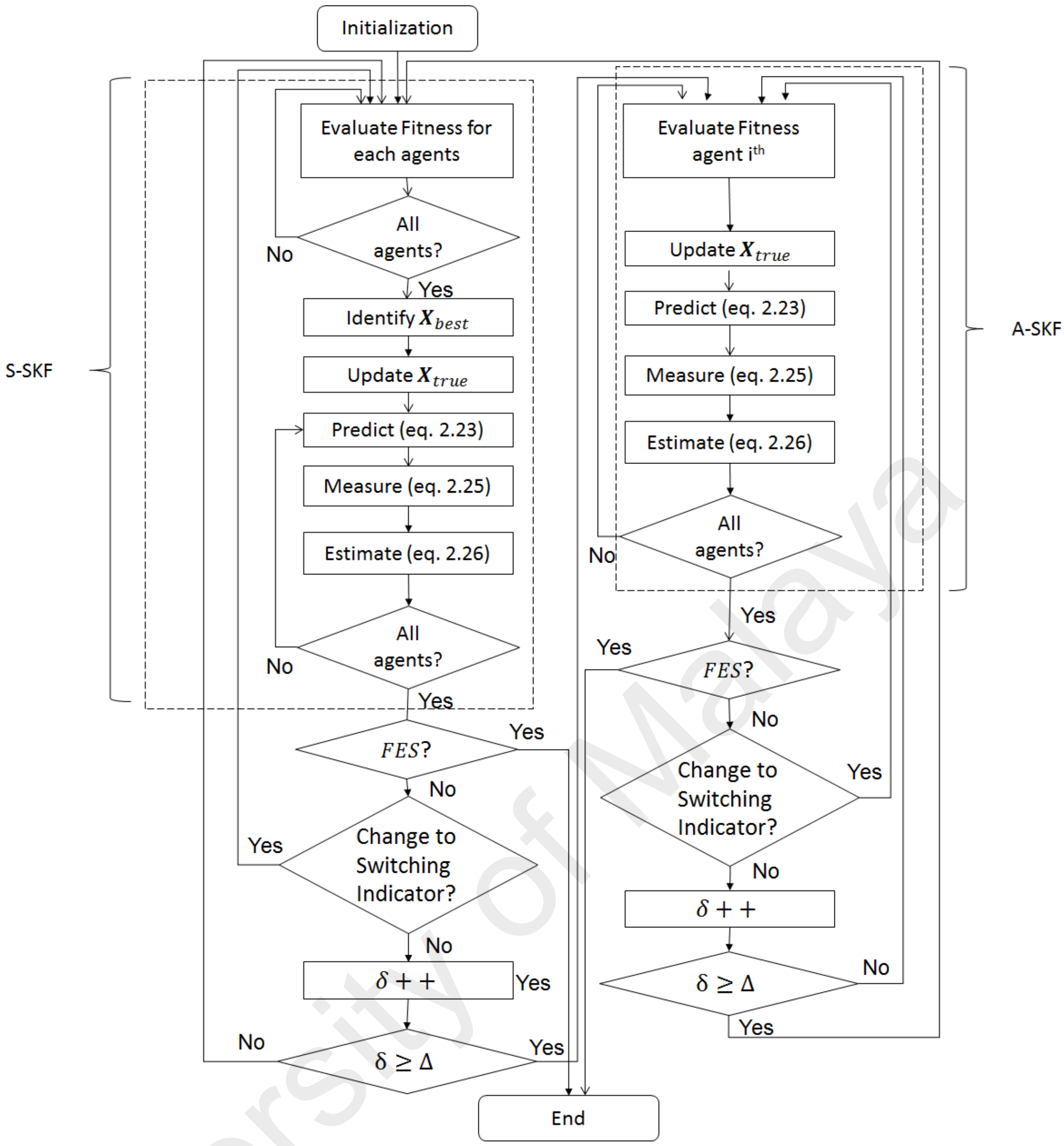

Figure 6.7: Flowchart of ASw-SKF$_s^b$

6.4 Experiments, Results and Discussion

6.4.1 Experimental Parameter Settings

The same experimental settings as chapter 4 and chapter 5 are used here. The effect of the switching indicator, fit^* or D^p, the starting strategy, synchronous or asynchronous, and the value of Δ are among the things studied. The Δ value tested are $\Delta = \{5\%, 10\%, 15\%, \ldots, 95\%\}$. These values are the percentage of number of fitness evaluation over the FES.

The results from the experiments are only accepted and presented in this section if the switching happens for more than 50% of the test functions. The number of switching of for each experiment conducted here are compiled in Appendix C.

6.4.2 Statistical Analysis

6.4.2.1 fit^* as the Switching Indicator

ASw-PSO $_a^{fit^*}$ - In this experiment ASw-PSO $_a^{fit^*}$, which is adaptive switching iteration that starts with asynchronous update and uses fit^* as the switching indicator is studied.

Based on the number of switch, only results from the tests with $\Delta = \{5\%, 10\%, 15\%, 20\%\}$ are studied here. The average fitness error values are tabulated in Table 6.1. The smallest fitness error value for each function is marked with **boldface**. No dominant algorithm is observed. The smallest results are spread among the PSO variants tested.

Based on the values in Table 6.1, Wilcoxon signed rank test is conducted. The results of the test are tabulated in Table 6.2. Wilcoxon signed rank test shows that ASw-PSO $_a^{fit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$ are able to perform as good as S-PSO and A-PSO. The statistic values for ASw-PSO $_a^{fit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$ are above 152. However, ASw-PSO $_a^{fit^*}$ with $\Delta = 20\%$ is not able to perform as good as A-PSO, the statistic value is 120 which is equivalent to critical value of 120. Thus, A-PSO is better with significance level of 2%.

Table 6.1:Average Fitness Error of ASw-PSO $_{a}^{fit^*}$

Function ID	S-PSO	A-PSO	Δ			
			5%	10%	15%	20%
f1	6.670E+06	**5.200E+06**	6.220E+06	9.580E+06	7.430E+06	8.150E+06
f2	2.879E+02	**1.389E+02**	2.454E+02	2.177E+02	1.598E+02	1.800E+02
f3	3.663E+02	2.945E+02	3.531E+02	4.156E+02	3.384E+02	**2.515E+02**
f4	1.746E+02	1.608E+02	1.761E+02	**1.500E+02**	1.553E+02	1.660E+02
f5	**2.085E+01**	2.086E+01	2.086E+01	2.088E+01	2.088E+01	2.085E+01
f6	**1.033E+01**	1.071E+01	1.099E+01	1.058E+01	1.053E+01	1.146E+01
f7	1.058E-02	9.766E-03	1.197E-02	1.048E-02	1.165E-02	**7.718E-03**
f8	1.917E+01	1.857E+01	1.877E+01	1.907E+01	**1.831E+01**	1.871E+01
f9	**5.871E+01**	6.879E+01	6.625E+01	6.643E+01	6.895E+01	6.325E+01
f10	5.584E+02	6.090E+02	5.614E+02	**5.255E+02**	5.324E+02	6.117E+02
f11	**2.639E+03**	2.839E+03	2.881E+03	2.866E+03	2.726E+03	2.833E+03
f12	1.893E+00	1.658E+00	1.693E+00	**1.632E+00**	1.734E+00	1.694E+00
f13	**4.086E-01**	4.446E-01	4.200E-01	4.242E-01	4.442E-01	4.472E-01
f14	2.850E-01	3.454E-01	3.053E-01	2.969E-01	3.309E-01	**2.811E-01**
f15	7.404E+00	7.254E+00	7.594E+00	6.599E+00	**6.512E+00**	7.269E+00
f16	1.126E+01	**1.122E+01**	1.127E+01	1.129E+01	1.125E+01	1.136E+01
f17	6.780E+05	6.340E+05	**5.760E+05**	5.950E+05	6.010E+05	6.880E+05
f18	7.474E+03	4.828E+03	5.646E+03	4.322E+04	**4.073E+03**	5.897E+03
f19	8.054E+00	7.416E+00	9.664E+00	8.070E+00	7.989E+00	**7.306E+00**
f20	6.018E+02	5.209E+02	**5.039E+02**	5.498E+02	6.370E+02	5.310E+02
f21	1.360E+05	1.660E+05	**1.220E+05**	1.480E+05	**1.220E+05**	1.750E+05
f22	2.559E+02	**2.294E+02**	2.573E+02	2.952E+02	2.844E+02	2.357E+02
f23	3.158E+02	3.159E+02	3.158E+02	3.159E+02	3.159E+02	**3.158E+02**
f24	2.329E+02	**2.293E+02**	2.310E+02	2.308E+02	2.328E+02	2.311E+02
f25	2.087E+02	2.091E+02	2.087E+02	2.086E+02	2.089E+02	**2.081E+02**
f26	1.071E+02	1.071E+02	1.038E+02	1.138E+02	**1.037E+02**	1.109E+02
f27	5.512E+02	5.556E+02	5.837E+02	**5.198E+02**	5.565E+02	5.668E+02
f28	1.103E+03	1.142E+03	**1.078E+03**	1.147E+03	1.105E+03	1.104E+03
f29	2.370E+06	1.600E+06	**7.630E+05**	3.150E+06	2.400E+06	2.140E+06
f30	3.970E+03	**3.391E+03**	3.757E+03	3.565E+03	3.551E+03	3.401E+03

Table 6.2: Wilcoxon Signed Rank Test Statistical Values for ASw-PSO $_{a}^{fit^*}$

S-PSO vs ASw-PSO $_{a}^{fit^*}$			A-PSO vs ASw-PSO $_{a}^{fit^*}$		
Δ	*R*+	*R*−	Δ	*R*+	*R*−
5%	164	301	5%	265	200
10%	262	203	10%	292	173
15%	211	254	15%	236	229
20%	217	248	20%	345	<u>120</u>

ASw-PSO $_{s}^{fit^*}$ - In this experiment adaptive switching PSO, ASw-PSO $_{s}^{fit^*}$ that starts with synchronous update and uses fit^* as the switching indicator is studied. Based on the average number of switching, only the results from $\Delta = \{5\%, 10\%, 15\%, 20\%\}$ are used here.

Table 6.3 shows the average fitness error values of ASw-PSO $_{s}^{fit^*}$ compared to S-PSO and A-PSO. Similar as the experiment before, no dominant strategy is observed.

Wilcoxon sign ranked test is conducted for pairwise comparison between ASw-PSO $_{s}^{fit^*}$ and S-PSO and also A-PSO using the average fitness values in Table 6.3. The findings of Wilcoxon test in Table 6.4 show that ASw-PSO $_{s}^{fit^*}$ with $\Delta = \{5\%, 20\%\}$ are slightly better than S-PSO but statistically the performance is on par. The same settings of adaptive switching PSO also provide performances that are on par with A-PSO. The statistic values of the settings are bigger than 152. The ASw-PSO $_{s}^{fit^*}$ with $\Delta = \{10\%, 15\%\}$ does not perform as good as A-PSO, with significance level of 5% and 2% respectively. On the other hand, ASw-PSO $_{s}^{fit^*}$ with $\Delta = \{10\%, 15\%\}$ are statistically performing as good as S-PSO.

Table 6.3: Average Error of ASw-PSO $_{s}^{fit^*}$

Function ID	S-PSO	A-PSO	Δ			
			5%	10%	15%	20%
f1	6.670E+06	**5.200E+06**	7.890E+06	5.840E+06	8.020E+06	7.680E+06
f2	2.879E+02	1.389E+02	2.701E+02	**1.379E+02**	2.838E+02	1.573E+02
f3	3.663E+02	**2.945E+02**	3.342E+02	5.172E+02	4.651E+02	2.987E+02
f4	1.746E+02	1.608E+02	**1.589E+02**	1.780E+02	1.687E+02	1.723E+02
f5	**2.085E+01**	2.086E+01	2.087E+01	2.085E+01	2.088E+01	2.087E+01
f6	**1.033E+01**	1.071E+01	1.094E+01	1.068E+01	1.094E+01	1.082E+01
f7	1.058E-02	**9.766E-03**	1.338E-02	1.189E-02	1.280E-02	1.099E-02
f8	1.917E+01	1.857E+01	1.940E+01	1.878E+01	**1.778E+01**	1.991E+01
f9	**5.871E+01**	6.879E+01	6.574E+01	6.578E+01	6.099E+01	6.733E+01
f10	**5.584E+02**	6.090E+02	5.821E+02	6.355E+02	6.745E+02	5.891E+02
f11	**2.639E+03**	2.839E+03	2.730E+03	2.905E+03	2.877E+03	2.780E+03
f12	1.893E+00	**1.658E+00**	1.720E+00	1.666E+00	1.799E+00	1.738E+00
f13	**4.086E-01**	4.446E-01	4.314E-01	4.243E-01	4.564E-01	4.590E-01
f14	2.850E-01	3.454E-01	**2.809E-01**	2.810E-01	3.353E-01	2.832E-01
f15	7.404E+00	7.254E+00	7.203E+00	**6.339E+00**	8.076E+00	7.353E+00
f16	1.126E+01	1.122E+01	1.128E+01	1.137E+01	1.137E+01	**1.122E+01**
f17	6.780E+05	6.340E+05	7.350E+05	6.090E+05	**5.970E+05**	6.010E+05
f18	7.474E+03	**4.828E+03**	9.363E+03	5.543E+03	7.240E+03	7.422E+03
f19	8.054E+00	7.416E+00	7.439E+00	1.108E+01	8.509E+00	**7.322E+00**
f20	6.018E+02	**5.209E+02**	5.618E+02	6.841E+02	5.981E+02	5.655E+02
f21	1.360E+05	1.660E+05	**1.330E+05**	1.380E+05	1.360E+05	2.110E+05
f22	2.559E+02	2.294E+02	2.424E+02	2.718E+02	2.698E+02	**2.225E+02**
f23	3.158E+02	3.159E+02	3.158E+02	3.159E+02	**3.158E+02**	3.158E+02
f24	2.329E+02	**2.293E+02**	2.308E+02	2.315E+02	2.298E+02	2.330E+02
f25	2.087E+02	2.091E+02	**2.084E+02**	2.089E+02	2.090E+02	2.089E+02
f26	1.071E+02	1.071E+02	**1.037E+02**	1.104E+02	1.138E+02	1.171E+02
f27	5.512E+02	5.556E+02	**4.969E+02**	5.606E+02	5.656E+02	5.582E+02
f28	1.103E+03	1.142E+03	1.117E+03	1.208E+03	1.138E+03	**1.063E+03**
f29	2.370E+06	1.600E+06	**6.320E+05**	2.370E+06	1.590E+06	2.190E+06
f30	3.970E+03	**3.391E+03**	3.406E+03	3.921E+03	4.063E+03	3.844E+03

Table 6.4: Wilcoxon Signed Rank Test Statistical Values for ASw-PSO $_{s}^{fit^*}$

S-PSO vs ASw-PSO $_{s}^{fit^*}$			A-PSO vs ASw-PSO $_{s}^{fit^*}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	194	271	5%	244	221
10%	267	168	10%	344	121
15%	272	163	15%	327	138
20%	188	247	20%	309	156

ASw-GSA $_{a}^{fit^*}$ - The ASw-GSA $_{a}^{fit^*}$ is investigated here with the best fitness value found so far is used as switching indicator. Switching occurs in all value of Δ for ASw-GSA $_{a}^{fit^*}$. Therefore, the results from the entire experiments are taken and studied here. Expectedly, the number of switching decreases with increment of Δ value.

The average fitness error value for the test functions of each algorithms is tabulated in Table 6.5. The minimum which is the best value for each test function is highlighted with **boldface**. From the results, it can be seen than synchronous update is the best strategy for GSA. The best average error is mostly found by S-GSA. S-GSA is outperformed by other strategies only in four functions, f8, f16, f26, and f27.

Pairwise comparison using Wilcoxon signed rank test shows that none of the ASw-GSA $_{a}^{fit^*}$ tested is better than S-GSA, while ASw-GSA $_{a}^{fit^*}$ with $\Delta = \{5\%, 10\%\}$ is better than A-GSA with level of significance 1% and 5% respectively. The statistical values of Wilcoxon signed rank test are tabulated in Table 6.6.

Table 6.5: Average Error of ASw-GSA $_{a}^{fit^*}$

Function ID	S-GSA	A-GSA	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	**1.300E+07**	7.110E+08	2.150E+08	6.740E+08	7.910E+08	7.590E+08	7.500E+08	7.220E+08	7.870E+08	7.360E+08	7.230E+08	7.330E+08
f2	**8.603E+03**	5.940E+10	9.210E+08	3.050E+10	5.190E+10	5.790E+10	5.780E+10	5.840E+10	5.680E+10	5.850E+10	5.800E+10	5.990E+10
f3	**5.784E+04**	9.770E+04	6.180E+04	6.050E+04	7.510E+04	8.320E+04	8.820E+04	9.240E+04	8.830E+04	9.290E+04	8.790E+04	9.300E+04
f4	**3.017E+02**	1.013E+04	1.830E+03	4.840E+03	9.380E+03	1.010E+04	1.050E+04	1.050E+04	1.050E+04	1.040E+04	1.030E+04	1.020E+04
f5	**2.000E+01**	2.095E+01	2.000E+01	2.050E+01	2.080E+01	2.100E+01	2.090E+01	2.100E+01	2.100E+01	2.100E+01	2.100E+01	2.100E+01
f6	**1.907E+01**	3.895E+01	3.620E+01	3.900E+01	3.920E+01	3.920E+01	3.890E+01	3.910E+01	3.920E+01	3.910E+01	3.920E+01	3.900E+01
f7	**0.000E+00**	5.439E+02	4.230E+01	3.270E+02	5.050E+02	5.240E+02	5.440E+02	5.280E+02	5.530E+02	5.420E+02	5.360E+02	5.520E+02
f8	1.405E+02	3.285E+02	**1.400E+02**	1.490E+02	2.340E+02	3.190E+02	3.210E+02	3.210E+02	3.210E+02	3.250E+02	3.300E+02	3.350E+02
f9	**1.624E+02**	3.781E+02	1.650E+02	1.650E+02	2.000E+02	3.480E+02	3.510E+02	3.530E+02	3.610E+02	3.560E+02	3.580E+02	3.660E+02
f10	**3.370E+03**	7.018E+03	4.280E+03	5.480E+03	6.650E+03	7.090E+03	7.200E+03	7.050E+03	7.220E+03	7.130E+03	7.080E+03	7.030E+03
f11	**4.058E+03**	7.155E+03	4.720E+03	6.230E+03	7.270E+03	7.220E+03	7.200E+03	7.180E+03	7.210E+03	7.220E+03	7.090E+03	7.290E+03
f12	**4.870E-04**	2.450E+00	4.390E-01	1.940E+00	2.540E+00	2.590E+00	2.520E+00	2.640E+00	2.400E+00	2.550E+00	2.510E+00	2.610E+00
f13	**3.017E-01**	6.146E+00	1.320E+00	5.600E+00	6.190E+00	6.230E+00	6.140E+00	6.290E+00	6.190E+00	6.150E+00	6.170E+00	6.260E+00
f14	**2.433E-01**	1.751E+02	2.610E+01	1.450E+02	1.780E+02	1.900E+02	1.850E+02	1.860E+02	1.810E+02	1.810E+02	1.840E+02	1.790E+02
f15	**3.659E+00**	3.470E+05	2.750E+01	1.940E+03	1.330E+05	2.320E+05	2.590E+05	2.350E+05	2.360E+05	2.660E+05	2.560E+05	3.520E+05
f16	1.363E+01	1.309E+01	1.310E+01	1.320E+01	1.320E+01	1.310E+01	1.310E+01	1.310E+01	1.310E+01	1.310E+01	1.310E+01	1.310E+01
f17	**5.310E+05**	1.840E+07	1.230E+07	1.880E+07	2.290E+07	2.340E+07	2.110E+07	2.180E+07	2.290E+07	2.360E+07	1.950E+07	1.990E+07
f18	**3.817E+02**	9.810E+08	1.230E+08	6.070E+08	1.080E+09	1.170E+09	1.100E+09	1.060E+09	1.180E+09	1.120E+09	1.110E+09	1.130E+09
f19	**1.153E+02**	2.924E+02	1.390E+02	2.280E+02	2.930E+02	2.820E+02	2.830E+02	2.820E+02	3.010E+02	2.970E+02	2.710E+02	2.850E+02
f20	**4.521E+04**	7.100E+04	6.620E+04	6.300E+04	7.290E+04	7.130E+04	7.430E+04	7.520E+04	6.800E+04	7.870E+04	7.380E+04	6.570E+04
f21	**1.550E+05**	4.760E+06	4.340E+06	4.820E+06	5.440E+06	4.890E+06	5.090E+06	4.700E+06	4.400E+06	5.080E+06	4.410E+06	4.640E+06
f22	**9.562E+02**	1.300E+03	1.200E+03	1.390E+03	1.360E+03	1.400E+03	1.450E+03	1.300E+03	1.350E+03	1.330E+03	1.310E+03	1.390E+03
f23	**2.130E+02**	6.697E+02	2.170E+02	4.220E+02	6.970E+02	6.990E+02	6.990E+02	7.090E+02	6.650E+02	6.730E+02	7.290E+02	6.860E+02
f24	**2.000E+02**	2.726E+02	2.060E+02	2.140E+02	2.280E+02	2.510E+02	2.650E+02	2.650E+02	2.670E+02	2.650E+02	2.690E+02	2.760E+02
f25	**2.000E+02**	2.249E+02	2.010E+02	2.020E+02	2.060E+02	2.150E+02	2.200E+02	2.220E+02	2.230E+02	2.230E+02	2.230E+02	2.250E+02
f26	1.868E+02	1.064E+02	1.070E+02	1.070E+02	1.070E+02	1.070E+02	1.070E+02	1.070E+02	1.070E+02	1.070E+02	1.070E+02	1.070E+02
f27	1.179E+03	8.293E+02	8.420E+02	8.820E+02	8.820E+02	8.670E+02	8.840E+02	8.750E+02	8.810E+02	8.650E+02	8.820E+02	8.430E+02
f28	**1.257E+03**	4.703E+03	1.840E+03	3.410E+03	4.880E+03	4.860E+03	4.870E+03	4.690E+03	4.890E+03	4.900E+03	4.920E+03	4.860E+03
f29	**2.001E+02**	1.170E+08	8.470E+07	1.200E+08	1.430E+08	1.390E+08	1.550E+08	1.390E+08	1.310E+08	1.480E+08	1.400E+08	1.360E+08
f30	**1.096E+04**	7.470E+05	9.220E+05	8.820E+05	1.050E+06	9.160E+05	8.840E+05	8.140E+05	8.750E+05	9.100E+05	8.860E+05	8.870E+05

Table 6.5: Average Error of ASw-GSA $_{a}^{fit^*}$ (continued...)

Function ID	S-GSA	A-GSA	Δ								
			55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	**1.300E+07**	7.110E+08	6.870E+08	7.410E+08	6.550E+08	7.370E+08	7.090E+08	7.220E+08	6.910E+08	7.050E+08	6.900E+08
f2	**8.603E+03**	5.940E+10	5.950E+10	5.900E+10	5.740E+10	5.910E+10	5.620E+10	5.770E+10	5.880E+10	5.900E+10	5.890E+10
f3	**5.784E+04**	9.770E+04	9.570E+04	9.340E+04	9.590E+04	9.040E+04	9.420E+04	9.760E+04	9.430E+04	9.490E+04	9.070E+04
f4	**3.017E+02**	1.013E+04	1.020E+04	1.000E+04	9.840E+03	9.670E+03	1.020E+04	1.010E+04	9.960E+03	1.010E+04	9.790E+03
f5	**2.000E+01**	2.095E+01	2.100E+01	2.100E+01	2.100E+01	2.100E+01	2.090E+01	2.100E+01	2.100E+01	2.100E+01	2.090E+01
f6	**1.907E+01**	3.895E+01	3.890E+01	3.910E+01	3.880E+01	3.910E+01	3.900E+01	3.890E+01	3.860E+01	3.900E+01	3.910E+01
f7	**0.000E+00**	5.439E+02	5.470E+02	4.960E+02	5.180E+02	5.450E+02	5.250E+02	5.380E+02	5.440E+02	5.310E+02	5.270E+02
f8	1.405E+02	3.285E+02	3.310E+02	3.320E+02	3.300E+02	3.320E+02	3.300E+02	3.320E+02	3.320E+02	3.340E+02	3.300E+02
f9	**1.624E+02**	3.781E+02	3.730E+02	3.690E+02	3.740E+02	3.670E+02	3.700E+02	3.640E+02	3.660E+02	3.660E+02	3.710E+02
f10	**3.370E+03**	7.018E+03	7.080E+03	7.200E+03	7.020E+03	7.180E+03	7.040E+03	7.040E+03	6.890E+03	7.010E+03	7.170E+03
f11	**4.058E+03**	7.155E+03	7.220E+03	7.200E+03	7.160E+03	7.080E+03	7.170E+03	7.030E+03	7.200E+03	7.160E+03	7.110E+03
f12	**4.870E-04**	2.450E+00	2.500E+00	2.510E+00	2.520E+00	2.450E+00	2.500E+00	2.460E+00	2.490E+00	2.480E+00	2.540E+00
f13	**3.017E-01**	6.146E+00	6.230E+00	6.110E+00	6.200E+00	6.080E+00	6.140E+00	6.110E+00	6.190E+00	6.050E+00	6.290E+00
f14	**2.433E-01**	1.751E+02	1.820E+02	1.800E+02	1.810E+02	1.810E+02	1.840E+02	1.810E+02	1.770E+02	1.770E+02	1.750E+02
f15	**3.659E+00**	3.470E+05	2.920E+05	2.870E+05	3.340E+05	3.640E+05	3.230E+05	3.170E+05	3.280E+05	3.580E+05	3.630E+05
f16	1.363E+01	1.309E+01	1.310E+01	1.310E+01	1.310E+01	1.310E+01	1.310E+01	1.310E+01	1.310E+01	**1.300E+01**	1.310E+01
f17	**5.310E+05**	1.840E+07	1.860E+07	2.120E+07	1.950E+07	1.950E+07	1.930E+07	2.020E+07	2.040E+07	2.060E+07	1.870E+07
f18	**3.817E+02**	9.810E+08	1.140E+09	1.060E+09	9.820E+08	9.730E+08	1.070E+09	9.980E+08	1.010E+09	1.050E+09	1.050E+09
f19	**1.153E+02**	2.924E+02	2.770E+02	2.780E+02	2.780E+02	2.790E+02	2.790E+02	2.730E+02	2.690E+02	2.780E+02	2.820E+02
f20	**4.521E+04**	7.100E+04	6.980E+04	7.260E+04	6.400E+04	6.730E+04	7.000E+04	6.630E+04	6.520E+04	5.330E+04	6.420E+04
f21	**1.550E+05**	4.760E+06	4.750E+06	3.930E+06	4.730E+06	4.650E+06	4.340E+06	4.040E+06	4.170E+06	3.850E+06	3.870E+06
f22	**9.562E+02**	1.300E+03	1.350E+03	1.360E+03	1.320E+03	1.260E+03	1.340E+03	1.330E+03	1.320E+03	1.320E+03	1.310E+03
f23	**2.130E+02**	6.697E+02	7.150E+02	6.790E+02	7.050E+02	6.790E+02	6.890E+02	6.690E+02	6.880E+02	6.880E+02	6.830E+02
f24	**2.000E+02**	2.726E+02	2.750E+02	2.760E+02	2.730E+02	2.730E+02	2.740E+02	2.740E+02	2.740E+02	2.750E+02	2.730E+02
f25	**2.000E+02**	2.249E+02	2.250E+02	2.250E+02	2.250E+02	2.250E+02	2.250E+02	2.260E+02	2.260E+02	2.250E+02	2.250E+02
f26	1.868E+02	1.064E+02	1.070E+02	**1.060E+02**	1.070E+02	**1.060E+02**	1.070E+02	**1.060E+02**	1.070E+02	1.070E+02	1.070E+02
f27	1.179E+03	8.293E+02	8.530E+02	**8.200E+02**	8.240E+02	8.210E+02	8.580E+02	8.580E+02	8.380E+02	8.700E+02	8.590E+02
f28	**1.257E+03**	4.703E+03	4.920E+03	4.750E+03	4.700E+03	4.690E+03	4.730E+03	4.750E+03	4.760E+03	4.700E+03	4.850E+03
f29	**2.001E+02**	1.170E+08	1.410E+08	1.290E+08	1.350E+08	1.370E+08	1.300E+08	1.430E+08	1.360E+08	1.400E+08	1.280E+08
f30	**1.096E+04**	7.470E+05	7.970E+05	8.380E+05	8.130E+05	8.890E+05	8.230E+05	8.410E+05	8.310E+05	7.820E+05	9.280E+05

Table 6.6: Wilcoxon Signed Rank Test Statistical Values for ASw-GSA $_{a}^{fit^*}$

S-GSA vs. ASw-GSA $_{a}^{fit^*}$			A-GSA vs ASw-GSA $_{a}^{fit^*}$		
Δ	R^+	R^-	Δ	R^+	R^-
5%	427	38	5%	32	433
10%	440	25	10%	127	338
15%	442	23	15%	282	183
20%	443	22	20%	315	150
25%	443	22	25%	325	140
30%	443	22	30%	285	180
35%	443	22	35%	291	174
40%	443	22	40%	340	125
45%	443	22	45%	291	174
50%	443	22	50%	366	99
55%	443	22	55%	317	148
60%	443	22	60%	288	177
65%	443	22	65%	218	247
70%	442	23	70%	215	250
75%	443	22	75%	276	189
80%	443	22	80%	244	221
85%	443	22	85%	241	224
90%	443	22	90%	242	223
95%	443	22	95%	253	212

ASw-GSA $_{s}^{fit^*}$ - In this section ASw-GSA $_{s}^{fit^*}$ is investigated, the population starts with synchronous update and fit^* is used as switching indicator. Only results from ASw-GSA $_{s}^{fit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$ are analysed here. This is due to switching does not happen in more than half of the functions for the other value of Δ.

The average error of ASw-GSA $_{s}^{fit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$ are compared with S-GSA and A-GSA in Table 6.7. The best average fitness error values are distributed between S-GSA, A-GSA, and ASw-GSA $_{s}^{fit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$. The ASw-GSA $_{s}^{fit^*}$ with $\Delta = \{5\%\}$, found more number of the best average error value compared to S-GSA and A-GSA for unimodal functions.

The Wilcoxon signed rank test is conducted on the results of the experiment. The statistic value of Wilcoxon signed rank test is shown in Table 6.8. Even though, ASw-GSA $_{s}^{fit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$ are found to performed better than S-GSA, statistically they are on par with S-GSA with the statistic values are bigger than 152. ASw-GSA $_{s}^{fit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$ are significantly better than A-GSA with statistic values of lesser than 109, thus the level of significance is 1%.

Table 6.7: Average Error of ASw-GSA $_{s}^{fit^*}$

Function ID	S-GSA	A-GSA	Δ		
			5%	10%	15%
f1	1.300E+07	7.110E+08	**1.120E+07**	1.220E+07	1.370E+07
f2	8.603E+03	5.940E+10	**8.300E+03**	8.370E+03	8.467E+03
f3	**5.784E+04**	9.770E+04	7.249E+04	7.463E+04	6.160E+04
f4	3.017E+02	1.013E+04	**2.651E+02**	2.824E+02	2.700E+02
f5	**2.000E+01**	2.095E+01	2.010E+01	**2.000E+01**	**2.000E+01**
f6	**1.907E+01**	3.895E+01	1.951E+01	1.979E+01	1.946E+01
f7	**0.000E+00**	5.439E+02	**0.000E+00**	**0.000E+00**	**0.000E+00**
f8	1.405E+02	3.285E+02	**1.380E+02**	1.402E+02	1.385E+02
f9	**1.624E+02**	3.781E+02	1.682E+02	1.628E+02	1.632E+02
f10	3.370E+03	7.018E+03	3.287E+03	3.344E+03	**3.270E+03**
f11	4.058E+03	7.155E+03	4.056E+03	4.000E+03	**3.963E+03**
f12	**4.870E-04**	2.450E+00	6.921E-04	1.005E-03	5.545E-04
f13	3.017E-01	6.146E+00	3.248E-01	3.187E-01	**2.928E-01**
f14	2.433E-01	1.751E+02	2.579E-01	2.420E-01	**2.410E-01**
f15	**3.659E+00**	3.470E+05	3.708E+00	3.791E+00	3.924E+00
f16	1.363E+01	**1.309E+01**	1.325E+01	1.331E+01	1.333E+01
f17	**5.310E+05**	1.840E+07	5.930E+05	5.370E+05	5.500E+05
f18	3.817E+02	9.810E+08	4.665E+02	3.321E+02	**3.231E+02**
f19	1.153E+02	2.924E+02	9.168E+01	9.319E+01	**8.926E+01**
f20	**4.521E+04**	7.100E+04	7.607E+04	8.051E+04	6.247E+04
f21	1.550E+05	4.760E+06	1.670E+05	1.670E+05	**1.510E+05**
f22	9.562E+02	1.300E+03	9.114E+02	**8.926E+02**	8.987E+02
f23	2.130E+02	6.697E+02	**2.000E+02**	2.041E+02	2.043E+02
f24	**2.000E+02**	2.726E+02	2.000E+02	**2.000E+02**	**2.000E+02**
f25	**2.000E+02**	2.249E+02	**2.000E+02**	**2.000E+02**	**2.000E+02**
f26	1.868E+02	**1.064E+02**	1.814E+02	1.783E+02	1.846E+02
f27	1.179E+03	8.293E+02	**8.194E+02**	8.333E+02	1.103E+03
f28	1.257E+03	4.703E+03	1.128E+03	1.419E+03	**1.019E+03**
f29	**2.001E+02**	1.170E+08	2.001E+02	2.001E+02	**2.001E+02**
f30	**1.096E+04**	7.470E+05	1.334E+04	1.191E+04	1.190E+04

Table 6.8: Wilcoxon Signed Rank Test Statistical Values for ASw-GSA $_s^{fit^*}$

S-GSA vs. ASw-GSA $_s^{fit^*}$			A-GSA vs ASw-GSA $_s^{fit^*}$		
Δ	R^+	R^-	Δ	R^+	R^-
5%	225.5	239.5	5%	<u>30</u>	435
10%	216	249	10%	<u>33</u>	432
15%	172	263	15%	<u>22</u>	443

ASw-SKF $_a^{fit^*}$ - The adaptive switching SKF, ASw-SKF $_a^{fit^*}$ that started with asynchronous and used $\boldsymbol{X}_{true}$ as the switching indicator is considered here. It is found that switching rarely happens for ASw-SKF $_a^{fit^*}$. Switching only occurs for five functions; f12, f13, f14, f24, and f26 for several values of Δ. Due to lack of switches, the readings from the experiments conducted here are ignored.

In chapter 4 it was seen that the diversity of A-SKF oscillated and not decreasing smoothly. The ability to preserve diversity allowed improvement of $\boldsymbol{X}_{true}$, thus, preventing switching within ASw-SKF $_a^{fit^*}$.

ASw-SKF $_s^{fit^*}$ - Here SKF with adaptive switching, ASw-SKF $_s^{fit^*}$ that starts with synchronous update is studied. For all values of Δ tested, switching happens in more than 50% of the test functions. Therefore, the results from all the test are taken. For four functions, f1, f17, f20 and f21, no switching happens regardless of the Δ value.

The average fitness error values of ASw-SKF $_s^{fit^*}$ are tabulated in Table 6.9. These values are used for Wilcoxon signed rank test. A-SKF was able to find more number of fitter solution compared to others.

Table 6.9: Average Error of ASw-SKF $_{s}^{fit^*}$

Function ID	S-SKF	A-SKF	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	4.860E+05	1.100E+07	4.940E+05	3.140E+05	2.650E+05	2.700E+05	3.150E+05	3.050E+05	3.680E+05	4.380E+05	3.070E+05	3.000E+05
f2	2.450E+08	1.290E+06	1.840E+07	1.800E+06	2.130E+06	7.690E+06	5.760E+06	**5.260E+05**	6.090E+06	4.170E+06	6.040E+06	4.090E+06
f3	1.841E+04	9.901E+03	1.345E+04	1.347E+04	1.445E+04	1.096E+04	1.750E+04	1.310E+04	1.507E+04	**9.735E+03**	1.205E+04	1.425E+04
f4	3.646E+01	1.177E+02	3.385E+01	2.622E+01	3.948E+01	3.037E+01	3.324E+01	2.741E+01	1.977E+01	**1.934E+01**	4.199E+01	3.177E+01
f5	2.002E+01	2.001E+01	2.001E+01	2.000E+01	2.000E+01	2.001E+01	2.000E+01	2.001E+01	2.001E+01	2.000E+01	**2.000E+01**	2.001E+01
f6	2.195E+01	1.817E+01	**1.742E+01**	1.855E+01	1.813E+01	1.910E+01	1.817E+01	1.859E+01	1.882E+01	1.827E+01	1.863E+01	1.876E+01
f7	1.635E-01	**8.444E-02**	1.456E-01	2.725E-01	2.120E-01	1.426E-01	2.134E-01	1.701E-01	2.008E-01	1.168E-01	2.684E-01	2.133E-01
f8	5.878E+00	5.473E+00	**2.714E+00**	2.863E+00	2.743E+00	3.440E+00	3.763E+00	3.423E+00	4.228E+00	3.000E+00	4.257E+00	3.235E+00
f9	9.087E+01	**7.526E+01**	8.897E+01	8.692E+01	8.942E+01	9.697E+01	9.029E+01	9.004E+01	9.100E+01	8.829E+01	8.765E+01	9.018E+01
f10	2.263E+02	1.620E+02	1.215E+02	1.284E+02	1.017E+02	1.164E+02	9.813E+01	1.216E+02	**8.248E+01**	1.178E+02	1.105E+02	1.560E+02
f11	2.640E+03	2.585E+03	2.807E+03	2.962E+03	2.693E+03	2.991E+03	2.838E+03	2.849E+03	**2.548E+03**	2.608E+03	2.752E+03	2.683E+03
f12	3.592E-01	**2.099E-01**	2.426E-01	2.728E-01	2.803E-01	2.911E-01	2.665E-01	3.019E-01	3.015E-01	2.625E-01	3.283E-01	3.138E-01
f13	4.443E-01	**3.567E-01**	4.664E-01	4.519E-01	4.136E-01	4.237E-01	4.023E-01	4.197E-01	4.128E-01	4.636E-01	4.363E-01	4.664E-01
f14	2.593E-01	**2.273E-01**	2.774E-01	2.590E-01	2.732E-01	2.771E-01	2.565E-01	2.825E-01	2.554E-01	2.610E-01	2.670E-01	2.682E-01
f15	2.192E+01	**1.640E+01**	2.415E+01	2.167E+01	1.923E+01	2.182E+01	2.067E+01	2.087E+01	2.162E+01	2.080E+01	2.081E+01	3.037E+01
f16	1.060E+01	1.067E+01	1.055E+01	1.059E+01	1.066E+01	1.050E+01	1.062E+01	1.079E+01	1.079E+01	1.054E+01	1.057E+01	1.062E+01
f17	1.050E+05	1.170E+06	1.240E+05	1.080E+05	1.110E+05	1.520E+05	9.313E+04	8.178E+04	1.260E+05	1.070E+05	1.310E+05	1.360E+05
f18	1.150E+07	8.560E+06	4.954E+04	6.822E+04	1.790E+05	5.419E+04	**4.931E+03**	2.024E+04	4.535E+04	1.170E+05	1.550E+05	2.046E+04
f19	2.050E+01	1.985E+01	1.816E+01	2.986E+01	2.747E+01	2.101E+01	2.047E+01	2.442E+01	2.410E+01	2.151E+01	2.298E+01	2.571E+01
f20	2.984E+04	**2.415E+04**	3.352E+04	3.256E+04	3.387E+04	3.122E+04	3.130E+04	3.083E+04	3.583E+04	3.253E+04	3.762E+04	3.268E+04
f21	2.610E+05	5.550E+05	1.840E+05	2.590E+05	1.820E+05	1.700E+05	2.010E+05	2.080E+05	**1.430E+05**	3.070E+05	2.150E+05	1.740E+05
f22	6.217E+02	**4.973E+02**	6.412E+02	5.928E+02	5.745E+02	6.347E+02	5.901E+02	5.971E+02	6.284E+02	6.152E+02	6.412E+02	6.154E+02
f23	3.181E+02	**3.161E+02**	3.164E+02	3.167E+02	3.168E+02	3.163E+02	3.166E+02	3.166E+02	3.161E+02	3.161E+02	3.165E+02	3.166E+02
f24	2.310E+02	**2.292E+02**	2.312E+02	2.324E+02	2.327E+02	2.322E+02	2.321E+02	2.308E+02	2.334E+02	2.319E+02	2.307E+02	2.325E+02
f25	2.151E+02	2.143E+02	2.150E+02	2.139E+02	2.139E+02	2.151E+02	2.148E+02	2.143E+02	2.133E+02	2.139E+02	2.140E+02	2.148E+02
f26	1.204E+02	1.204E+02	1.204E+02	1.171E+02	1.237E+02	1.171E+02	1.171E+02	1.104E+02	1.105E+02	1.237E+02	1.104E+02	1.237E+02
f27	5.985E+02	**5.476E+02**	6.574E+02	6.755E+02	6.775E+02	7.114E+02	6.030E+02	7.393E+02	6.269E+02	6.426E+02	6.603E+02	6.577E+02
f28	1.574E+03	1.610E+03	1.654E+03	1.618E+03	1.514E+03	1.653E+03	1.791E+03	1.509E+03	1.575E+03	**1.394E+03**	1.698E+03	1.408E+03
f29	2.477E+03	1.189E+03	1.143E+03	1.236E+03	1.179E+03	1.099E+03	1.218E+03	1.254E+03	1.197E+03	1.136E+03	**1.097E+03**	2.000E+03
f30	5.438E+03	3.848E+03	4.056E+03	3.773E+03	**3.487E+03**	3.699E+03	4.388E+03	3.871E+03	3.876E+03	4.682E+03	3.885E+03	4.521E+03

Table 6.9: Average Error of ASw-SKF $_s^{fit^*}$ (continued...)

Function ID	S-SKF	A-SKF	Δ 55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	4.860E+05	1.100E+07	3.340E+05	2.470E+05	2.840E+05	4.190E+05	3.590E+05	**2.280E+05**	4.220E+05	2.570E+05	2.720E+05
f2	2.450E+08	1.290E+06	1.100E+07	8.000E+06	2.400E+07	7.360E+06	6.790E+07	1.230E+07	1.670E+07	3.410E+07	1.460E+08
f3	1.841E+04	9.901E+03	1.487E+04	1.487E+04	1.244E+04	1.346E+04	1.465E+04	1.269E+04	1.453E+04	1.411E+04	1.739E+04
f4	3.646E+01	1.177E+02	3.044E+01	1.942E+01	2.946E+01	2.178E+01	2.961E+01	3.092E+01	3.314E+01	2.753E+01	2.192E+01
f5	2.002E+01	2.001E+01	2.001E+01	2.000E+01	**2.000E+01**	**2.000E+01**	2.001E+01	2.001E+01	2.001E+01	2.000E+01	2.001E+01
f6	2.195E+01	1.817E+01	1.835E+01	1.745E+01	1.907E+01	1.987E+01	2.377E+01	2.228E+01	2.500E+01	2.539E+01	2.206E+01
f7	1.635E-01	**8.444E-02**	1.107E-01	1.564E-01	3.208E-01	3.424E-01	2.208E+00	1.284E+04	4.261E+04	4.262E+04	5.616E+00
f8	5.878E+00	5.473E+00	2.983E+00	3.196E+00	3.390E+00	3.282E+00	3.099E+00	3.079E+00	3.756E+00	3.427E+00	4.687E+00
f9	9.087E+01	**7.526E+01**	8.580E+01	9.019E+01	8.904E+01	8.666E+01	8.634E+01	8.004E+01	9.165E+01	8.510E+01	9.532E+01
f10	2.263E+02	1.620E+02	1.228E+02	1.343E+02	1.276E+02	1.218E+02	1.482E+02	1.460E+02	1.384E+02	1.244E+02	1.856E+02
f11	2.640E+03	2.585E+03	2.757E+03	2.682E+03	2.641E+03	2.660E+03	2.721E+03	2.642E+03	2.709E+03	2.585E+03	2.912E+03
f12	3.592E-01	**2.099E-01**	2.870E-01	2.579E-01	2.796E-01	2.889E-01	2.789E-01	2.721E-01	2.782E-01	2.809E-01	3.094E-01
f13	4.443E-01	**3.567E-01**	4.483E-01	4.287E-01	4.408E-01	4.235E-01	4.604E-01	4.259E-01	4.509E-01	4.089E-01	4.538E-01
f14	2.593E-01	**2.273E-01**	2.697E-01	2.879E-01	2.927E-01	2.690E-01	2.733E-01	2.613E-01	2.735E-01	2.733E-01	2.675E-01
f15	2.192E+01	**1.640E+01**	2.295E+01	1.415E+02	2.055E+01	2.277E+01	2.153E+01	2.330E+01	2.314E+01	8.733E+01	2.130E+01
f16	1.060E+01	1.067E+01	1.090E+01	1.072E+01	1.048E+01	1.070E+01	1.075E+01	1.050E+01	1.047E+01	**1.036E+01**	1.066E+01
f17	1.050E+05	1.170E+06	1.400E+05	1.100E+05	**8.077E+04**	1.490E+05	1.320E+05	1.280E+05	1.060E+05	1.230E+05	1.220E+05
f18	1.150E+07	8.560E+06	3.881E+04	8.202E+03	1.830E+05	1.110E+05	7.730E+05	9.106E+04	1.400E+06	1.350E+05	1.050E+06
f19	2.050E+01	1.985E+01	**1.733E+01**	2.021E+01	2.373E+01	2.295E+01	2.653E+01	2.405E+01	2.296E+01	2.776E+01	2.575E+01
f20	2.984E+04	**2.415E+04**	3.605E+04	3.413E+04	3.656E+04	3.082E+04	3.969E+04	3.456E+04	3.441E+04	3.306E+04	3.287E+04
f21	2.610E+05	5.550E+05	2.050E+05	1.650E+05	2.860E+05	2.420E+05	1.880E+05	2.900E+05	2.220E+05	2.000E+05	2.000E+05
f22	6.217E+02	**4.973E+02**	5.737E+02	6.108E+02	6.115E+02	6.361E+02	6.011E+02	5.802E+02	5.774E+02	6.197E+02	5.781E+02
f23	3.181E+02	**3.161E+02**	3.167E+02	3.162E+02	3.169E+02	3.175E+02	3.163E+02	3.172E+02	3.169E+02	3.169E+02	3.171E+02
f24	2.310E+02	**2.292E+02**	2.324E+02	2.321E+02	2.309E+02	2.314E+02	2.321E+02	2.335E+02	2.312E+02	2.335E+02	2.326E+02
f25	2.151E+02	2.143E+02	2.140E+02	2.145E+02	2.144E+02	2.138E+02	2.145E+02	2.137E+02	**2.129E+02**	2.153E+02	2.146E+02
f26	1.204E+02	1.204E+02	1.171E+02	1.104E+02	1.270E+02	1.237E+02	1.304E+02	1.238E+02	1.171E+02	**1.071E+02**	1.237E+02
f27	5.985E+02	**5.476E+02**	7.049E+02	6.777E+02	6.694E+02	7.208E+02	6.250E+02	6.602E+02	6.759E+02	6.489E+02	6.496E+02
f28	1.574E+03	1.610E+03	1.586E+03	1.614E+03	1.557E+03	1.526E+03	1.574E+03	1.689E+03	1.523E+03	1.591E+03	1.568E+03
f29	2.477E+03	1.189E+03	1.760E+03	1.240E+03	1.259E+03	1.110E+03	1.212E+03	1.199E+03	1.230E+03	1.685E+03	1.860E+03
f30	5.438E+03	3.848E+03	3.882E+03	3.760E+03	3.922E+03	4.782E+03	4.019E+03	5.687E+03	3.847E+03	5.430E+03	4.409E+03

The statistical values of Wilcoxon signed rank test are shown in Table 6.10. ASw-SKF $_{s}^{fit^*}$ with $\Delta = \{10\%, 15\%, 25\%, 30\%, 35\%, 40\%, 55\%, 60\%, 65\%\}$ are significantly better than the original SKF, S-SKF with level of significance at least 10%. On the other hand, comparison of ASw-SKF $_{s}^{fit^*}$ with A-SKF's performance shows that the adaptive switching and asynchronous iteration strategy are statistically on par with each other.

Table 6.10: Wilcoxon Signed Rank Test Statistical Values for ASw-SKF $_{s}^{fit^*}$

S-SKF vs. ASw-SKF $_{s}^{fit^*}$			A-SKF vs ASw-SKF $_{s}^{fit^*}$		
Δ	R^+	R^-	Δ	R^+	R^-
5%	180	285	5%	260	205
10%	<u>141</u>	324	10%	261	204
15%	<u>151</u>	314	15%	234	231
20%	162	303	20%	247	218
25%	<u>101</u>	364	25%	284	181
30%	<u>87</u>	378	30%	242	223
35%	<u>140</u>	325	35%	238	227
40%	<u>128</u>	337	40%	224	241
45%	160	305	45%	262	203
50%	156	309	50%	287	178
55%	<u>135</u>	330	55%	257	208
60%	<u>146</u>	319	60%	263	202
65%	<u>124</u>	341	65%	283	182
70%	159	306	70%	262	203
75%	173	292	75%	291	174
80%	223	242	80%	299	166
85%	161	304	85%	258	207
90%	168	297	90%	270	195
95%	173	292	95%	310	155

6.4.2.2 D^p as the Switching Indicator

ASw-PSO$_a^{D^p}$ ***and*** **ASw-PSO$_s^{D^p}$** – No switching is observed for adaptive switching PSOs that adopt D^p as the switching indicator. This condition is too rigid; a slight movement of the particles change D^p and prevents the particles to switch their iteration strategy.

ASw-GSA$_a^{D^p}$ - No switching is observed for ASw-GSA$_a^{D^p}$, which is an adaptive switching GSA that starts with asynchronous update and D^p as the switching indicator. This is expected. Based on the observation in chapter 4, the position diversity of A-GSA kept oscillating throughout the search, $D^p(t+1) \neq D^p(t)$. Therefore, the switching counter, δ, is not incremented and $\delta < \Delta$.

ASw-GSA$_s^{D^p}$ - Switching occurs in more than 50% of the test functions for ASw-GSA$_s^{D^p}$ with $\Delta = \{5\%\}$. The results for the test using $\Delta = \{5\%\}$ is taken and compared with S-GSA and A-GSA and presented in Table 6.11. ASw-GSA$_s^{D^p}$ found the smallest error value for 15 functions, where it performed the best for all unimodal functions.

The Wilcoxon signed rank test is conducted based on the results in Table 6.11. The statistical value from the test is tabulated in Table 6.12. Statistically ASw-GSA$_s^{D^p}$ with $\Delta = \{5\%\}$ is on par with S-GSA, with statistical value of 220 which is bigger than 152. The statistic value for comparison of ASw-GSA$_s^{D^p}$ with A-GSA is 22, thus, it is significantly better than A-GSA with level of significance of 1%.

Table 6.11: Average Error of ASw-GSA$_s^{D^p}$

Function ID	S-GSA	A-GSA	Δ
			5%
f1	1.300E+07	7.110E+08	**1.090E+07**
f2	8.603E+03	5.940E+10	**8.538E+03**
f3	5.784E+04	9.770E+04	**5.585E+04**
f4	**3.017E+02**	1.013E+04	3.353E+02
f5	**2.000E+01**	2.095E+01	**2.000E+01**
f6	1.907E+01	3.895E+01	**1.905E+01**
f7	**0.000E+00**	5.439E+02	**0.000E+00**
f8	**1.405E+02**	3.285E+02	1.430E+02
f9	**1.624E+02**	3.781E+02	1.637E+02
f10	**3.370E+03**	7.018E+03	3.389E+03
f11	**4.058E+03**	7.155E+03	4.111E+03
f12	**4.870E-04**	2.450E+00	8.648E-04
f13	**3.017E-01**	6.146E+00	3.031E-01
f14	2.433E-01	1.751E+02	**2.423E-01**
f15	3.659E+00	3.470E+05	**3.642E+00**
f16	1.363E+01	**1.309E+01**	1.356E+01
f17	**5.310E+05**	1.840E+07	5.500E+05
f18	3.817E+02	9.810E+08	**3.716E+02**
f19	1.153E+02	2.924E+02	**1.118E+02**
f20	**4.521E+04**	7.100E+04	4.612E+04
f21	**1.550E+05**	4.760E+06	1.640E+05
f22	9.562E+02	1.300E+03	**8.821E+02**
f23	2.130E+02	6.697E+02	**2.041E+02**
f24	**2.000E+02**	2.726E+02	**2.000E+02**
f25	**2.000E+02**	2.249E+02	**2.000E+02**
f26	1.868E+02	**1.064E+02**	1.814E+02
f27	1.179E+03	**8.293E+02**	1.162E+03
f28	1.257E+03	4.703E+03	**1.225E+03**
f29	**2.001E+02**	1.170E+08	2.001E+02
f30	**1.096E+04**	7.470E+05	1.208E+04

Table 6.12: Wilcoxon Signed Rank Test Statistical Values for ASw-GSA$_s^{D^p}$

S-GSA vs. ASw-GSA$_s^{D^p}$			A-GSA vs ASw-GSA$_s^{D^p}$		
Δ	R^+	R^-	Δ	R^+	R^-
5%	220	245	5%	<u>22</u>	443

ASw-SKF$_a^{D^p}$ - In ASw-SKF$_a^{D^p}$, diversity is used as the switching indicator and asynchronous update is used as the initial iteration strategy. The average number of switching shows that ASw-SKF$_a^{D^p}$ is able to switch, but, in small number of functions with a very low probability. Based on A-SKF's diversity, this is predictable. The diversity of asynchronously update SKF reduced, but, it oscillated around a small value till the end of the search. Thus, the result of ASw-SKF$_a^{D^p}$ is ignored.

ASw-SKF$_s^{D^p}$ - ASw-SKF$_s^{D^p}$ starts with synchronous update and uses diversity as its switching condition. Unlike the version that starts with asynchronous update, switching is observed in more than half of the test function for all values of Δ tested.

The average error value of the ASw-SKF$_s^{D^p}$ is compared with S-SKF and A-SKF in Table 6.13. The minimum error found for each test function are highlighted with **boldface**. Asynchronous update is observed to perform the best among the iteration strategy tested.

Further analysis is performed using pairwise comparison using Wilcoxon signed rank test. The statistical value of Wilcoxon signed rank test is shown in Table 6.14. The test shows that the ASw-SKF$_s^{D^p}$ with all value of Δ are significantly on par with A-SKF. On the other hand, ASw-SKF$_s^{D^p}$ with $\Delta = \{5\%, 20\%, 25\%, 35\%, 45\%, 50\%, 55\%, 60\%, 75\%, 80\%, 85\%, 95\%\}$ are significantly better than S-SKF with significance level of at least 10% while other setting of Δ are giving performances that are on par with S-SKF.

Table 6.13: Average Error of ASw-$\boldsymbol{SKF_s^{Dp}}$

Function ID	S-SKF	A-SKF	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	4.860E+05	1.100E+07	2.730E+05	2.720E+05	2.670E+05	4.570E+05	4.760E+05	5.690E+05	4.670E+05	2.400E+05	1.860E+05	4.870E+05
f2	2.450E+08	**1.290E+06**	5.010E+06	1.110E+07	2.100E+06	1.860E+07	6.260E+06	3.200E+06	2.750E+06	2.370E+06	9.290E+06	7.760E+06
f3	1.841E+04	**9.901E+03**	1.510E+04	1.075E+04	1.453E+04	1.118E+04	1.181E+04	1.275E+04	1.438E+04	1.550E+04	1.608E+04	1.323E+04
f4	3.646E+01	1.177E+02	2.664E+01	2.887E+01	3.285E+01	2.587E+01	2.371E+01	3.683E+01	4.028E+01	**1.510E+01**	2.281E+01	3.772E+01
f5	2.002E+01	2.001E+01	2.000E+01	**2.000E+01**	2.000E+01	2.000E+01	**2.000E+01**	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01
f6	2.195E+01	1.817E+01	1.964E+01	5.261E+01	5.274E+01	5.257E+01	5.275E+01	5.268E+01	6.207E+01	6.207E+01	2.086E+01	**1.772E+01**
f7	1.635E-01	**8.444E-02**	2.346E-01	4.263E+04	1.400E-01	1.092E-01	9.136E-02	9.137E-02	1.450E+00	1.432E+00	1.778E+00	1.866E-01
f8	5.878E+00	5.473E+00	2.847E+00	3.285E+00	3.589E+00	3.331E+00	**2.552E+00**	3.023E+00	3.356E+00	3.405E+00	3.082E+00	2.668E+00
f9	9.087E+01	**7.526E+01**	8.258E+01	8.545E+01	9.382E+01	9.015E+01	8.860E+01	8.900E+01	8.419E+01	9.496E+01	8.606E+01	8.598E+01
f10	2.263E+02	1.620E+02	1.317E+02	1.370E+02	1.180E+02	**1.015E+02**	1.414E+02	1.160E+02	1.299E+02	1.477E+02	1.512E+02	1.385E+02
f11	2.640E+03	2.585E+03	**2.477E+03**	2.763E+03	2.700E+03	2.795E+03	2.871E+03	3.079E+03	2.616E+03	2.809E+03	2.544E+03	2.565E+03
f12	3.592E-01	**2.099E-01**	2.814E-01	2.982E-01	2.720E-01	2.969E-01	3.102E-01	3.126E-01	2.913E-01	2.546E-01	2.858E-01	2.706E-01
f13	4.443E-01	**3.567E-01**	4.222E-01	4.670E-01	4.061E-01	4.249E-01	4.227E-01	4.440E-01	4.435E-01	4.651E-01	4.323E-01	4.385E-01
f14	2.593E-01	**2.273E-01**	2.655E-01	2.928E-01	2.719E-01	2.652E-01	2.683E-01	2.707E-01	2.794E-01	2.738E-01	2.471E-01	2.645E-01
f15	2.192E+01	**1.640E+01**	2.452E+01	3.964E+01	2.351E+01	1.960E+01	2.181E+01	3.888E+01	8.631E+01	8.489E+01	2.050E+01	2.094E+01
f16	1.060E+01	1.067E+01	1.049E+01	1.049E+01	1.060E+01	1.065E+01	1.068E+01	1.072E+01	1.050E+01	1.083E+01	1.061E+01	1.061E+01
f17	1.050E+05	1.170E+06	1.160E+05	1.270E+05	7.553E+04	1.050E+05	1.380E+05	1.120E+05	1.110E+05	8.198E+04	1.220E+05	9.123E+04
f18	1.150E+07	8.560E+06	1.180E+06	2.877E+04	8.304E+04	4.719E+04	3.598E+03	1.070E+05	8.754E+04	6.177E+04	6.634E+04	9.837E+03
f19	2.050E+01	1.985E+01	1.980E+01	2.097E+01	1.891E+01	2.756E+01	2.182E+01	1.633E+01	2.414E+01	1.702E+01	2.402E+01	**1.443E+01**
f20	2.984E+04	**2.415E+04**	4.005E+04	3.429E+04	3.562E+04	2.832E+04	3.494E+04	3.793E+04	3.363E+04	3.549E+04	2.782E+04	3.805E+04
f21	2.610E+05	5.550E+05	1.580E+05	2.050E+05	1.600E+05	2.330E+05	1.480E+05	1.620E+05	1.960E+05	2.670E+05	1.680E+05	2.210E+05
f22	6.217E+02	**4.973E+02**	5.652E+02	6.111E+02	6.338E+02	6.303E+02	5.569E+02	5.905E+02	5.778E+02	6.045E+02	5.583E+02	5.436E+02
f23	3.181E+02	**3.161E+02**	3.167E+02	3.163E+02	3.166E+02	3.169E+02	3.165E+02	3.166E+02	3.171E+02	3.169E+02	3.165E+02	3.166E+02
f24	2.310E+02	**2.292E+02**	2.325E+02	2.303E+02	2.311E+02	2.331E+02	2.299E+02	2.327E+02	2.310E+02	2.325E+02	2.313E+02	2.314E+02
f25	2.151E+02	2.143E+02	2.128E+02	2.156E+02	2.146E+02	2.147E+02	2.149E+02	2.152E+02	2.145E+02	2.129E+02	2.133E+02	**2.123E+02**
f26	1.204E+02	1.204E+02	**1.071E+02**	1.138E+02	1.337E+02	1.171E+02	1.138E+02	1.105E+02	1.104E+02	1.237E+02	1.171E+02	1.072E+02
f27	5.985E+02	**5.476E+02**	6.730E+02	6.687E+02	7.126E+02	6.211E+02	6.854E+02	6.799E+02	5.926E+02	6.990E+02	6.806E+02	6.781E+02
f28	1.574E+03	1.610E+03	1.656E+03	1.592E+03	1.588E+03	1.536E+03	1.660E+03	1.711E+03	1.556E+03	1.573E+03	1.510E+03	1.765E+03
f29	2.477E+03	1.189E+03	1.164E+03	1.122E+03	1.223E+03	1.240E+03	1.116E+03	1.168E+03	**1.057E+03**	1.254E+03	1.310E+03	1.117E+03
f30	5.438E+03	3.848E+03	4.500E+03	**3.721E+03**	4.270E+03	4.382E+03	4.069E+03	3.955E+03	3.879E+03	3.819E+03	4.225E+03	4.043E+03

Table 6.13: Average Error of ASw-SKF_s^{Dp} (continued...)

Function ID	S-SKF	A-SKF	Δ								
			55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	4.860E+05	1.100E+07	3.250E+05	1.860E+05	2.840E+05	5.600E+05	2.840E+05	3.250E+05	4.830E+05	2.890E+05	**1.790E+05**
f2	2.450E+08	**1.290E+06**	6.760E+06	4.660E+07	6.740E+06	8.680E+06	4.710E+06	1.400E+07	3.030E+07	6.250E+07	8.270E+07
f3	1.841E+04	**9.901E+03**	1.425E+04	1.470E+04	1.217E+04	1.212E+04	1.423E+04	1.531E+04	1.382E+04	1.647E+04	1.506E+04
f4	3.646E+01	1.177E+02	3.989E+01	2.089E+01	2.732E+01	2.562E+01	3.075E+01	1.962E+01	2.260E+01	3.133E+01	2.446E+01
f5	2.002E+01	2.001E+01	2.000E+01	2.000E+01	**2.000E+01**	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.002E+01
f6	2.195E+01	1.817E+01	1.866E+01	1.817E+01	1.838E+01	1.828E+01	1.924E+01	1.798E+01	1.788E+01	1.807E+01	1.878E+01
f7	1.635E-01	**8.444E-02**	1.876E-01	1.297E-01	2.413E-01	2.549E-01	1.713E-01	1.775E-01	2.333E-01	3.232E-01	2.243E-01
f8	5.878E+00	5.473E+00	4.063E+00	3.528E+00	3.266E+00	3.866E+00	3.169E+00	3.194E+00	4.626E+00	3.182E+00	5.027E+00
f9	9.087E+01	**7.526E+01**	8.669E+01	8.090E+01	8.722E+01	8.206E+01	8.669E+01	8.094E+01	8.650E+01	9.482E+01	9.031E+01
f10	2.263E+02	1.620E+02	1.190E+02	1.441E+02	1.074E+02	1.169E+02	1.498E+02	1.158E+02	1.544E+02	1.456E+02	1.616E+02
f11	2.640E+03	2.585E+03	2.731E+03	2.785E+03	2.763E+03	2.848E+03	2.710E+03	2.738E+03	2.849E+03	2.944E+03	2.734E+03
f12	3.592E-01	**2.099E-01**	3.202E-01	2.816E-01	3.263E-01	2.643E-01	2.867E-01	2.671E-01	2.752E-01	3.272E-01	3.196E-01
f13	4.443E-01	**3.567E-01**	4.230E-01	4.581E-01	4.211E-01	4.506E-01	4.350E-01	3.950E-01	4.297E-01	4.218E-01	4.402E-01
f14	2.593E-01	**2.273E-01**	2.663E-01	2.715E-01	2.781E-01	2.890E-01	2.845E-01	2.576E-01	2.768E-01	2.745E-01	2.608E-01
f15	2.192E+01	**1.640E+01**	2.406E+01	2.158E+01	2.504E+01	2.337E+01	5.158E+02	2.229E+01	2.270E+01	2.205E+01	2.135E+01
f16	1.060E+01	1.067E+01	1.075E+01	1.080E+01	1.065E+01	1.072E+01	1.079E+01	1.054E+01	**1.046E+01**	1.070E+01	1.071E+01
f17	1.050E+05	1.170E+06	**6.405E+04**	9.196E+04	1.530E+05	1.230E+05	1.410E+05	1.340E+05	8.374E+04	1.030E+05	1.310E+05
f18	1.150E+07	8.560E+06	1.150E+05	3.487E+04	1.907E+04	1.000E+05	**2.968E+03**	6.086E+04	4.660E+05	6.850E+04	3.470E+06
f19	2.050E+01	1.985E+01	2.059E+01	2.902E+01	2.185E+01	1.664E+01	2.987E+01	2.696E+01	2.658E+01	2.500E+01	1.763E+01
f20	2.984E+04	**2.415E+04**	3.179E+04	3.601E+04	3.009E+04	3.233E+04	3.709E+04	3.719E+04	3.222E+04	3.286E+04	3.793E+04
f21	2.610E+05	5.550E+05	1.610E+05	2.040E+05	2.210E+05	1.970E+05	1.680E+05	1.950E+05	2.270E+05	1.930E+05	**1.450E+05**
f22	6.217E+02	**4.973E+02**	6.239E+02	6.342E+02	6.787E+02	6.481E+02	5.554E+02	6.076E+02	6.275E+02	6.272E+02	6.077E+02
f23	3.181E+02	**3.161E+02**	3.163E+02	3.164E+02	3.164E+02	3.162E+02	3.168E+02	3.173E+02	3.169E+02	3.172E+02	3.169E+02
f24	2.310E+02	**2.292E+02**	2.320E+02	2.306E+02	2.319E+02	2.319E+02	2.318E+02	2.315E+02	2.298E+02	2.317E+02	2.330E+02
f25	2.151E+02	2.143E+02	2.159E+02	2.142E+02	2.141E+02	2.139E+02	2.164E+02	2.134E+02	2.145E+02	2.131E+02	2.141E+02
f26	1.204E+02	1.204E+02	1.105E+02	1.204E+02	1.105E+02	1.237E+02	1.107E+02	1.170E+02	1.171E+02	1.271E+02	1.237E+02
f27	5.985E+02	**5.476E+02**	6.296E+02	7.278E+02	6.470E+02	5.923E+02	6.889E+02	6.207E+02	6.953E+02	7.433E+02	5.940E+02
f28	1.574E+03	1.610E+03	1.521E+03	**1.488E+03**	1.586E+03	1.531E+03	1.524E+03	1.591E+03	1.572E+03	1.686E+03	1.545E+03
f29	2.477E+03	1.189E+03	1.167E+03	1.508E+03	1.221E+03	1.534E+03	1.237E+03	1.479E+03	1.967E+03	2.786E+03	1.302E+03
f30	5.438E+03	3.848E+03	4.333E+03	4.081E+03	4.000E+03	4.590E+03	4.783E+03	4.784E+03	5.979E+03	5.092E+03	5.657E+03

Table 6.14: Wilcoxon Signed Rank Test Statistical Values for ASw-SKF$_s^{Dp}$

S-SKF vs. ASw-SKF$_s^{Dp}$			S-SKF vs. ASw-SKF$_s^{Dp}$		
Δ	R+	R−	Δ	R+	R−
5%	114	351	5%	232	233
10%	178	287	10%	243	222
15%	163	302	15%	274	191
20%	100	335	20%	276	189
25%	144	321	25%	273	192
30%	202	263	30%	272	193
35%	126	339	35%	245	220
40%	180	285	40%	260	205
45%	77	388	45%	248	217
50%	113	352	50%	222	243
55%	137	328	55%	257	208
60%	100	335	60%	283.5	181.5
65%	165	300	65%	265	200
70%	159	306	70%	267	198
75%	150	315	75%	278	187
80%	139	326	80%	259	206
85%	136	329	85%	266	199
90%	196	269	90%	297	168
95%	135	330	95%	282	183

6.4.2.3 Multiple Comparisons Among Algorithms

The best adaptive switching setting for each parent algorithms are selected here for the Friedman and Holm test. The selection is carried based on the findings of Wilcoxon test, where the setting that contributes to the most improvement with respect to the implementation of the parent algorithms in both synchronous and asynchronous strategy is chosen. For PSO, ASw-PSO$_a^{fit^*}$ with Δ= 5% is chosen, while ASw-GSA$_s^{fit^*}$ with Δ= 15% and ASw-SKF$_s^{Dp}$ with Δ= 45% are chosen for GSA and SKF respectively.

From Table 6.15 it can be seen that ASw-PSO$_a^{fit^*}$ is ranked the best. The statistics of Holm posthoc procedure with significance level of 5% is tabulated in Table 6.16. The statistics show that ASw-PSO$_a^{fit^*}$ is statistically on par with other algorithms and significantly better than A-GSA. Additionally, Holm procedure also shows that ASw-GSA$_s^{fit^*}$ is significantly better than A-GSA.

Table 6.15: Average Rankings of Friedman Test for Adaptive Switching

Algorithm	Ranking
Asw-PSO$_a^{fit*}$	3.7667
A-PSO	3.9667
S-PSO	4.3333
A-SKF	4.3333
ASw-SKF$_s^{Dp}$	4.6333
S-GSA	4.9
Asw-GSA$_s^{fit*}$	5.0667
S-SKF	5.3667
A-GSA	8.6333
p-value: 8.13×10^{-11}	

Table 6.16: Statistics of Holm Test for Adaptive Switching

i	algorithms	$z = (R_0-R_i)/SE$	p	Holm
36	Asw-PSO$_a^{fit*}$ vs. A-GSA	6.882506	0	0.001389
35	A-PSO vs. A-GSA	6.599663	0	0.001429
34	A-GSA vs. A-SKF	6.081118	0	0.001471
33	S-PSO vs. A-GSA	6.081118	0	0.001515
32	A-GSA vs. ASw-SKF$_s^{Dp}$	5.656854	0	0.001563
31	S-GSA vs. A-GSA	5.279731	0	0.001613
30	A-GSA vs. Asw-GSA$_s^{fit*}$	5.044028	0	0.001667
29	A-GSA vs. S-SKF	4.619764	0.000004	0.001724
28	Asw-PSO$_a^{fit*}$ vs. S-SKF	2.262742	0.023652	0.001786
27	A-PSO vs. S-SKF	1.979899	0.047715	0.001852
26	Asw-PSO$_a^{fit*}$ vs. Asw-GSA$_s^{fit*}$	1.838478	0.065992	0.001923
25	Asw-PSO$_a^{fit*}$ vs. S-GSA	1.602775	0.108984	0.002
24	A-PSO vs. Asw-GSA$_s^{fit*}$	1.555635	0.119795	0.002083
23	S-SKF vs. A-SKF	1.461354	0.143918	0.002174
22	S-PSO vs. S-SKF	1.461354	0.143918	0.002273
21	A-PSO vs. S-GSA	1.319933	0.186858	0.002381
20	Asw-PSO$_a^{fit*}$ vs. ASw-SKF$_s^{Dp}$	1.225652	0.22033	0.0025
19	Asw-GSA$_s^{fit*}$ vs. A-SKF	1.03709	0.299694	0.002632
18	S-PSO vs. Asw-GSA$_s^{fit*}$	1.03709	0.299694	0.002778
17	S-SKF vs. ASw-SKF$_s^{Dp}$	1.03709	0.299694	0.002941
16	A-PSO vs. ASw-SKF$_s^{Dp}$	0.942809	0.345779	0.003125
15	S-PSO vs. Asw-PSO$_a^{fit*}$	0.801388	0.422907	0.003333
14	S-GSA vs. A-SKF	0.801388	0.422907	0.003571
13	S-PSO vs. S-GSA	0.801388	0.422907	0.003846
12	Asw-PSO$_a^{fit*}$ vs. A-SKF	0.801388	0.422907	0.004167
11	S-GSA vs. S-SKF	0.659966	0.509275	0.004545
10	Asw-GSA$_s^{fit*}$ vs. ASw-SKF$_s^{Dp}$	0.612826	0.539991	0.005
9	S-PSO vs. A-PSO	0.518545	0.604078	0.005556
8	A-PSO vs. A-SKF	0.518545	0.604078	0.00625
7	A-SKF vs. ASw-SKF$_s^{Dp}$	0.424264	0.671373	0.007143
6	S-PSO vs. ASw-SKF$_s^{Dp}$	0.424264	0.671373	0.008333
5	Asw-GSA$_s^{fit*}$ vs. S-SKF	0.424264	0.671373	0.01
4	S-GSA vs. ASw-SKF$_s^{Dp}$	0.377124	0.706082	0.0125
3	A-PSO vs. Asw-PSO$_a^{fit*}$	0.282843	0.777297	0.016667
2	S-GSA vs. Asw-GSA$_s^{fit*}$	0.235702	0.813664	0.025
1	S-PSO vs. A-SKF	0	1	0.05

6.4.3 Fitness Error and Population's Diversity

The results of ASw-PSO $_{a}^{fit^*}$ with Δ= {5%}, ASw-GSA $_{s}^{fit^*}$ with Δ= {15%}, and ASw-SKF$_{s}^{Dp}$ with Δ= {45%} are analysed here.

6.4.3.1 Adaptive Switching PSO

The distribution of fitness error of ASw-PSO $_{a}^{fit^*}$ with Δ= {5%} is compared with S-PSO and A-PSO in the boxplots of Figure 6.8 to Figure 6.11. The boxplots are at almost the same level. In some functions, like f4, f18, f20, and f30 the spread of the box for ASw-PSO $_{a}^{fit^*}$ is smaller than S-PSO and A-PSO, while S-PSO and A-PSO have smaller in other functions.

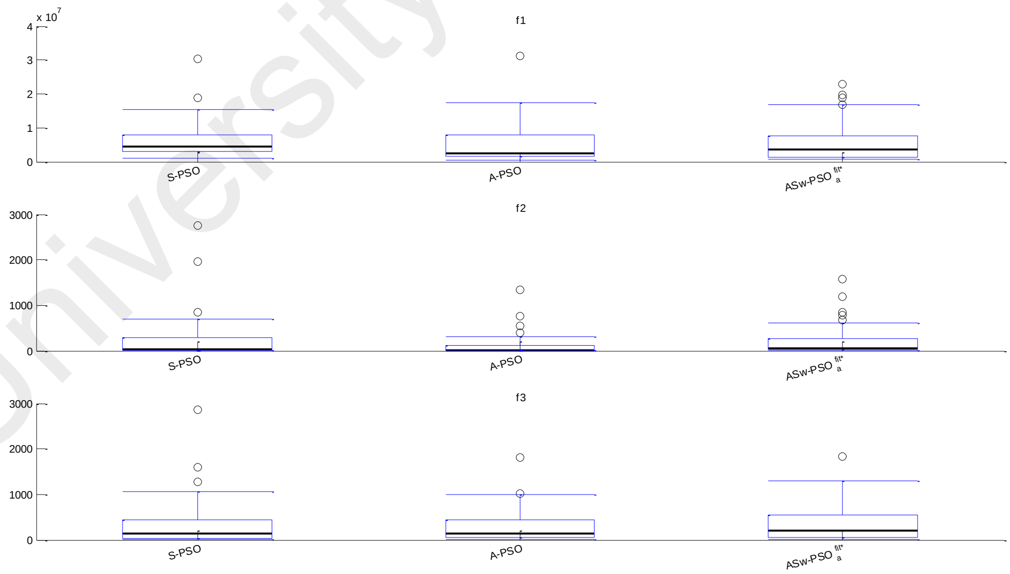

Figure 6.8: Fitness Error Distribution of Unimodal Functions for ASw-PSO $_{a}^{fit^*}$ with Δ= {5%}

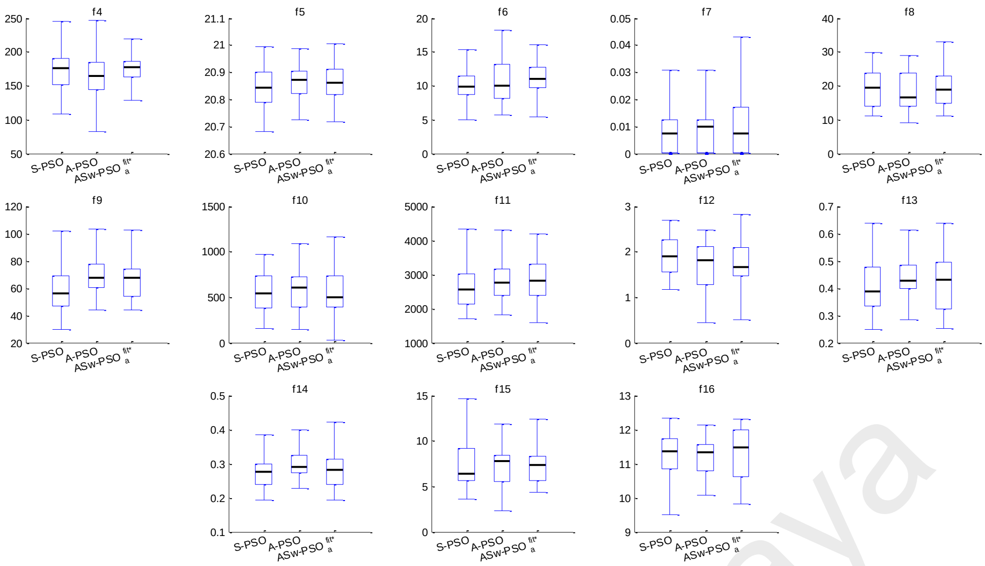

Figure 6.9: Fitness Error Distribution of Simple Multimodal Functions for ASw-PSO $_a^{fit^*}$ with Δ= {5%}

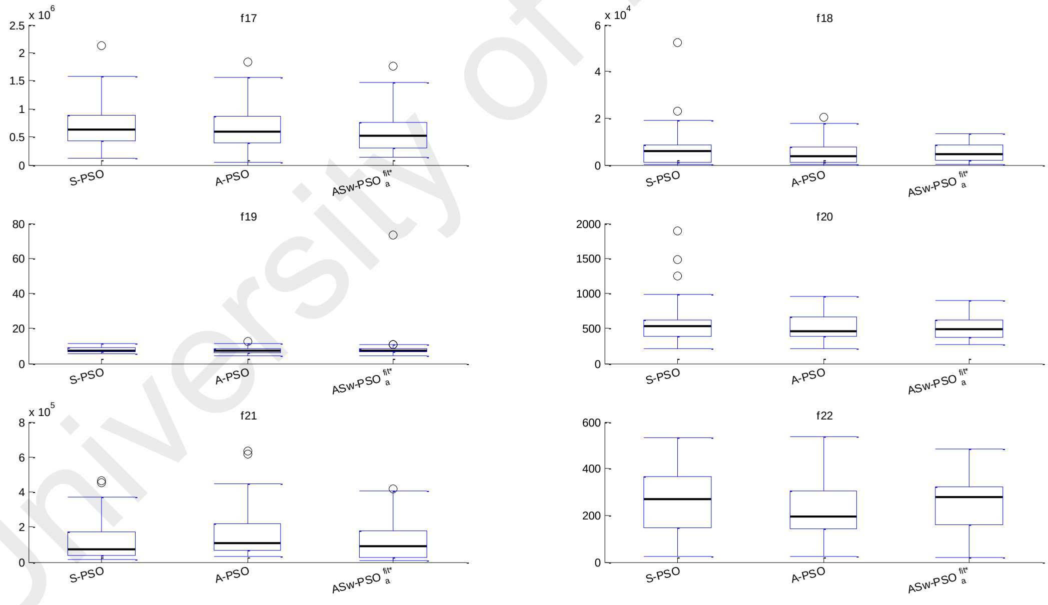

Figure 6.10: Fitness Error Distribution of Hybrid Functions for ASw-PSO $_a^{fit^*}$ with Δ= {5%}

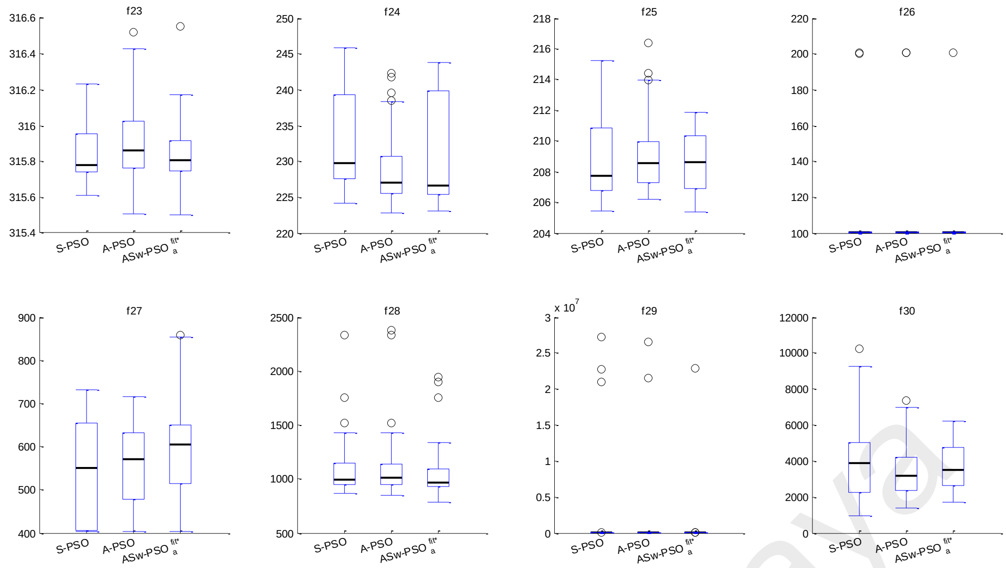

Figure 6.11: Fitness Error Distribution of Composite Functions for ASw-PSO $_{a}^{fit^*}$ with Δ= {5%}

Figure 6.12 shows the graphs of fitness error value over iteration for selected functions. The adaptive switching does not alter the particles' behaviour. Like S-PSO and A-PSO, the fitness errors of ASw-PSO $_{a}^{fit^*}$ decrease with iteration.

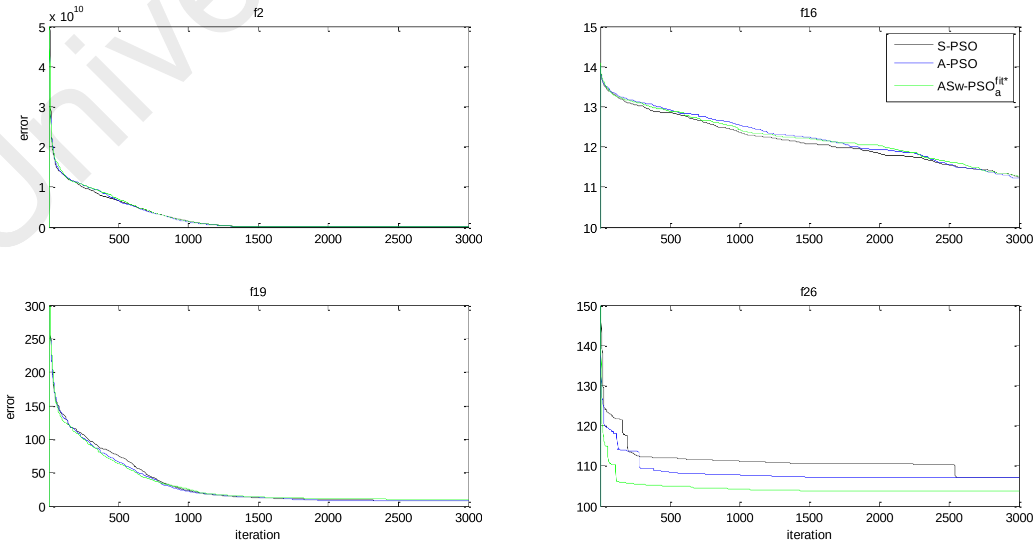

Figure 6.12: Fitness Error Rate of ASw-PSO $_{a}^{fit^*}$ with Δ= {5%}

The average position diversity of the population throughout the search are shown in Figure 6.13 to Figure 6.16. The position diversity of the population that adopts adaptive switching iteration strategy reduces at the same rate as synchronously update and asynchronously update populations.

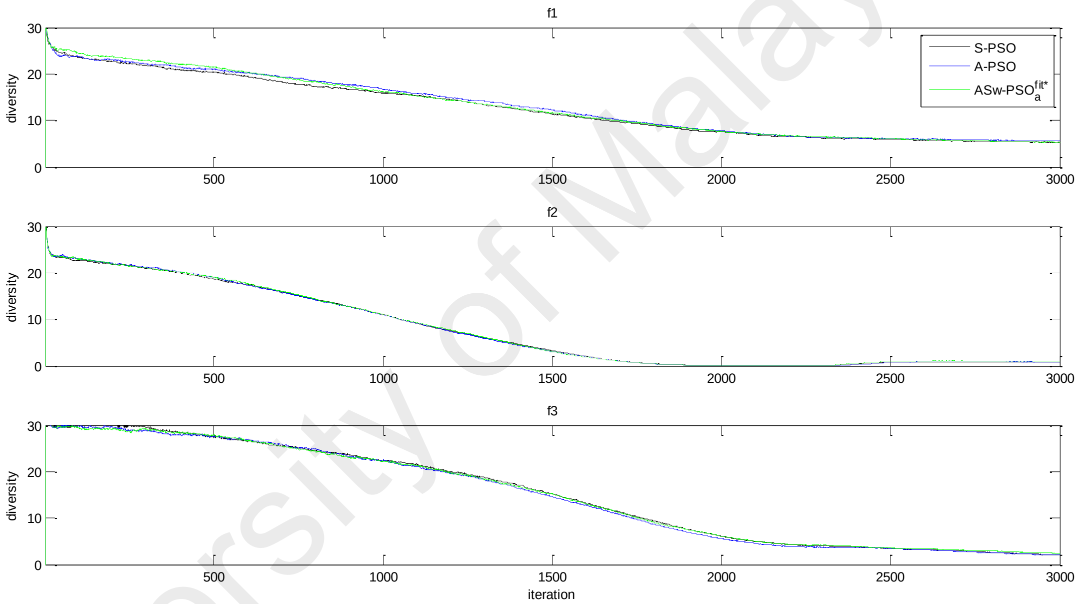

Figure 6.13: Rate of Position Diversity of Unimodal Functions for ASw-PSO $_a^{fit^*}$ with $\Delta = \{5\%\}$

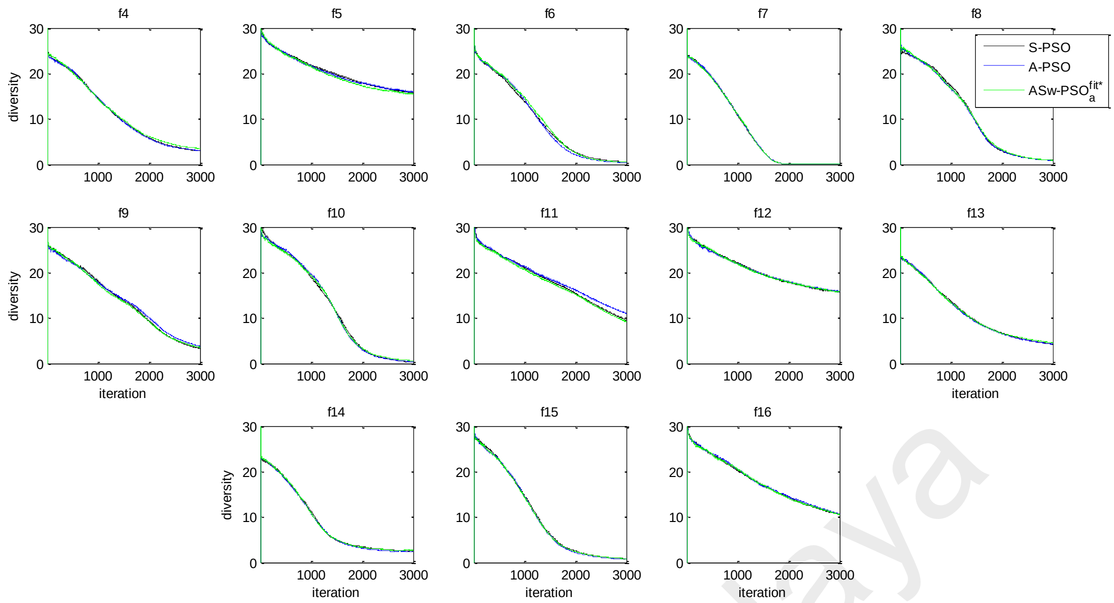

Figure 6.14: Rate of Position Diversity of Simple Multimodal Functions for ASw-PSO $_a^{fit^*}$ with $\Delta = \{5\%\}$

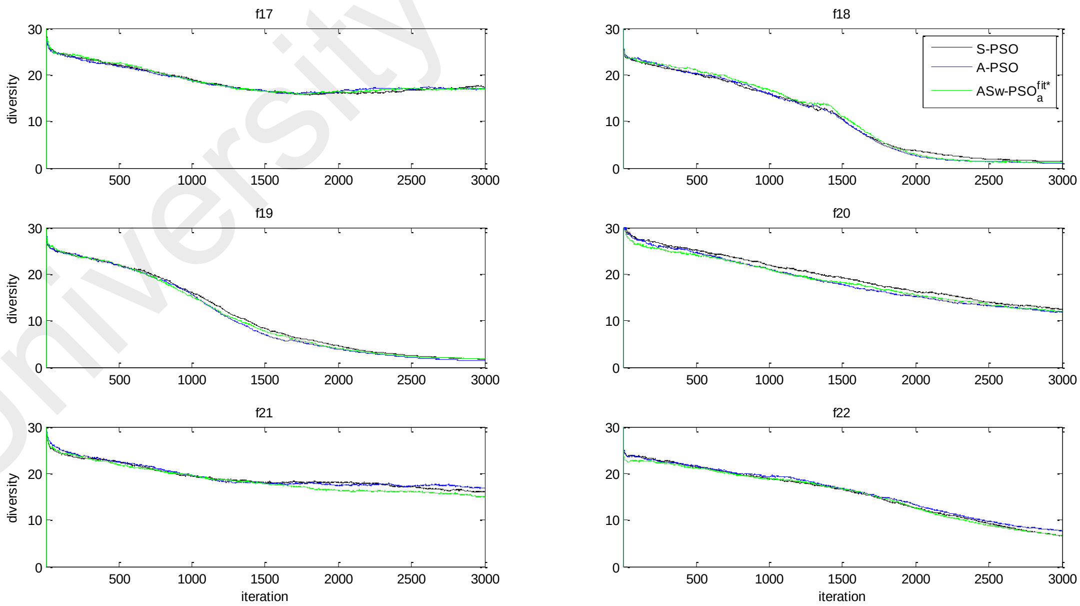

Figure 6.15: Rate of Position Diversity of Hybrid Functions for ASw-PSO $_a^{fit^*}$ with $\Delta = \{5\%\}$

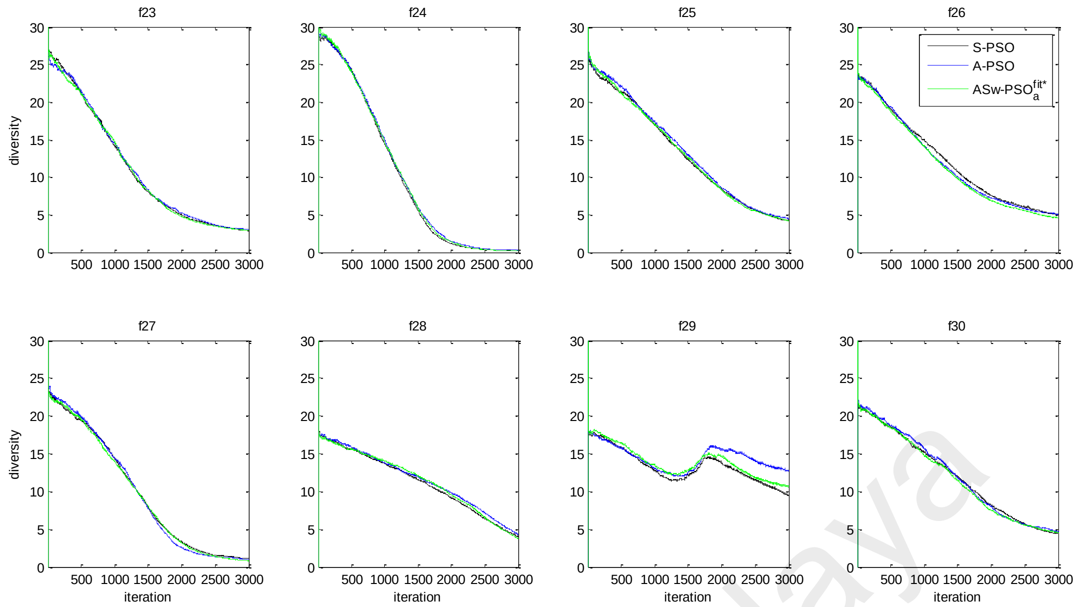

Figure 6.16: Rate of Position Diversity of Composite Functions for ASw-PSO $_{a}^{fit^*}$ with $\Delta = \{5\%\}$

6.4.3.2 Adaptive Switching GSA

An adaptive switching GSA that starts with synchronous update and uses large number of switching is able to achieve a performance that is significantly better than A-GSA and as good as S-GSA.

Figure 6.17 to Figure 6.20 show the error distribution of the algorithms using the boxplots. The boxplots of ASw-GSA $_{s}^{fit^*}$ with $\Delta = \{15\%\}$ are located at the same level as S-GSA and lower than A-GSA for most functions. The size of the boxes is as small as the boxplots of S-GSA showing the algorithms' consistent performance.

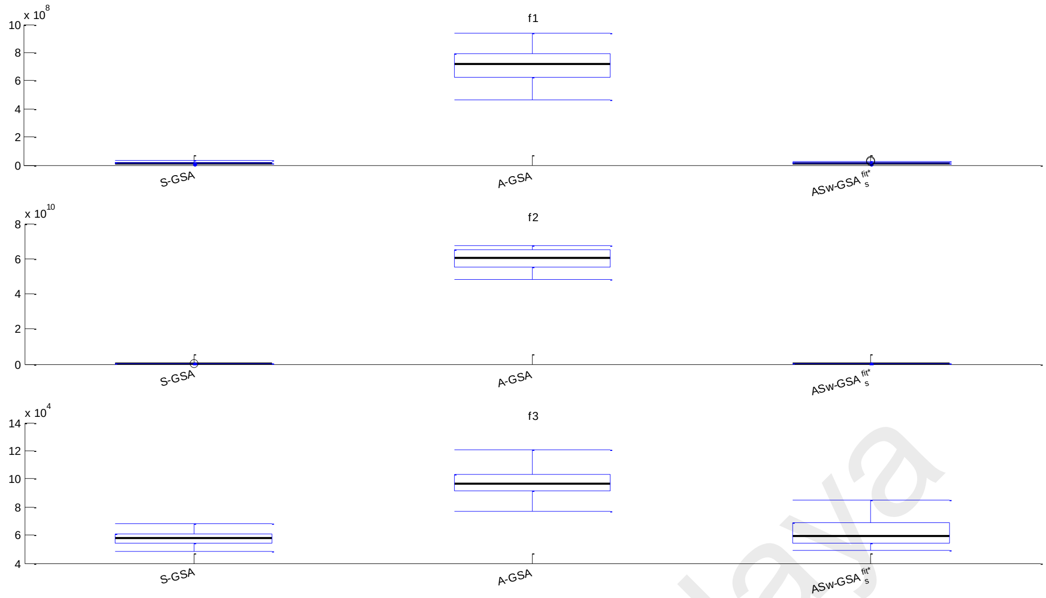

Figure 6.17: Fitness Error Distribution of Unimodal Functions for ASw-GSA$_{s}^{fit^*}$ with $\Delta = \{15\%\}$

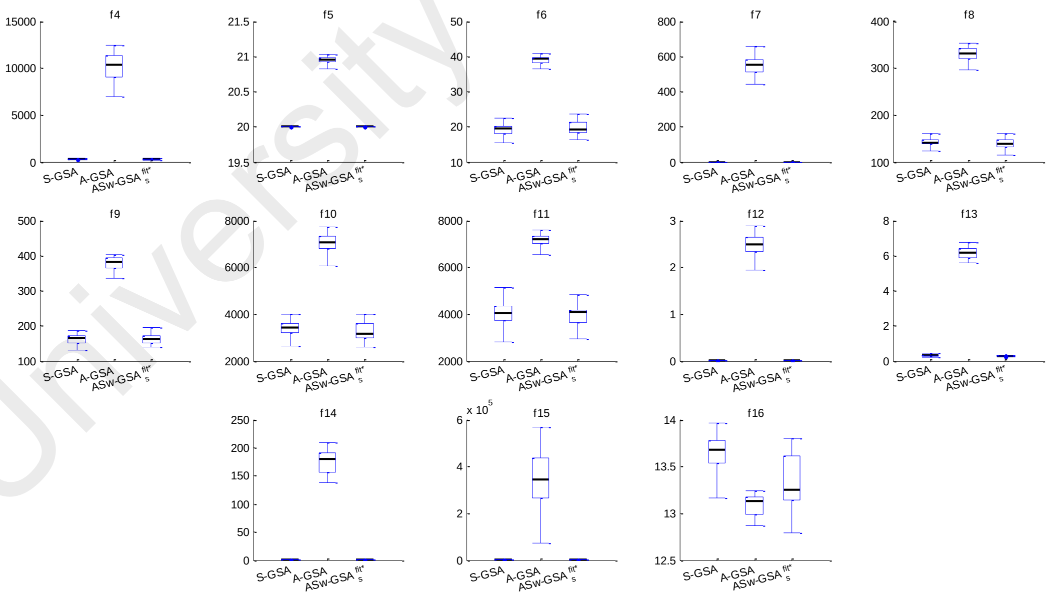

Figure 6.18: Fitness Error Distribution of Simple Multimodal Functions for ASw-GSA$_{s}^{fit^*}$ with $\Delta = \{15\%\}$

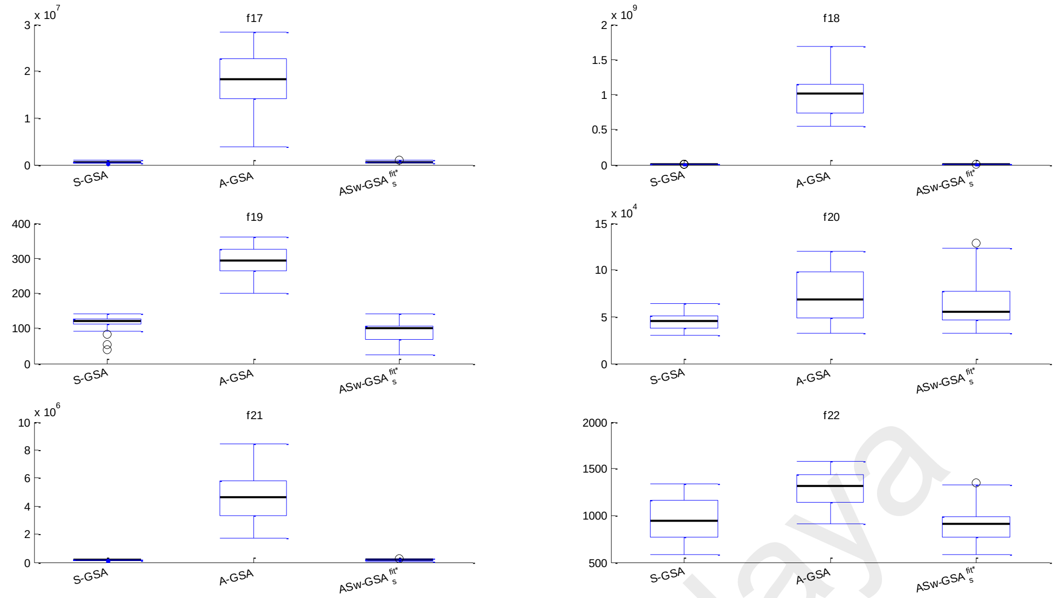

Figure 6.19: Fitness Error Distribution of Hybrid Functions for ASw-GSA $_s^{fit^*}$ with $\Delta = \{15\%\}$

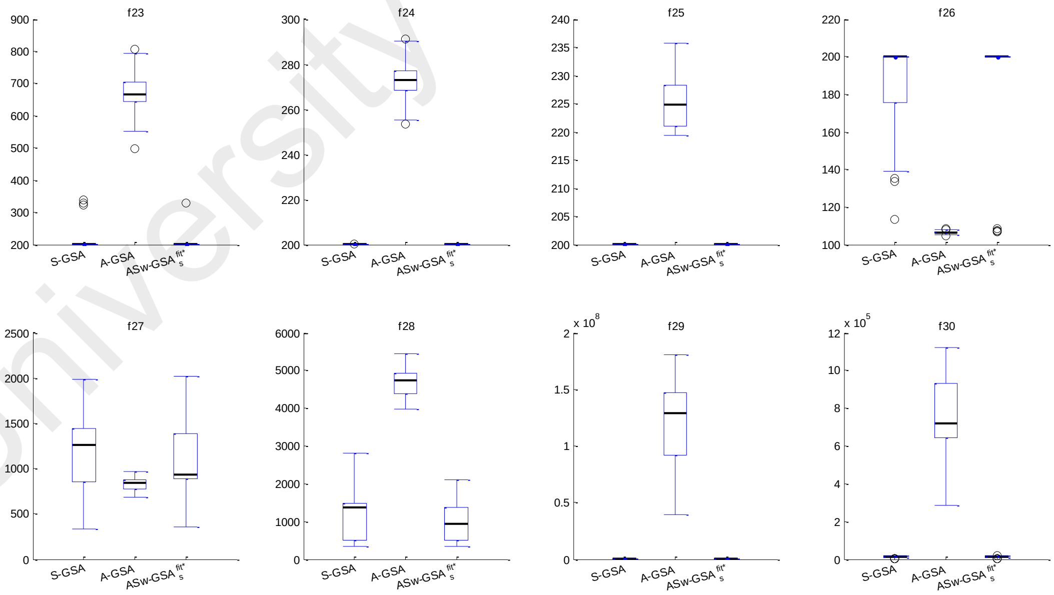

Figure 6.20: Fitness Error Distribution of Composite Functions for ASw-GSA $_s^{fit^*}$ with $\Delta = \{15\%\}$

Figure 6.21 shows the error rate of ASw-GSA $_s^{fit^*}$ with $\Delta = \{15\%\}$, S-GSA and A-GSA for selected test functions. The error rate of ASw-GSA $_s^{fit^*}$ with $\Delta = \{15\%\}$ decreases at a slower rate than S-GSA but faster than A-GSA. The final error value is between S-GSA and A-GSA.

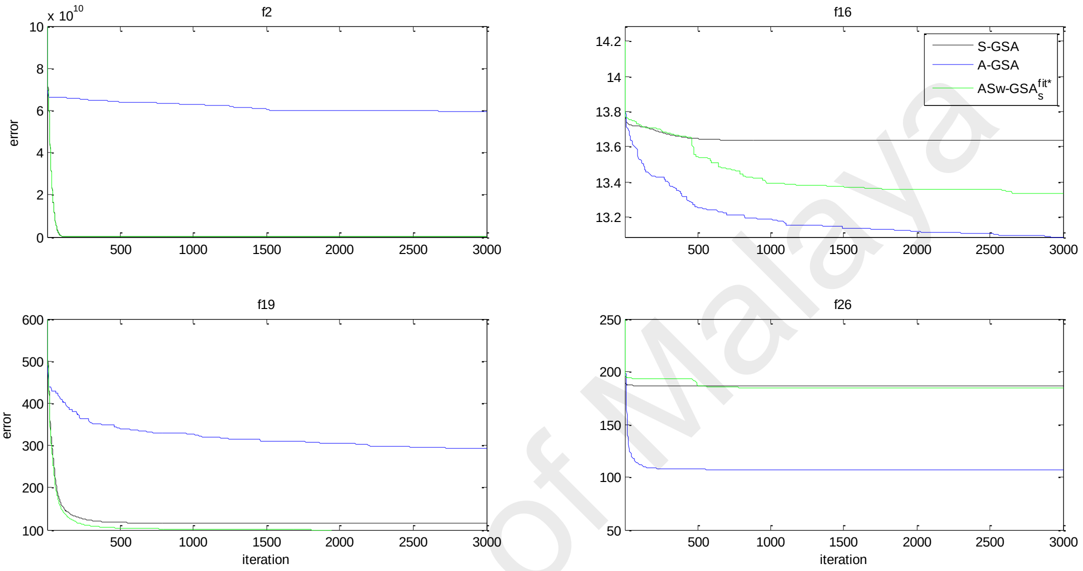

Figure 6.21: Fitness Error Rate of ASw-GSA $_s^{fit^*}$ with $\Delta = \{15\%\}$

Combination of synchronous update with asynchronous update in ASw-GSA $_s^{fit^*}$ with $\Delta = \{15\%\}$ changes the agents' diversity behaviour. This can be observed in Figure 6.22 to Figure 6.25. Initially the ASw-GSA $_s^{fit^*}$'s population's diversity decreases rapidly like synchronous update GSA. As the switching happens the diversity of the agents increased and similar to A-GSA, the diversity oscillates until the end of the search process.

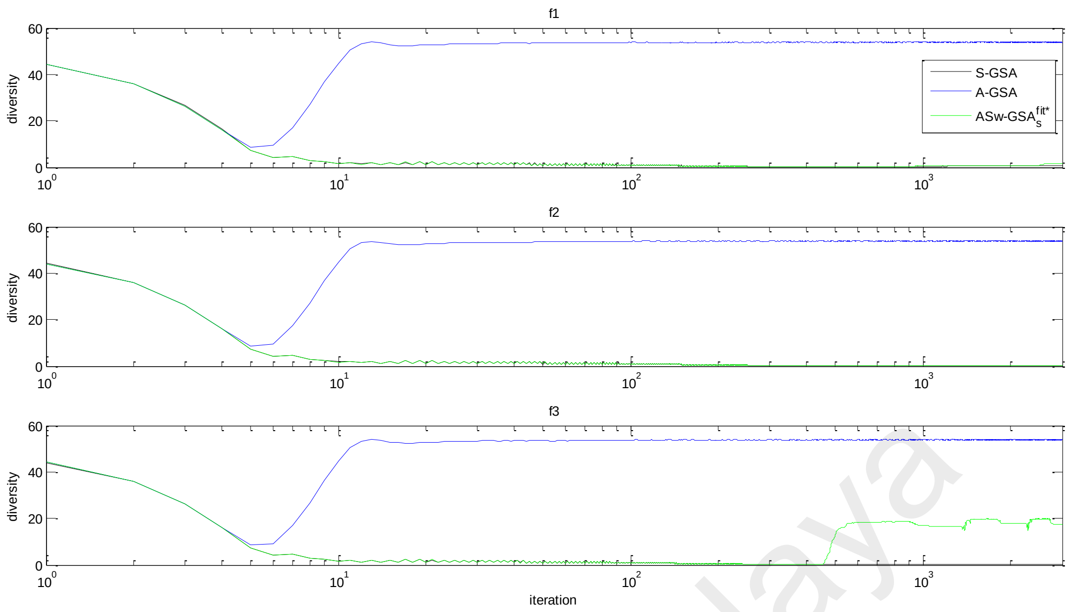

Figure 6.22: Rate of Position Diversity of Unimodal Functions for ASw-GSA $_s^{fit^*}$ with $\Delta = \{15\%\}$

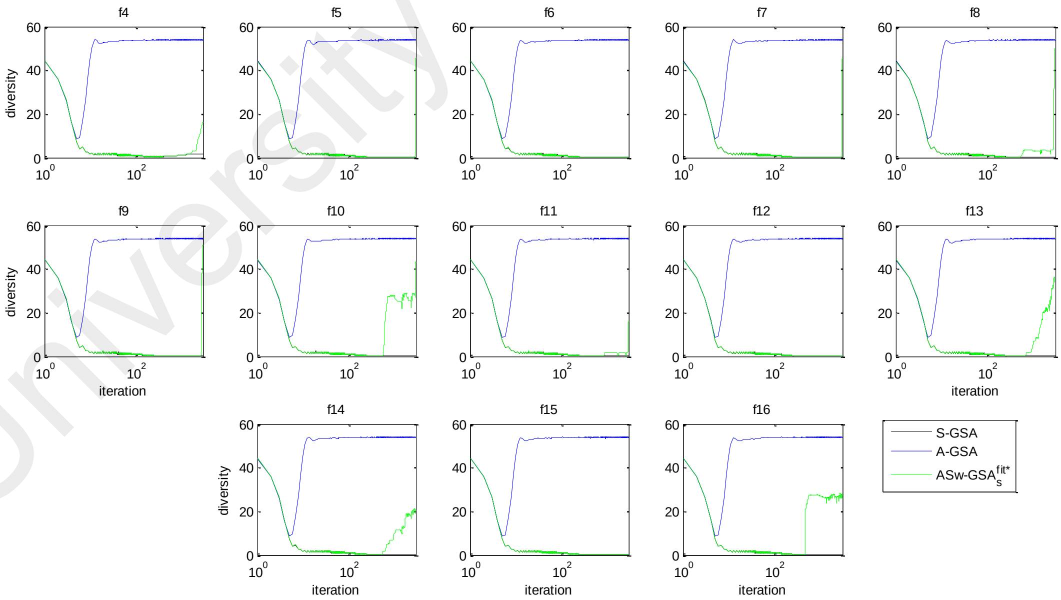

Figure 6.23: Rate of Position Diversity of Simple Multimodal Functions for ASw-GSA $_s^{fit^*}$ with $\Delta = \{15\%\}$

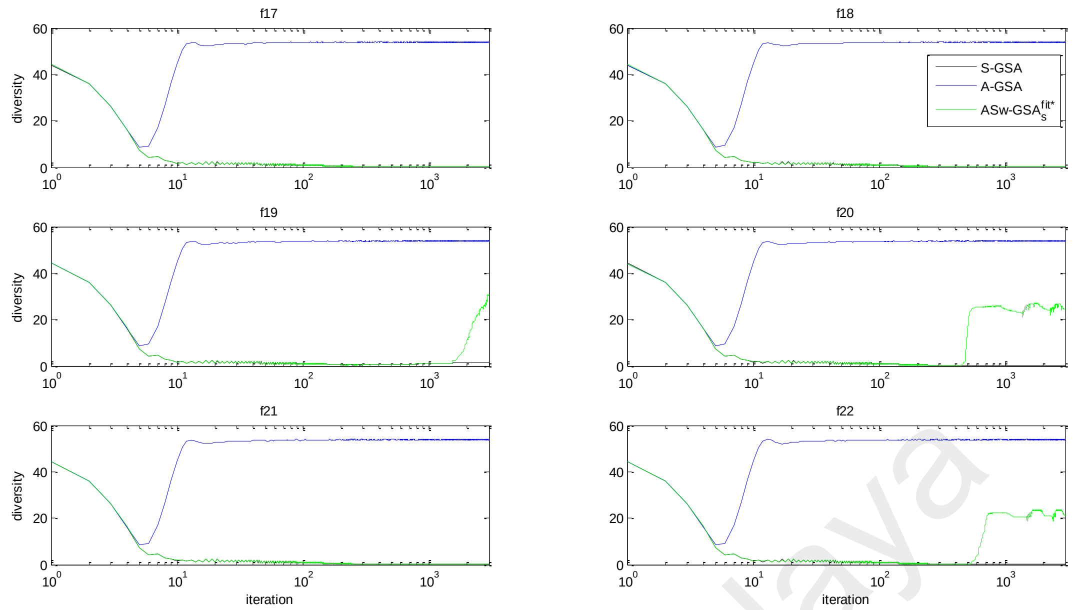

Figure 6.24: Rate of Position Diversity of Hybrid Functions for ASw-GSA $_s^{fit^*}$ with Δ= {15%}

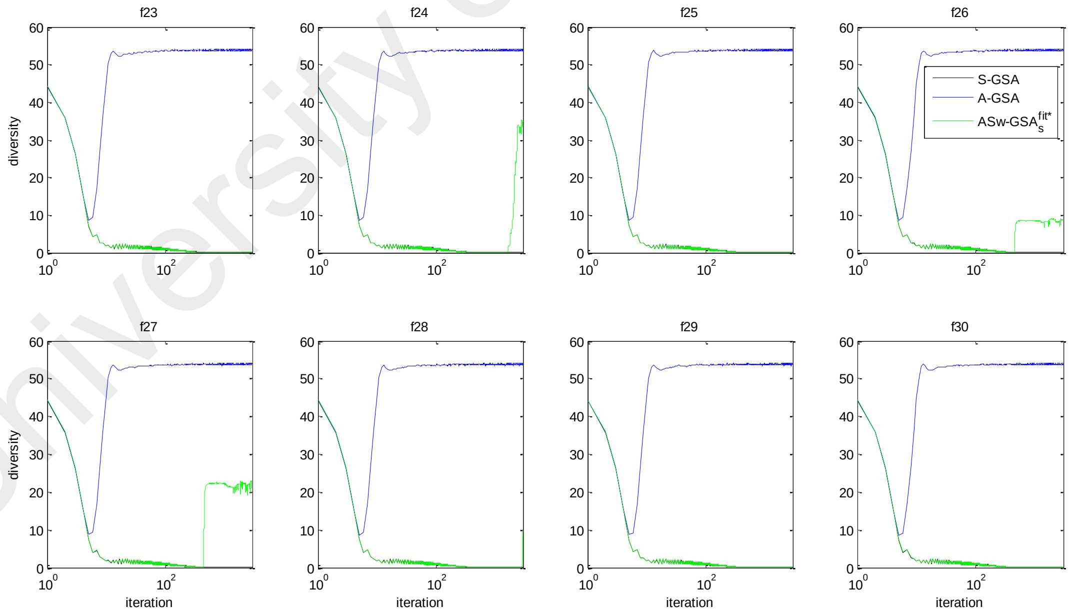

Figure 6.25: Rate of Position Diversity of Composite Functions for ASw-GSA $_s^{fit^*}$ with Δ= {15%}

6.4.3.3 Adaptive Switching SKF

The boxplots in Figure 6.26 to Figure 6.29, show that the distribution of the fitness error for ASw-SKF$_s^{Dp}$ with Δ= {45%}, S-SKF and A-SKF varies from one function to another. However, the boxplots of ASw-SKF$_s^{Dp}$ with Δ= {45%} are among the lowest and smallest.

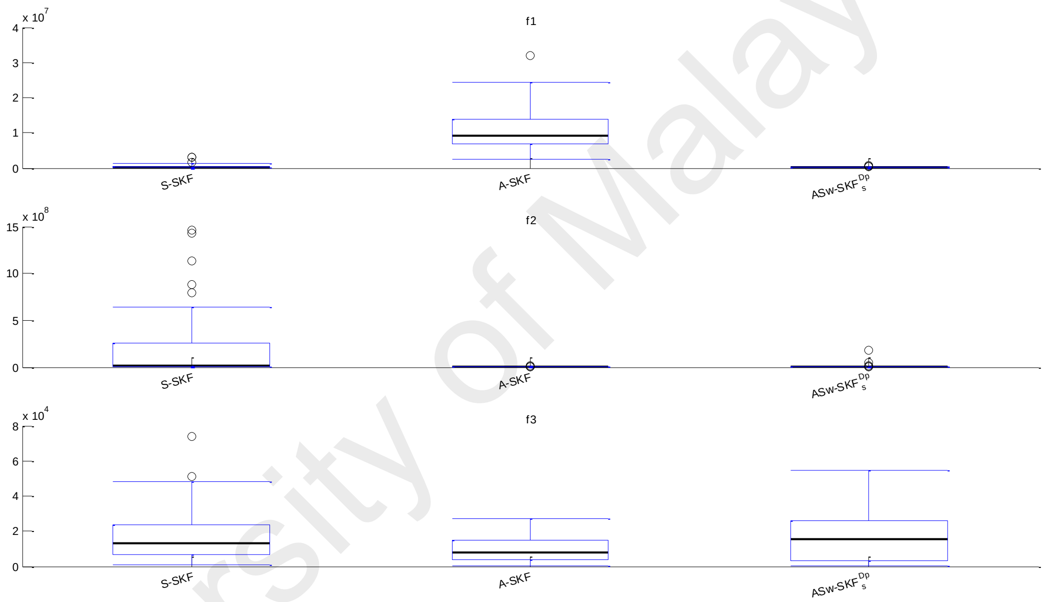

Figure 6.26: Fitness Error Distribution of Unimodal Functions for ASw-SKF$_s^{Dp}$ with Δ= {45%}

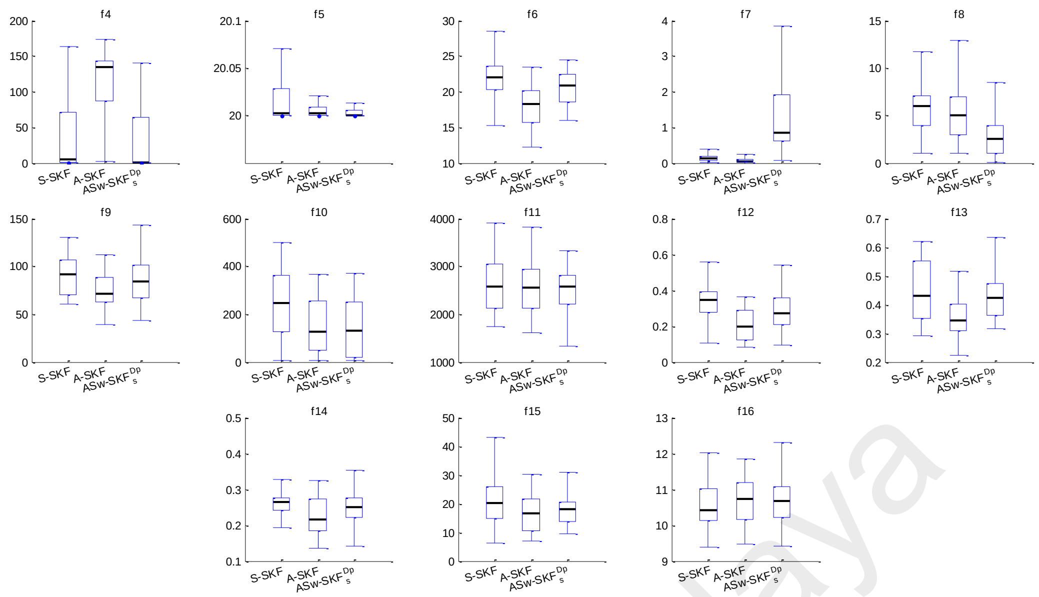

Figure 6.27: Fitness Error Distribution of Simple Multimodal Functions for ASw-SKF$_s^{Dp}$ with $\Delta = \{45\%\}$

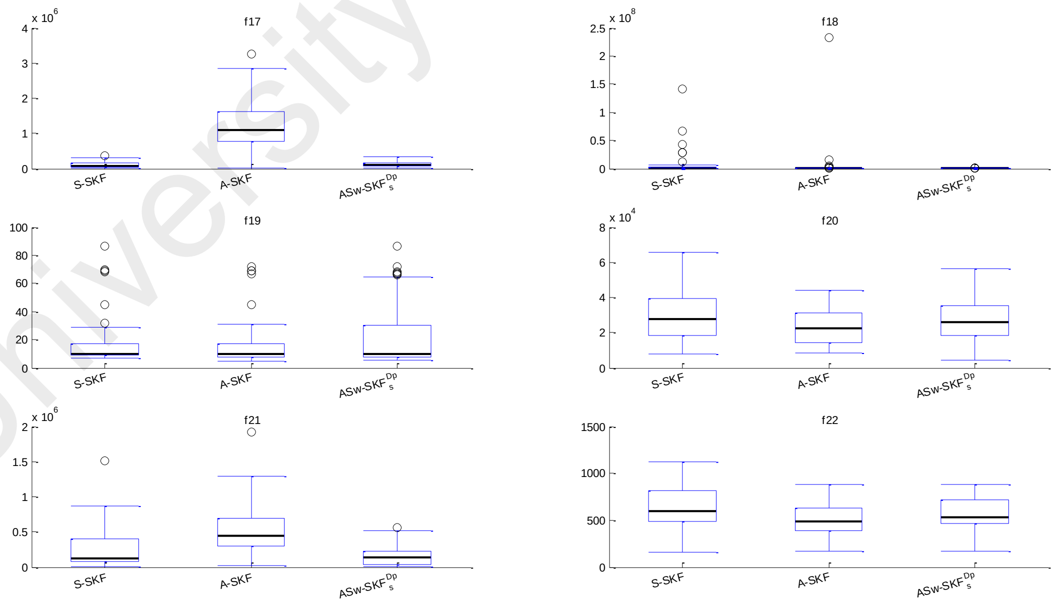

Figure 6.28: Fitness Error Distribution of Hybrid Functions for ASw-SKF$_s^{Dp}$ with $\Delta = \{45\%\}$

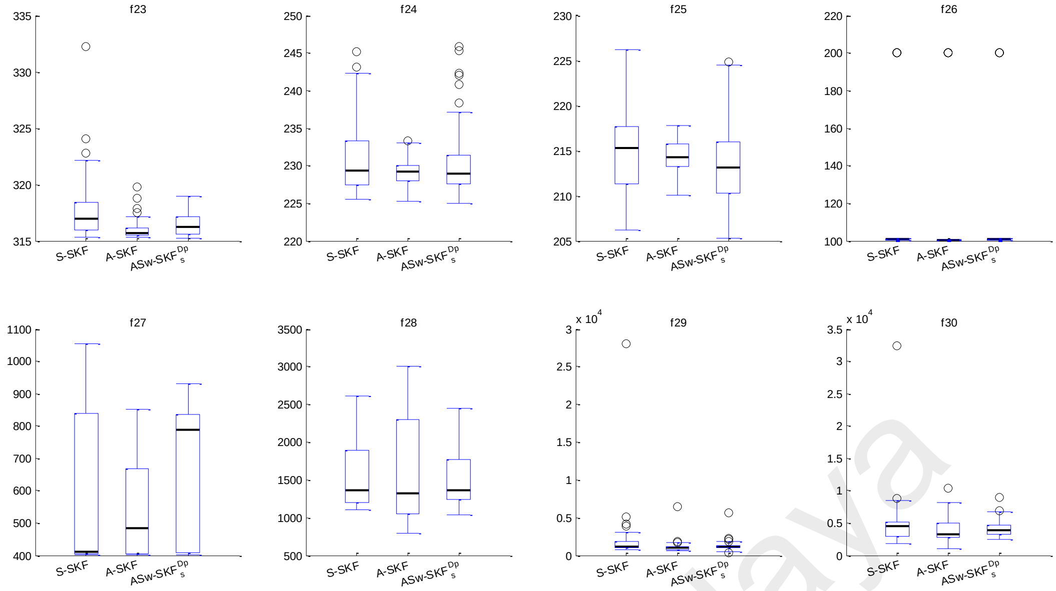

Figure 6.29: Fitness Error Distribution of Composite Functions for ASw-SKF$_s^{Dp}$ with Δ= {45%}

The error rate of ASw-SKF$_s^{Dp}$ with Δ= {45%} is compared with S-SKF and A-SKF in Figure 6.30. The error rate of ASw-SKF$_s^{Dp}$ with Δ= {45%} decreases as rapid as S-SKF. This is predictable as the population started with synchronous iteration strategy, and switching can only occur after the 1350th iteration (45% of FES), hence the agents' behaviour during the early phase of the search is similar to the agents of S-SKF.

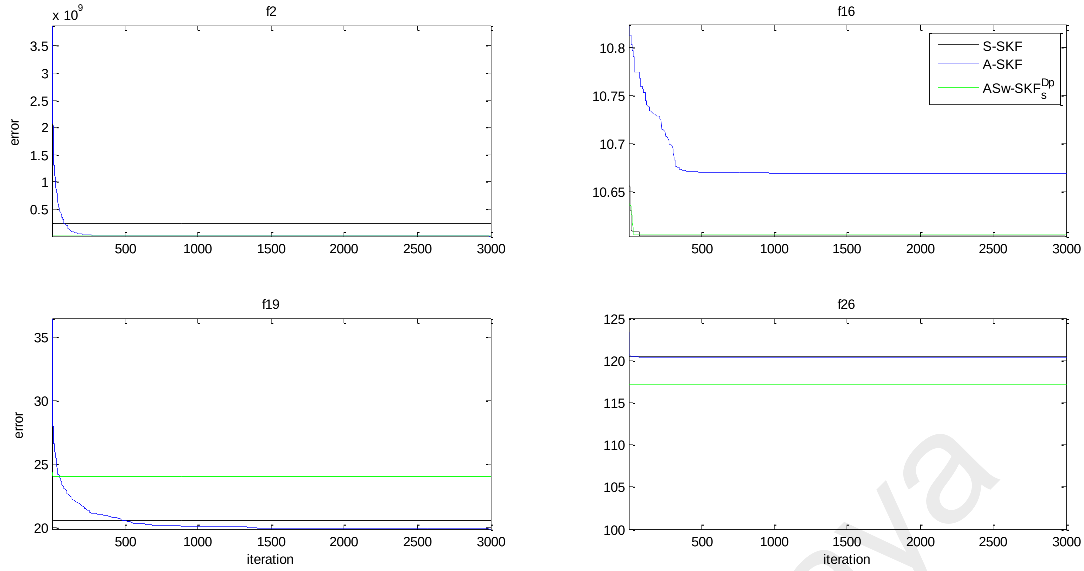

Figure 6.30: Fitness Error Rate of Unimodal Functions for ASw-SKF$_s^{Dp}$ with $\Delta = \{45\%\}$

The SKF population's behaviour is altered when adaptive switching iteration strategy is used. This is observed through the rate of position diversity in Figure 6.31 to Figure 6.34 for composite functions. Switching causes small disturbance to the agents' diversity. The disturbance contributes to better performance of SKF. The same is observed in ARPSO (Riget & Vesterstrøm, 2002), reinitialize PSO (Cheng et al., 2011) and others where disturbance to agents' convergence is used to improve performance.

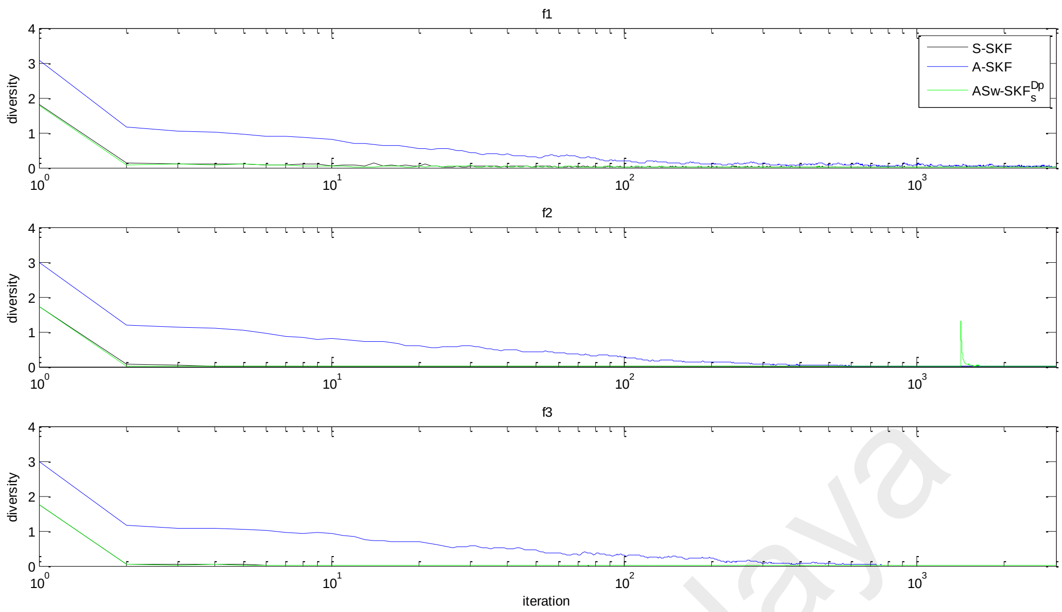

Figure 6.31: Rate of Position Diversity of Unimodal Functions for ASw-SKF$_s^{Dp}$ with $\Delta = \{45\%\}$

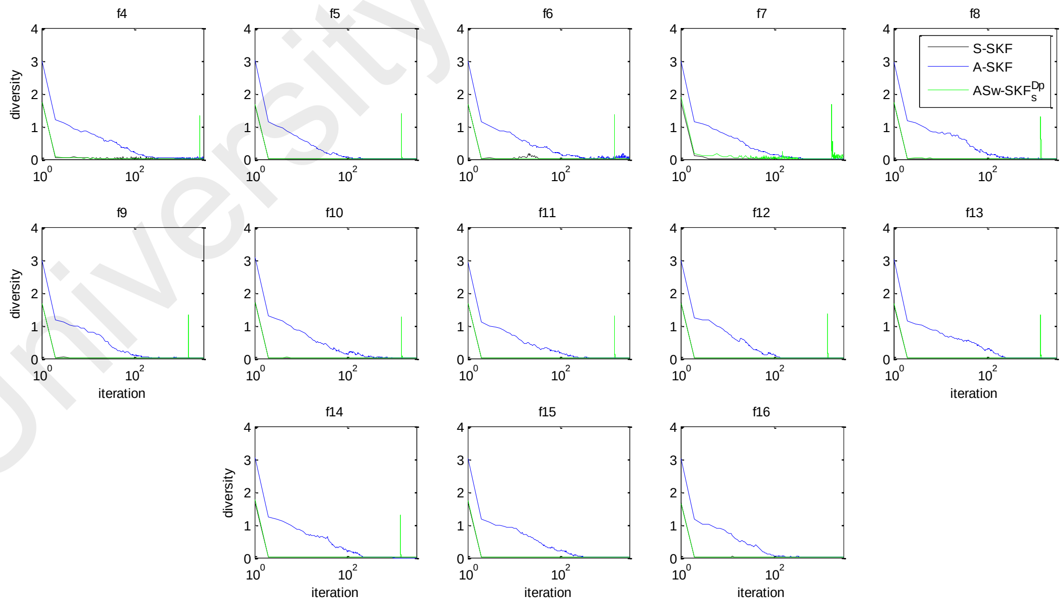

Figure 6.32: Rate of Position Diversity of Simple Multimodal Functions for ASw-SKF$_s^{Dp}$ with $\Delta = \{45\%\}$

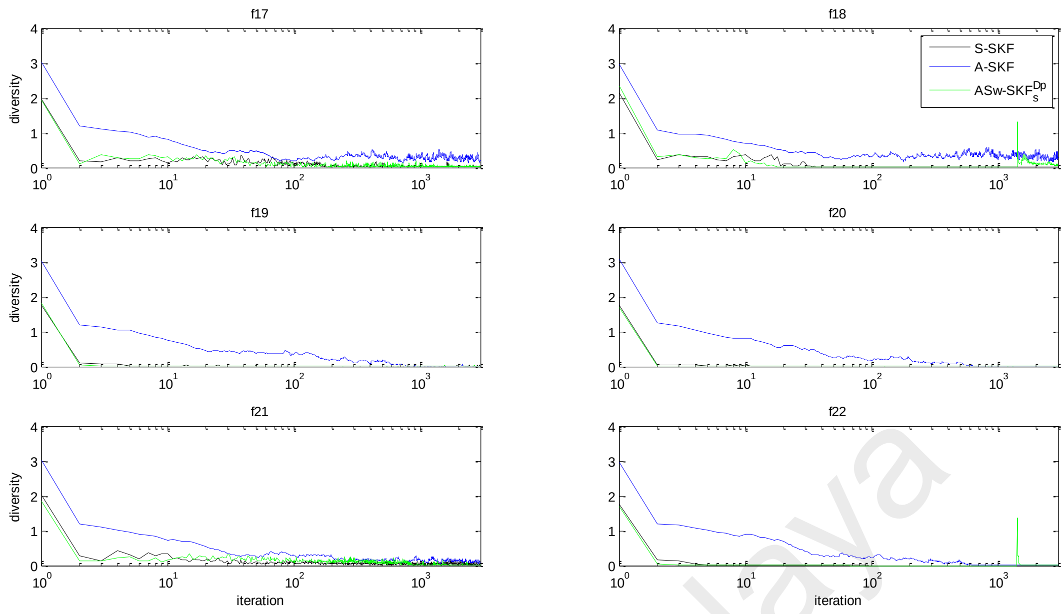

Figure 6.33: Rate of Position Diversity of Hybrid Functions for ASw-SKF$_s^{Dp}$ with $\Delta=\{45\%\}$

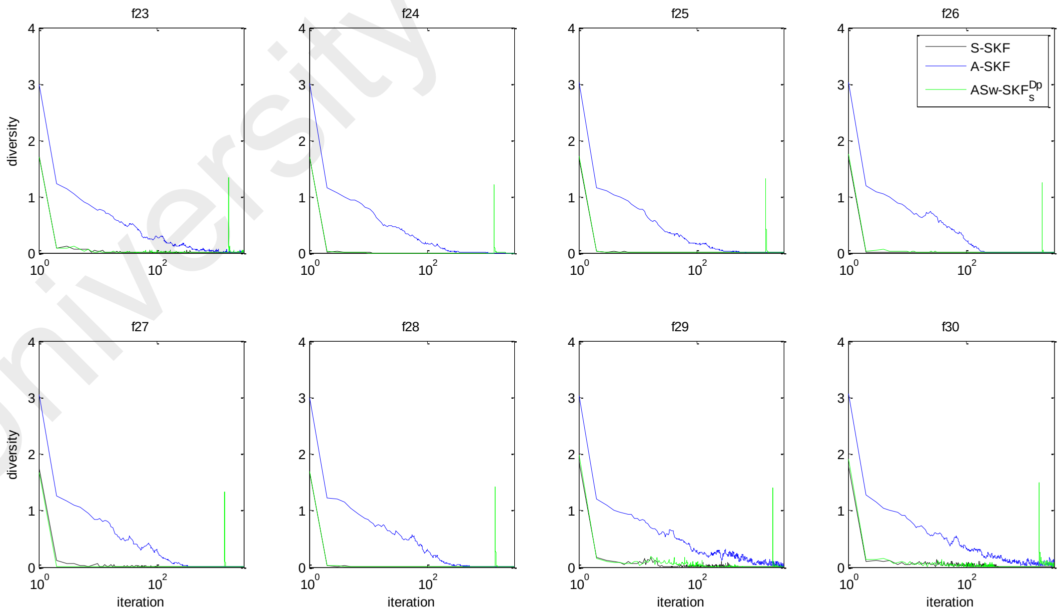

Figure 6.34: Rate of Position Diversity of Composite Functions for ASw-SKF$_s^{Dp}$ with $\Delta=\{45\%\}$

6.5 Conclusion

In adaptive switching strategy, decision on when to switch is made according to information of the population's condition. The information is needed so that switching is only conducted when the population is trapped in premature convergence or unable to further improve its performance.

Adaptive switching PSO is able to perform as good as PSO with traditional iteration strategies; S-PSO and A-PSO. However, adaptive switching iteration strategy is not altering the particles behavior. Therefore, it is not able to ensure better performance. Nonetheless, it is observed that $fit^*$is a better choice for the switching indicator of adaptive switching PSO.

Synchronous update as the initial iteration strategy and higher number of switches give a better adaptive switching GSA, which is able to give a performance as good as the synchronous GSA. Switching leads to an adaptive switching GSA that is better than A-GSA.

Adaptive switching SKF is able to perform better than S-SKF which is also the original SKF. Switching causes small disturbance to SKF's position diversity. The disturbance significantly contributes towards better performance of SKF. Both fit^* and D^p can be used as the switching indicator. Adaptive switching SKF must starts with synchronous update. The oscillating diversity, D^p, and changing fit^* of asynchronously updated SKF prevent switching when asynchronous update is used as the initial strategy.

Table 6.17 summarizes the performance of adaptive switching iteration strategy for each parent algorithms. The cell shaded grey indicate the ability of the proposed adaptive switching algorithm to outperform its parent algorithm that adopts either one of the two traditional iteration strategies.

Table 6.17: Overall Performance of Adaptive Switching Iteration Strategy

	S-PSO	**A-PSO**
ASw-PSO$_{\boldsymbol{a}}^{\boldsymbol{fit^*}}$	ASw-PSO$_a^{fit^*}$ with $\Delta=\{5\%, 10\%, 15\%, 20\%\}$ on par	ASw-PSO$_a^{fit^*}$ with $\Delta=\{5\%, 10\%, 15\%\}$ on par
ASw-PSO$_{\boldsymbol{s}}^{\boldsymbol{fit^*}}$	ASw-PSO$_s^{fit^*}$ with $\Delta=\{5\%, 10\%, 15\%, 20\%\}$ on par.	ASw-PSO$_s^{fit^*}$ with $\Delta=\{5\%, 20\%\}$ on par.
ASw-PSO$_{\boldsymbol{a}}^{\boldsymbol{Dp}}$	Invalid	Invalid
ASw-PSO$_{\boldsymbol{s}}^{\boldsymbol{fit^*}}$	Invalid	Invalid
	S-GSA	A-GSA
ASw-GSA$_{\boldsymbol{a}}^{\boldsymbol{fit^*}}$	Not as good	ASw-GSA$_a^{fit^*}$ with $\Delta=\{5\%, 10\%\}$ significantly better
ASw-GSA$_{\boldsymbol{s}}^{\boldsymbol{fit^*}}$	ASw-GSA$_s^{fit^*}$ with $\Delta=\{5\%, 10\%, 15\%\}$ on par	ASw-GSA$_s^{fit^*}$ with $\Delta=\{5\%, 10\%, 15\%\}$ significantly better
ASw-GSA$_{\boldsymbol{a}}^{\boldsymbol{Dp}}$	Invalid	Invalid
ASw-GSA$_{\boldsymbol{s}}^{\boldsymbol{Dp}}$	ASw-GSA$_s^{Dp}$ with $\Delta=\{5\%\}$ on par	ASw-GSA$_s^{Dp}$ with $\Delta=\{5\%\}$ significantly better

Table 6.17: Overall Performance of Adaptive Switching Iteration Strategy (continued…)

	S-SKF	A-SKF
ASw-SKF$_a^{fit^*}$	Invalid	Invalid
ASw-SKF$_s^{fit^*}$	ASw-SKF$_s^{fit^*}$ with $\Delta = \{10\%, 15\%, 25\%, 30\%, 35\%, 40\%, 55\%, 60\%, 65\%\}$ significantly better	On par
ASw-SKF$_a^{Dp}$	Invalid	Invalid
ASw-SKF$_s^{Dp}$	ASw-SKF$_s^{Dp}$ with $\Delta = \{5\%, 20\%, 25\%, 35\%, 45\%, 50\%, 55\%, 60\%, 75\%, 80\%, 85\%, 95\%\}$ significantly better	On par

CHAPTER 7: ADAPTIVE SWITCHING ITERATION STRATEGY WITH RANDOMNESS

7.1 Introduction

Another hybrid iteration strategy is proposed in this chapter, which is the adaptive switching iteration strategy with randomness. The proposed strategy is similar to adaptive switching strategy, however, in this new hybrid strategy, switching is allowed even when the switching indicator has some changes. This is achieved through randomness and by relaxing the condition to increment δ. In this chapter, the usage of randomness in metaheuristics are reviewed and the parent algorithms implemented using the proposed iteration strategy are presented together with the findings of the experiments.

7.2 Literature Review

Randomness is an important aspect in metaheuristics. It is embedded in the metaheuristics mechanism (Rahnamayan, Tizhoosh, & Salama, 2008). In PSO, the initial solutions are randomly generated and random numbers are used in its velocity update equation. The agents of GSA and SKF are also randomly generated. Random numbers are used in the calculation of force and velocity for GSA, while in SKF random numbers are used in its simulated measurement. Grey wolf optimizer (Mirjalili, Mirjalili, & Lewis, 2014), also starts with random population and its update mechanism is designed so that more random behavior is exhibit by the population. In Lion optimizer (M. Yazdani & Jolai, 2016), randomness is used during initialization and in many other stages of the search process. The same is observed in many other metaheuristic algorithms where the population starts with random distribution and random numbers are used in the formulation.

Additional randomness to original algorithm had also been proposed to improve the performance of metaheuristics. In (J. Zhang, Liu, Tan, & He, 2008), a black hole is randomly generated near to the PSO's current best particle, the randomly generated black hole enable the swarm to escape from premature convergence. A low-discrepancy quasi-random number sequence is proposed for GSA's agents initialization in (Altinoz, Yilmaz, & Weber, 2014). The new random generator provides a better distribution of the agents within the search space and increases the probability of finding optimal solution. A parameter-less SKF had been proposed in (Abdul Aziz et al., 2017), where the parameters of SKF is replaced with random values. This lift the need for parameter optimization for SKF.

Overall, the randomness is a popular approach in metaheuristics. It is able to produce candidate solutions, reduce bias and provide disturbance to the solutions (Barros, Federal, & Barros, 2012).

7.3 Adaptive Switching Iteration Strategy with Randomness

Like the adaptive switching iteration strategy, in adaptive switching with randomness, switching is conducted according to the condition of a switching indicator over a period of time. However, inspired by the positive effect of randomness, for the third hybrid iteration strategy the ratio of the switching indicator from one iteration to the next iteration is compared with a random value. The random value is ranged from zero to one and drawn from a uniform distribution, $rand \sim U([0,1])$. The random value is drawn every time the switching indicator is checked. The randomness is introduced to increase the probability of switching.

If fit^* is used as the switching indicator the switching counter, δ, is increased when;

$$\frac{fit^*(t+1)}{fit^*(t)} \geq rand \tag{7.1}$$

On the other hand, when D^p is used, the condition is

$$\frac{D^p(t+1)}{D^p(t)} \leq rand \tag{7.2}$$

When fit^* is used, δ is incremented if the ratio is bigger than or equivalent to a random value, whereas when D^p is used, the ratio need to be lesser than or equivalent to a random value. The difference is because, when fit^* is used, switch is more desired when no or marginal improvement is observed within a population, $fit^*(t+1) \geq fit^*(t)$. However, when D^p is used, switch is desired when the population is converging, $D^p(t+1) \leq D^p(t)$.

The general definition of adaptive switching iteration strategy with randomness is similar to Definition 6.1. The general flowchart is shown in Figure 7.1.

The details of the third hybrid iteration strategy implementation on the parent algorithms, such as initialization, information preserved during the switch and the stopping condition, are similar to what were discussed in chapter 6. Therefore, in the next subsection, only the flowcharts of the implementation of the new iteration strategy for each respective parent algorithms are presented.

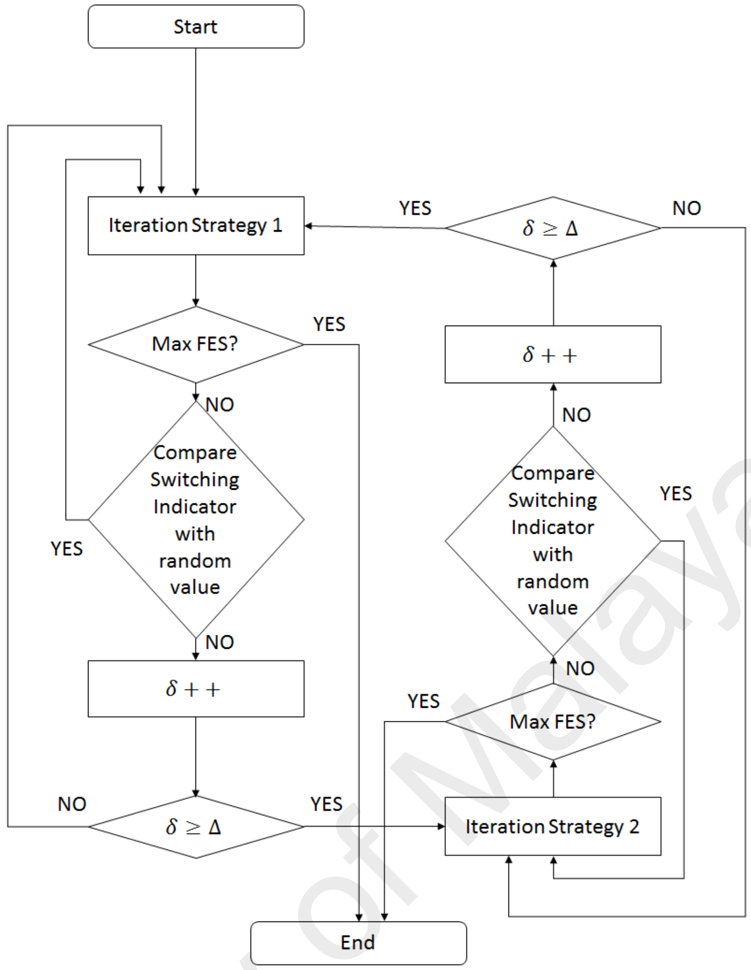

Figure 7.1: General Flowchart of Adaptive Switching with Randomness

7.3.1 PSO using Adaptive Switching Iteration Strategy with Randomness

Similar to random and adaptive switching strategy, either one of the traditional strategies can be the initial strategy of the third hybrid strategy. ASw-PSO$_a^{rfit^*}$ is the variant that uses asynchronous update as initial strategy and fit^* as the switching indicator, r in front of fit^* represents the integrated randomness. ASw-PSO$_a^{rD^p}$ uses D^p as its switching indicator, while both ASw-PSO$_s^{rfit^*}$ and ASw-PSO$_s^{rD^p}$ use synchronous update as the initial strategy. These variants of PSO are shown in Figure 7.2 to Figure 7.5.

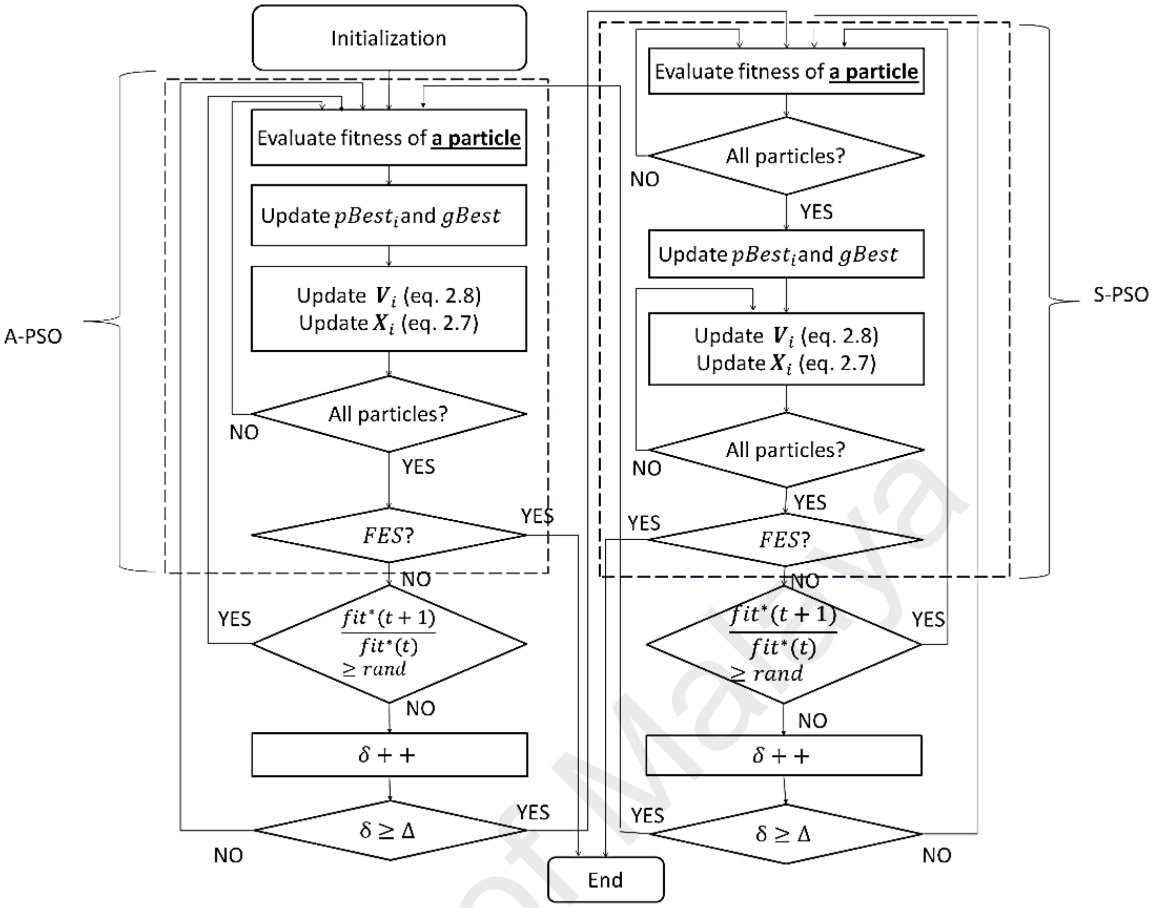

Figure 7.2: Flowchart of ASw-PSO$_a^{rfit^*}$

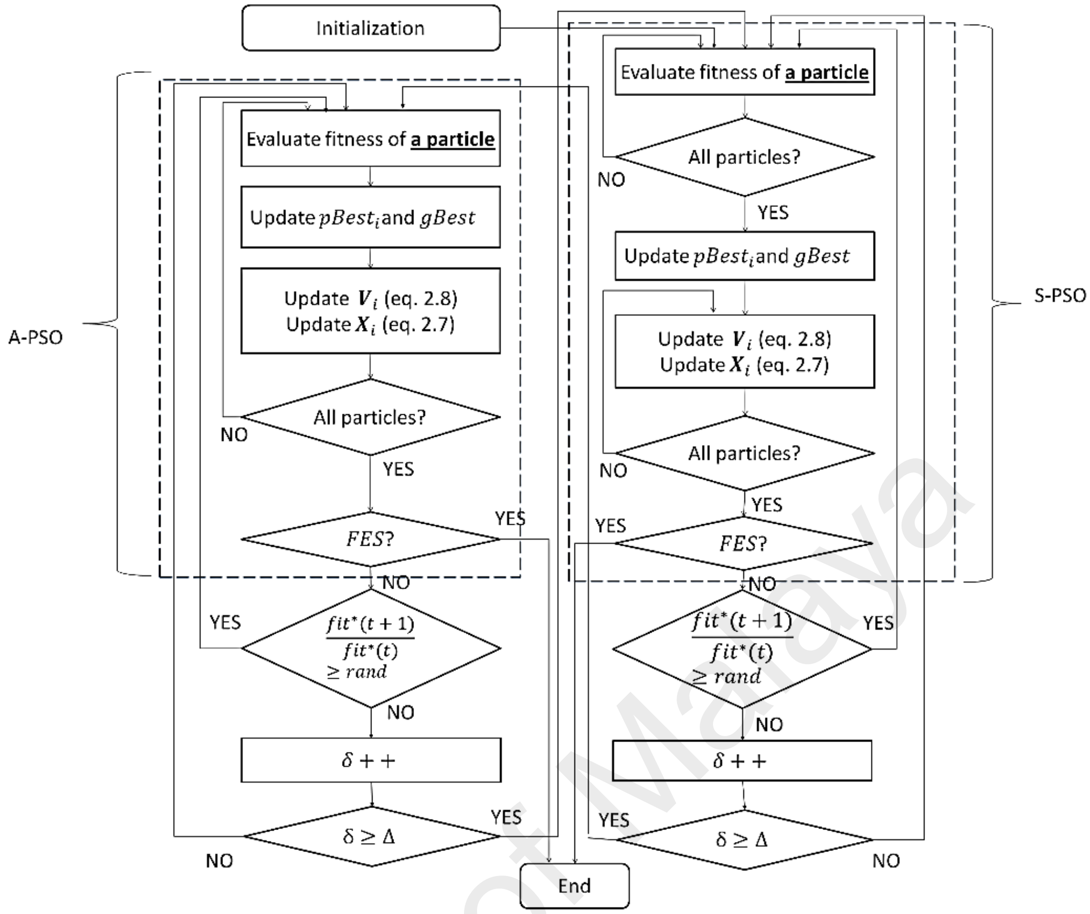

Figure 7.3: Flowchart of ASw-PSO$_s^{rfit^*}$

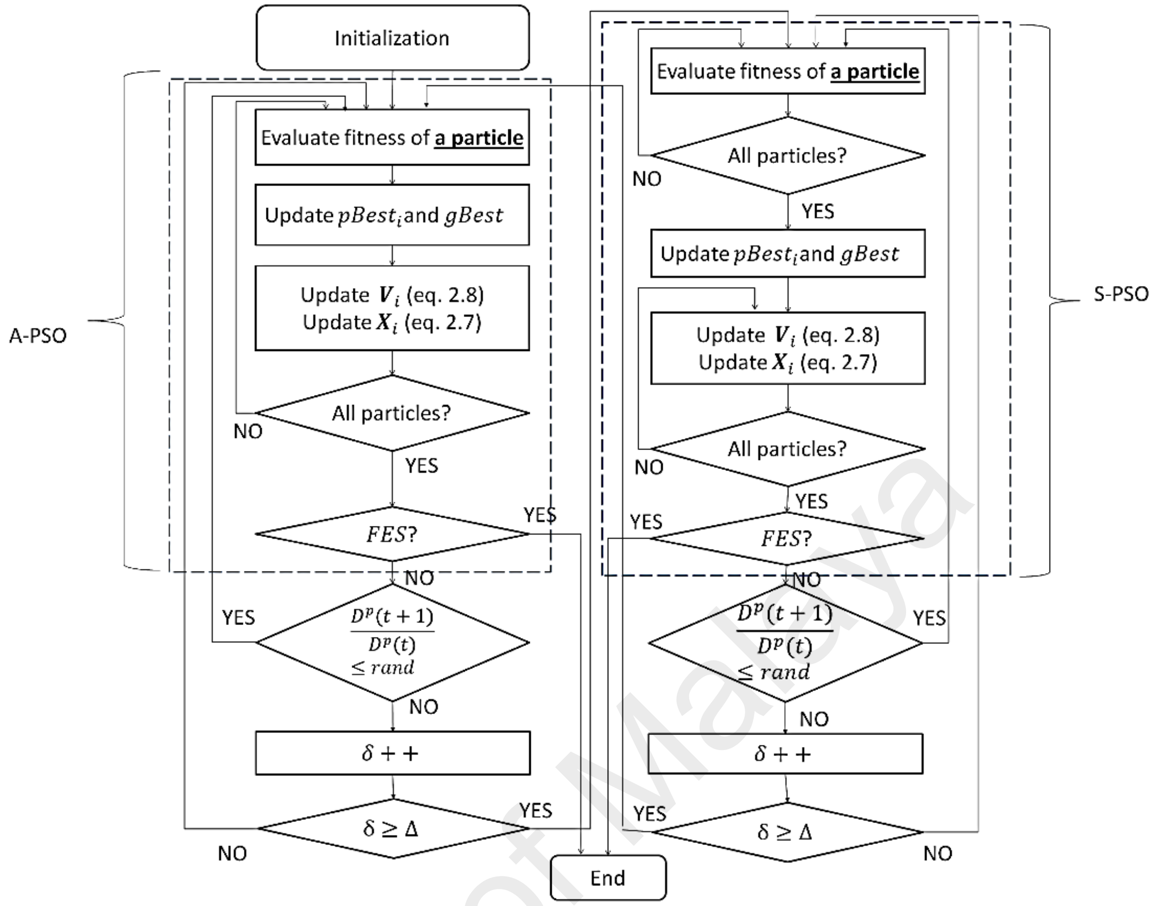

Figure 7.4: Flowchart of ASw-PSO$_a^{rD^p}$

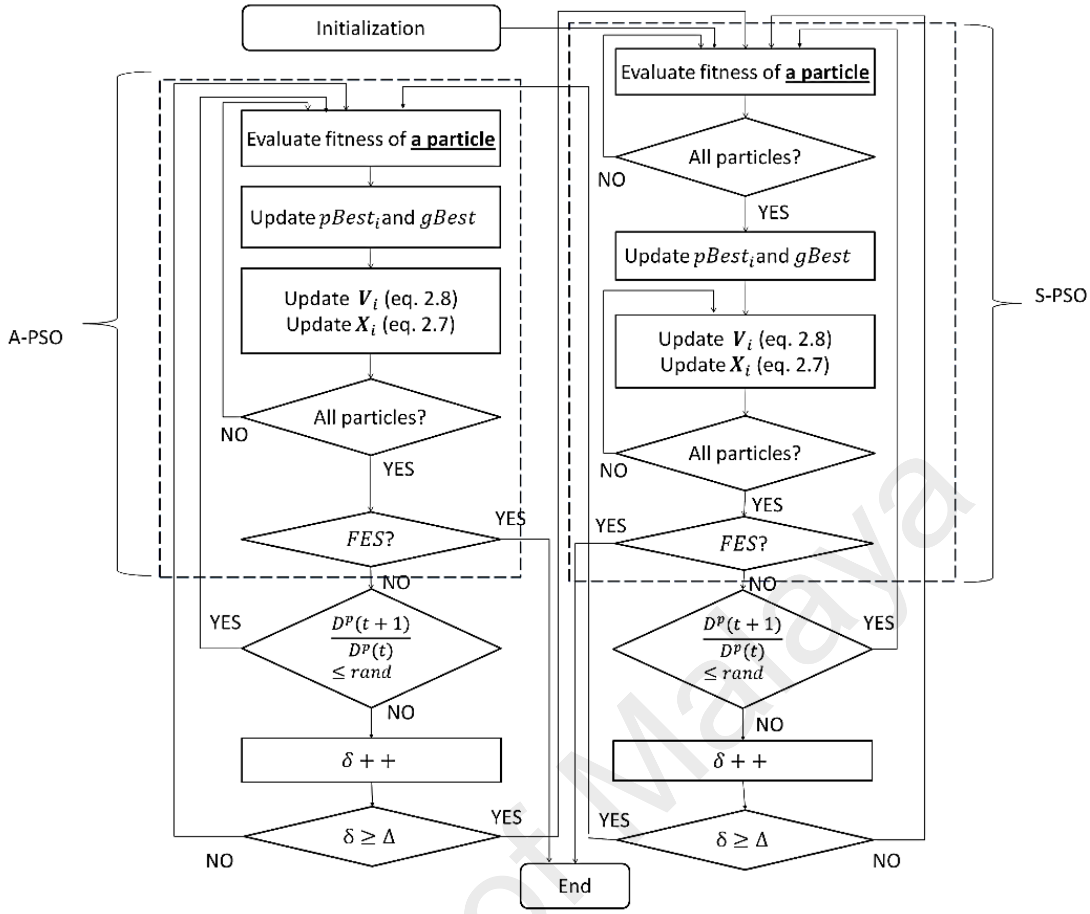

Figure 7.5: Flowchart of ASw-PSO$_s^{rD^p}$

7.3.2 GSA using Adaptive Switching Iteration Strategy with Randomness

In this section, the adaptive switching iteration strategy with randomness is applied to GSA. Same notations like what are used for PSO are applied. GSA that uses adaptive switching iteration strategy with randomness, starts with asynchronous update and uses fit^* as its switching indicator is represented as, ASw-GSA$_a^{rfit^*}$, whereas ASw-GSA$_a^{rD^p}$ represents the variant that uses D^p as its switching indicator. If the initial iteration strategy is synchronous update the variants are represented as ASw-GSA$_s^{rfit^*}$ or ASw-GSA$_s^{rD^p}$ according to the chosen switching indicator. These variants of GSA are shown in Figure 7.6 to Figure 7.9.

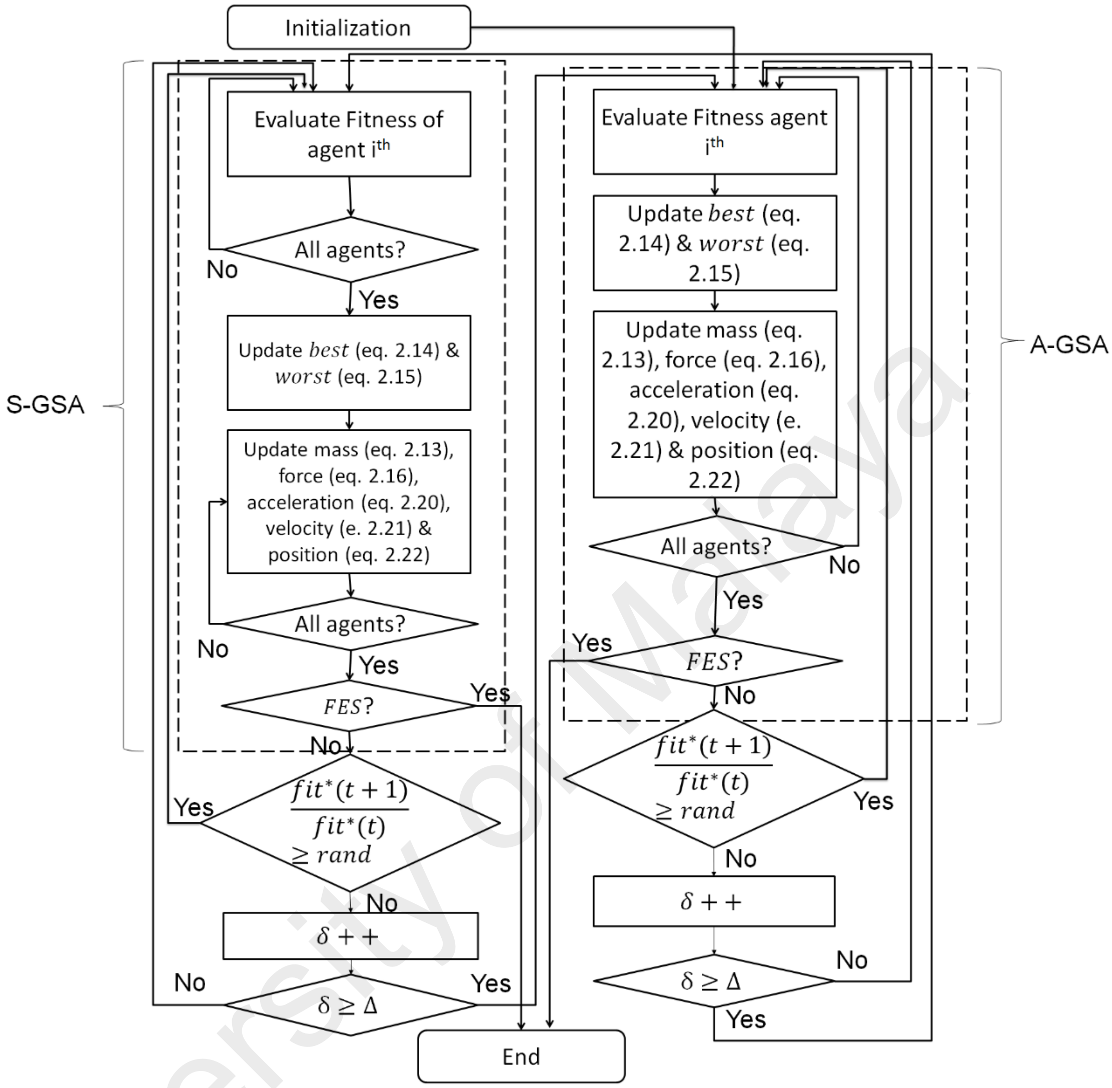

Figure 7.6: Flowchart of ASw-GSA$_a^{rfit^*}$

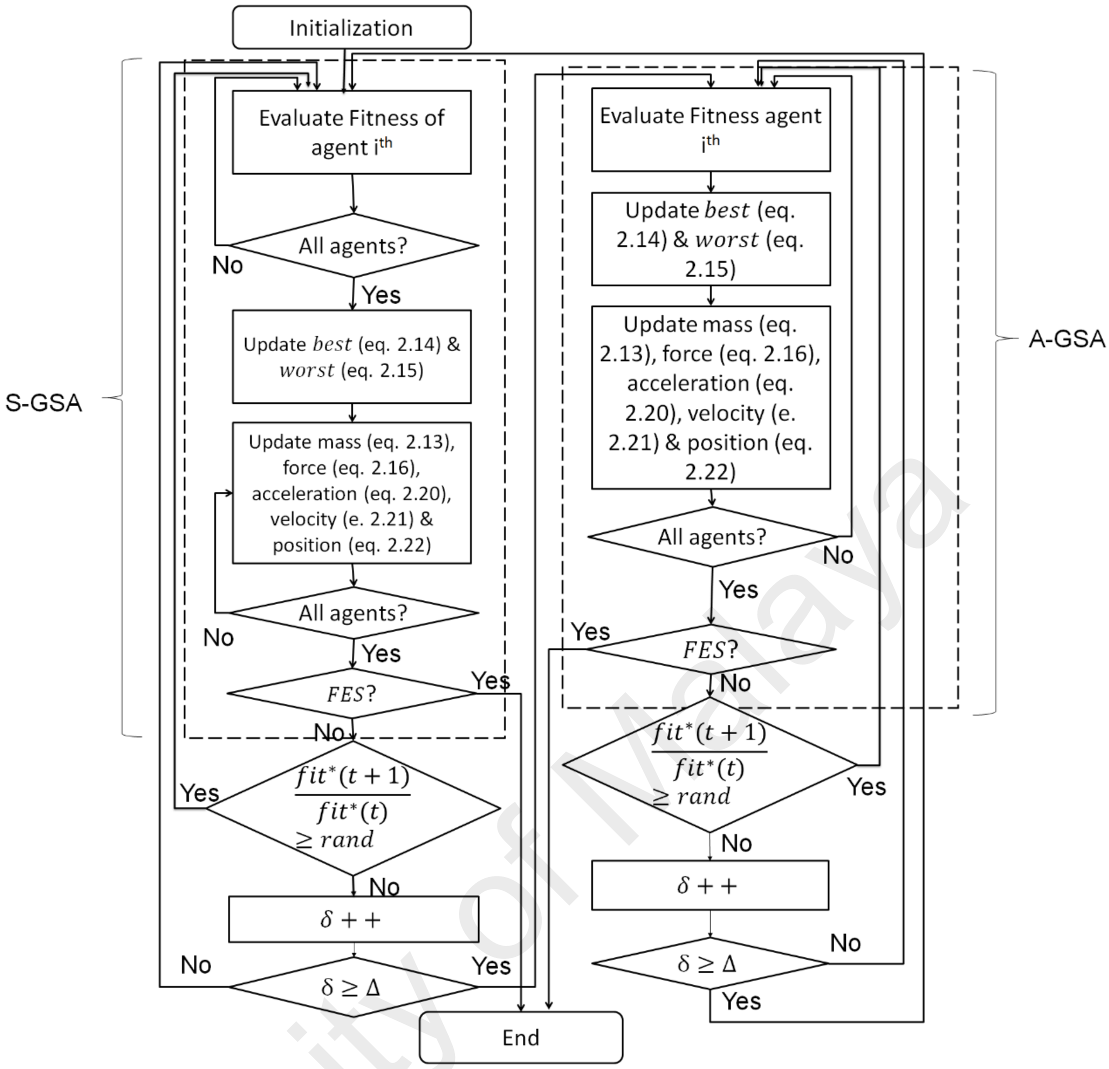

Figure 7.7: Flowchart of ASw-GSA$_s^{rfit^*}$

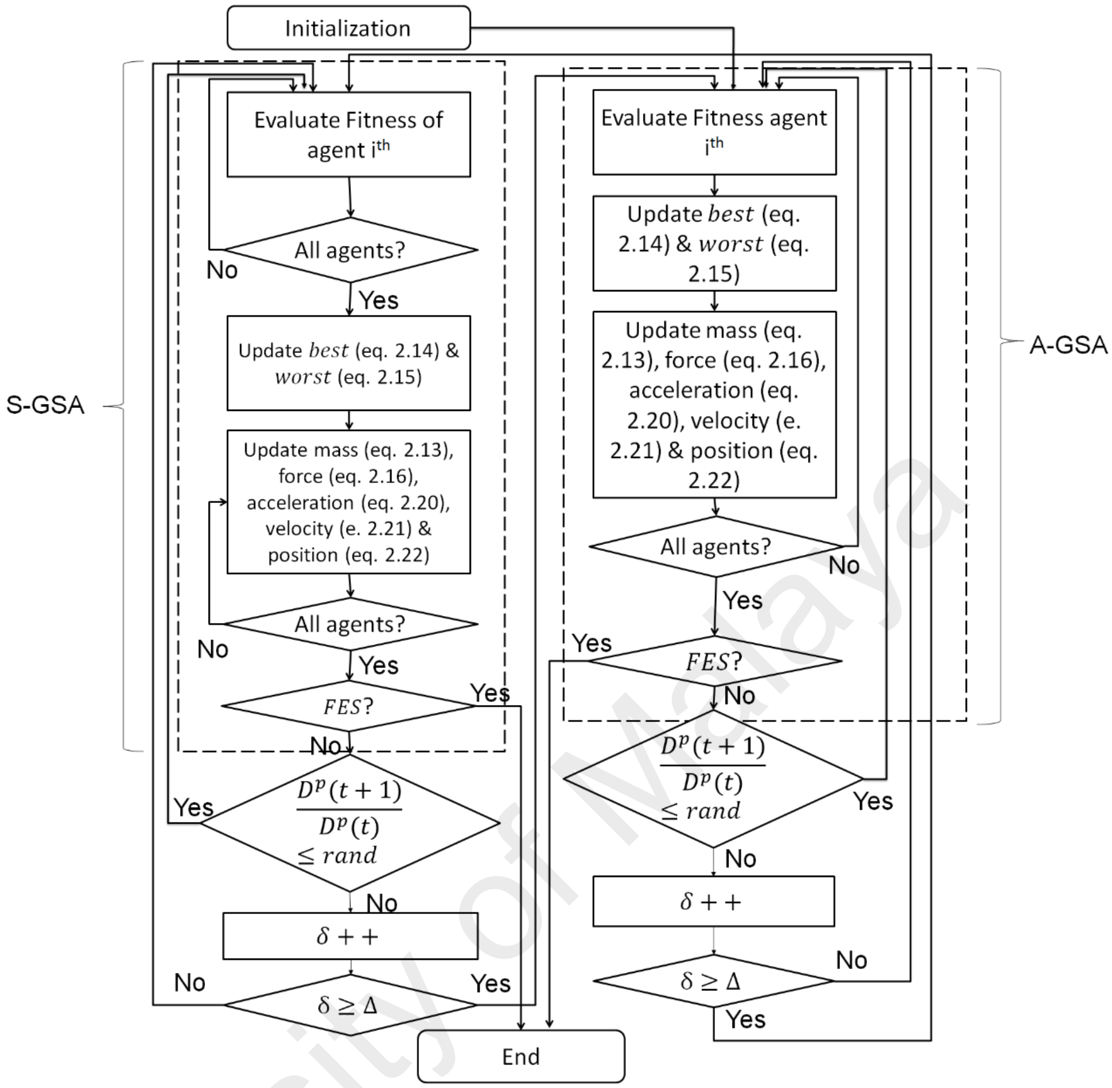

Figure 7.8: Flowchart of ASw-GSA$_a^{rD^p}$

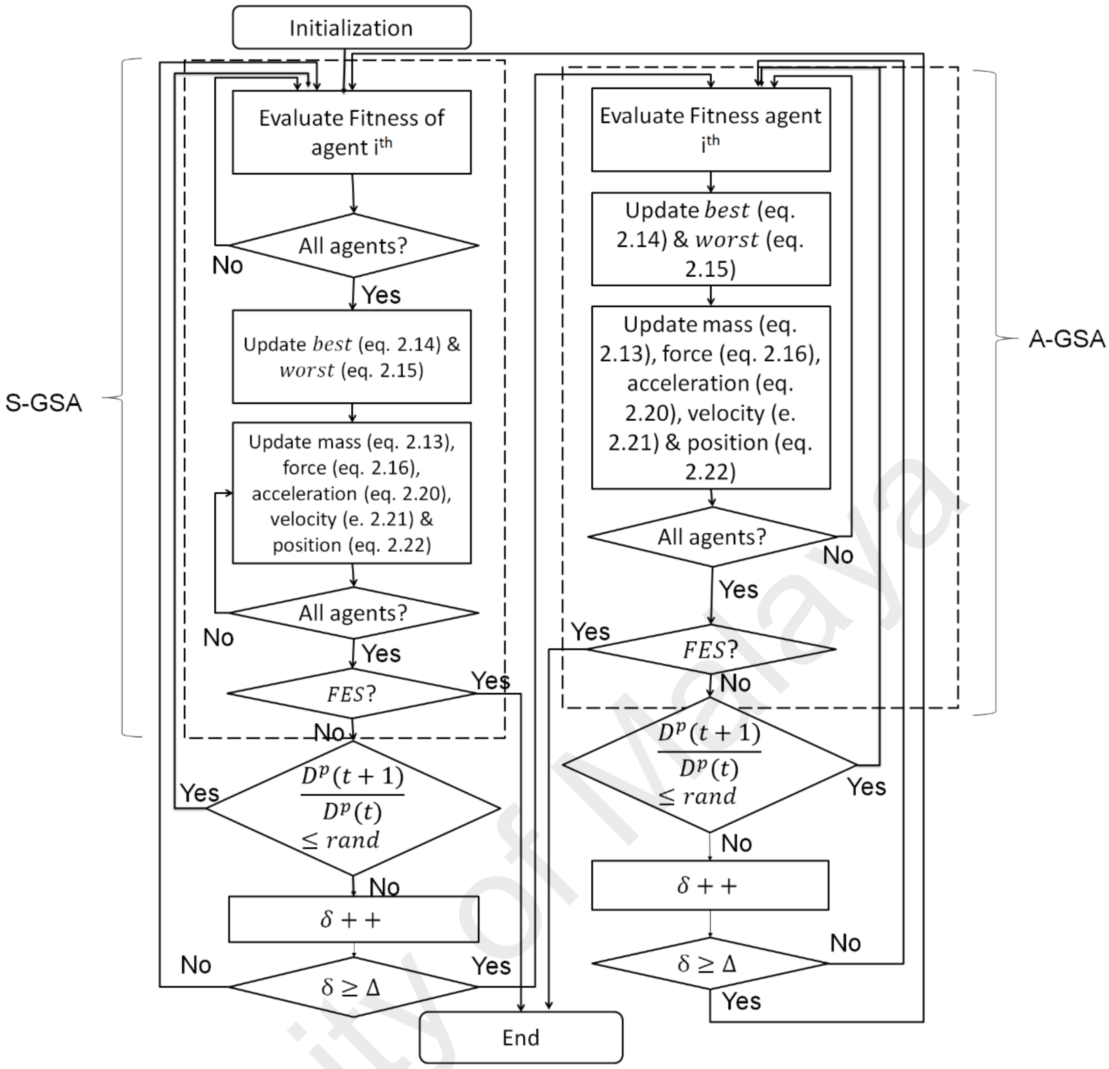

Figure 7.9: Flowchart of ASw-GSA$_s^{rD^p}$

7.3.3 SKF using Adaptive Switching Iteration Strategy with Randomness

ASw-SKF$_a^{rfit^*}$ is SKF algorithm that adopts adaptive switching iteration strategy with randomness that uses fit^* as its switching indicator and asynchronous update as the initial strategy, while ASw-SKF$_s^{rfit^*}$ starts with synchronous update. The variants that use D^p as the indicator are noted as; ASw-SKF$_a^{rD^p}$ and ASw-SKF$_s^{rD^p}$. The four new variants of SKF are presented in Figure 7.10 to Figure 7.13.

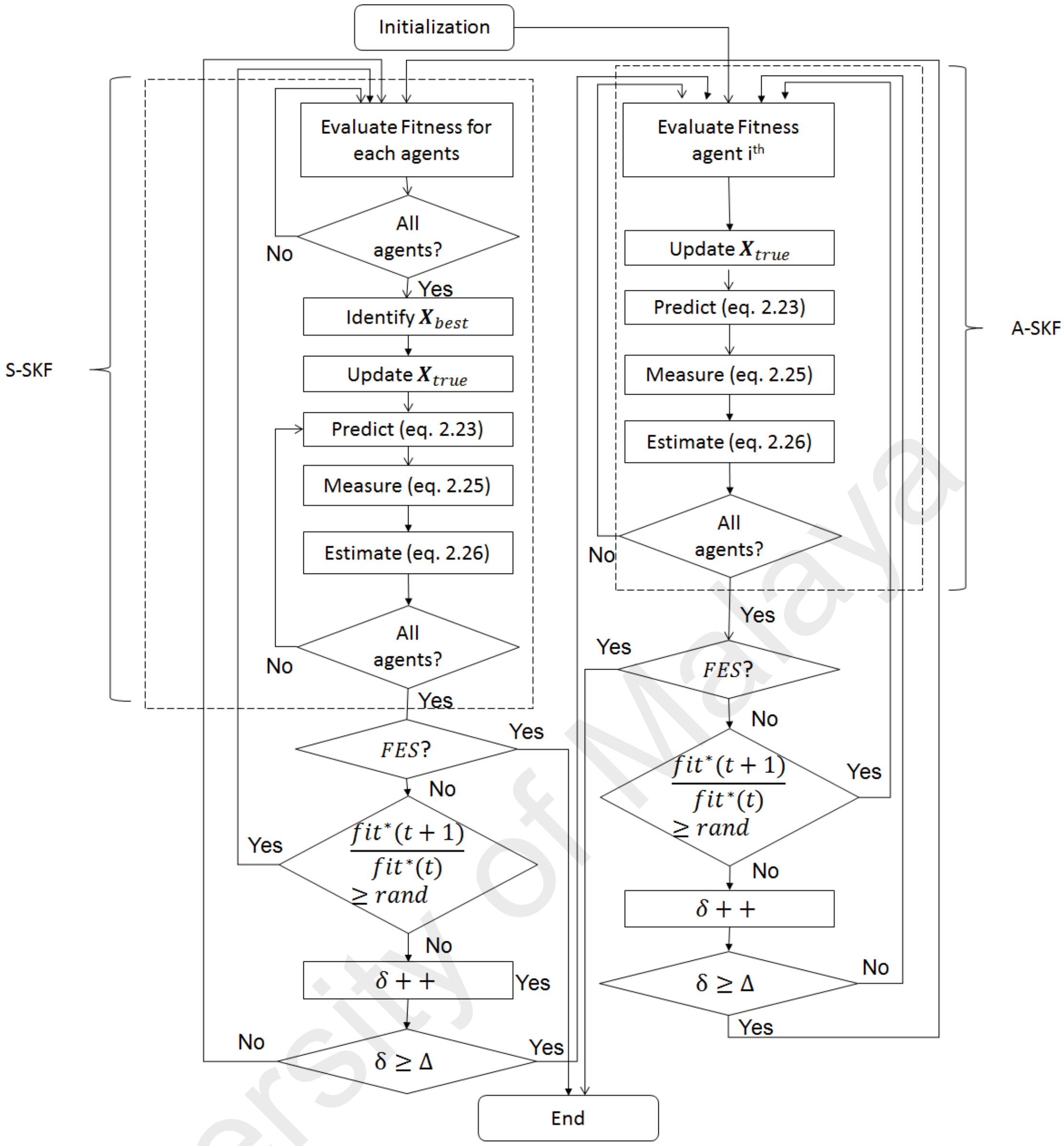

Figure 7.10: Flowchart of ASw-SKF$_a^{rfit^*}$

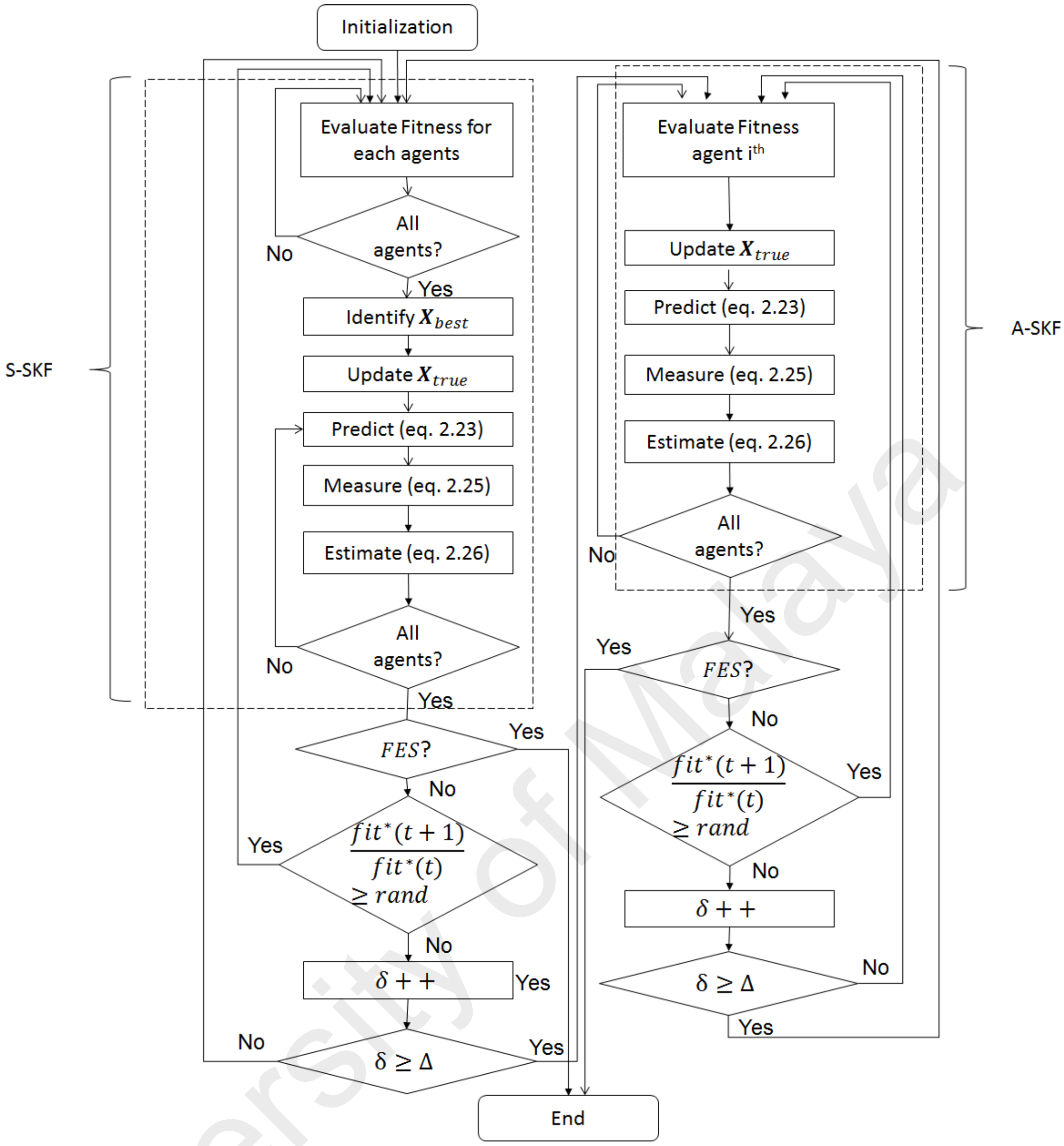

Figure 7.11: Flowchart of ASw-SKF$_s^{rfit^*}$

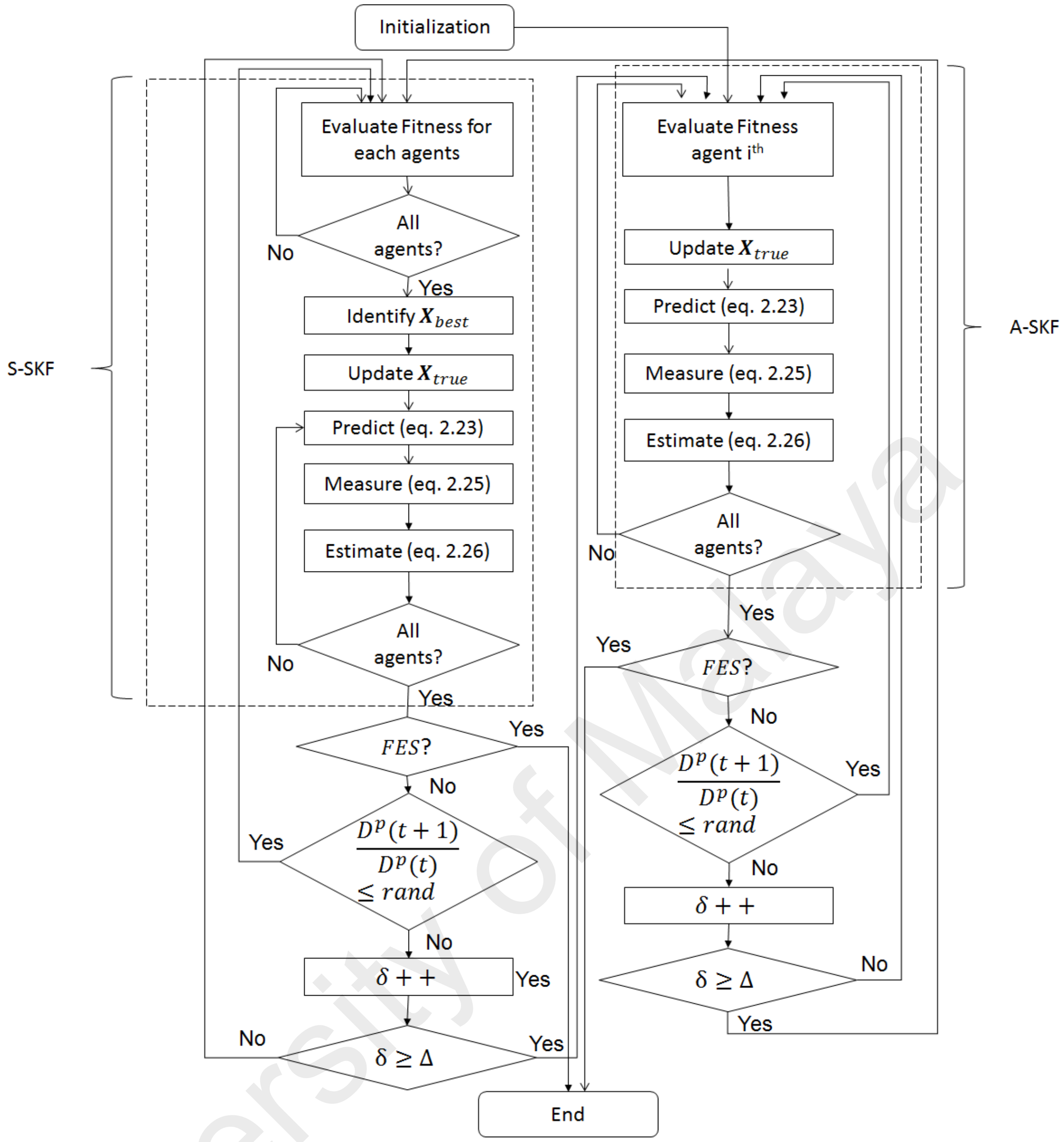

Figure 7.12: Flowchart of ASw-SKF$_a^{rD^p}$

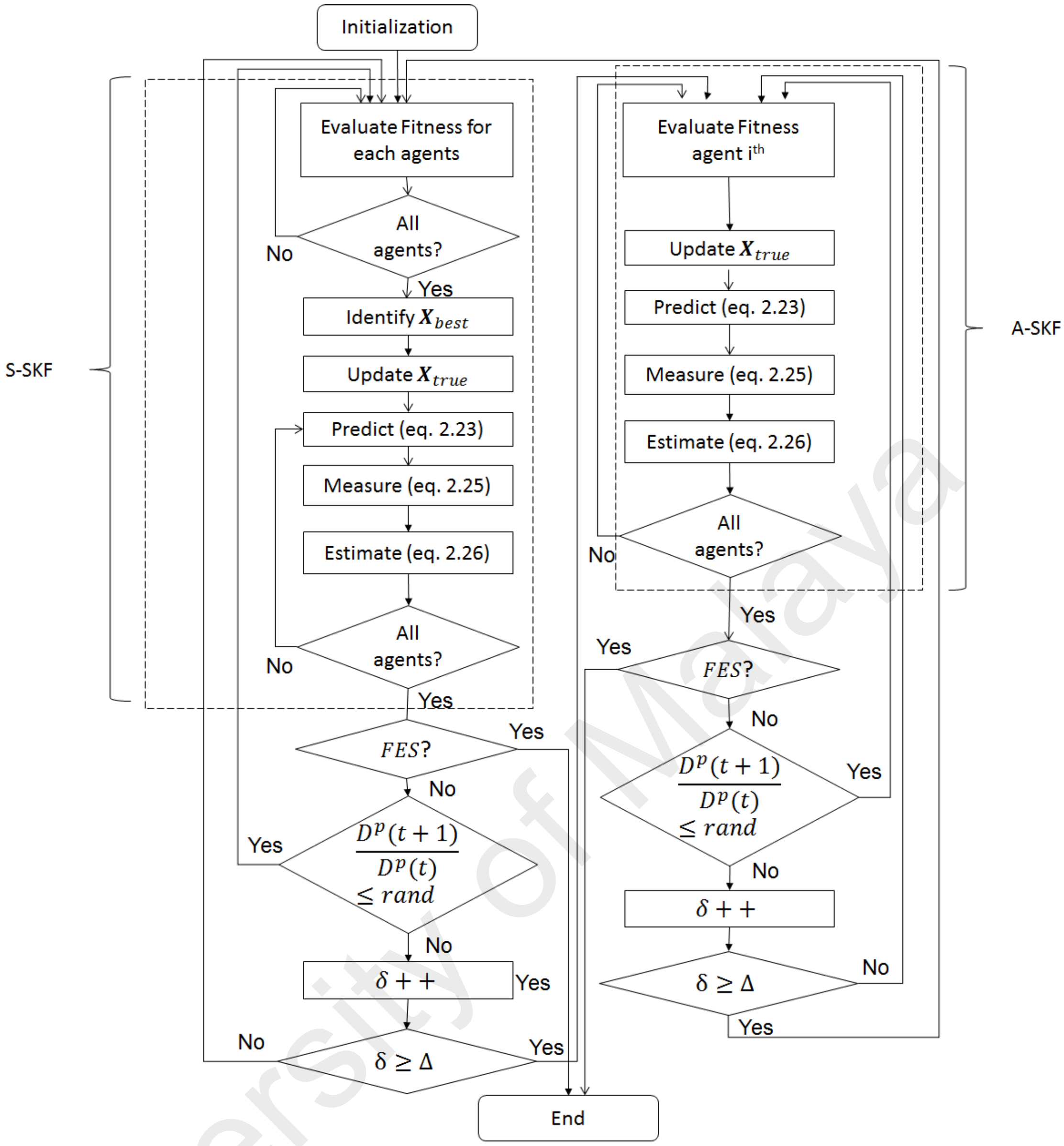

Figure 7.13: Flowchart of ASw-SKF$_s^{rD^p}$

7.4 Experiments, Results and Discussion

7.4.1 Experimental Parameter Settings

The experimental settings for the experiments conducted in this chapter are the same as chapter 4. Like chapter 6, the effect of Δ is tested using $\Delta = \{5\%, 10\%, 15\%, \ldots, 95\%\}$ and only results from experiments with more than 50% of switch are accepted. The number of switching of the experiments conducted here are compiled in Appendix D. Wilcoxon, Friedman and Holm ($\alpha = 5\%$) test are used for the statistical analysis.

7.4.2 Statistical Analysis

7.4.2.1 $\boldsymbol{fit}^{*}$ as the Switching Indicator

ASw-PSO$_{\boldsymbol{a}}^{\boldsymbol{rfit}^{*}}$ - Switching occurred in all setting of Δ and the switching is more frequent compared to PSO with adaptive switching iteration strategy.

The average fitness error values from the experiments are shown in Table 7.1. It can be seen that the smallest average fitness error values (in **boldface**) are fairly distributed among S-PSO, A-PSO and ASw-PSO$_{a}^{rfit^{*}}$ with various value of Δ. There is no dominant iteration strategy observed.

Further analysis was conducted using pairwise Wilcoxon sign rank test. The statistical value of the test is shown in Table 7.2. Comparison of S-PSO with ASw-PSO$_{a}^{rfit^{*}}$ shows that all value of Δ with exception of $\Delta = \{15\%\}$ has statistical values bigger than 152, which indicates statistically on par performance. With $\Delta = \{15\%\}$ ASw-PSO$_{a}^{rfit^{*}}$ fails to neither outperform nor match the performance of S-PSO. ASw-PSO$_{a}^{rfit^{*}}$ with $\Delta = \{10\%, 15\%, 25\%, 35\%, 65\%, 75\%, 80\%\}$ are not as good as A-PSO, with significance level of at least 10%.

Table 7.1: Average Error of ASw-PSO$_{a}^{rfit^*}$

Function ID	S-PSO	A-PSO	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	6.670E+06	**5.200E+06**	7.520E+06	7.740E+06	6.930E+06	7.260E+06	7.650E+06	5.500E+06	9.680E+06	7.670E+06	8.340E+06	7.430E+06
f2	2.879E+02	1.389E+02	1.404E+02	1.366E+02	2.828E+02	1.893E+02	1.649E+02	1.482E+02	**8.268E+01**	1.191E+02	1.957E+02	1.331E+02
f3	3.663E+02	2.945E+02	4.841E+02	3.323E+02	3.721E+02	3.958E+02	3.729E+02	4.387E+02	5.781E+02	2.750E+02	4.445E+02	3.842E+02
f4	1.746E+02	1.608E+02	1.599E+02	1.695E+02	**1.512E+02**	1.591E+02	1.679E+02	1.646E+02	1.644E+02	1.585E+02	1.753E+02	1.718E+02
f5	2.085E+01	2.086E+01	2.084E+01	2.086E+01	2.088E+01	2.085E+01	**2.084E+01**	2.086E+01	2.085E+01	2.089E+01	2.085E+01	2.088E+01
f6	1.033E+01	1.071E+01	1.122E+01	1.076E+01	1.192E+01	1.172E+01	1.074E+01	1.125E+01	1.040E+01	1.063E+01	1.029E+01	1.131E+01
f7	1.058E-02	9.766E-03	5.912E-03	1.215E-02	1.352E-02	1.074E-02	1.026E-02	8.206E-03	1.279E-02	**4.353E-03**	1.173E-02	9.180E-03
f8	1.917E+01	1.857E+01	1.755E+01	1.874E+01	1.901E+01	1.930E+01	1.914E+01	1.960E+01	2.063E+01	1.940E+01	**1.718E+01**	1.973E+01
f9	5.871E+01	6.879E+01	6.106E+01	6.137E+01	6.179E+01	6.245E+01	6.424E+01	**5.843E+01**	6.228E+01	6.876E+01	6.859E+01	6.374E+01
f10	5.584E+02	6.090E+02	6.274E+02	5.539E+02	6.191E+02	5.351E+02	5.796E+02	5.778E+02	6.279E+02	6.036E+02	5.168E+02	**5.005E+02**
f11	2.639E+03	2.839E+03	2.813E+03	2.733E+03	2.724E+03	2.614E+03	3.119E+03	2.816E+03	2.948E+03	**2.549E+03**	2.668E+03	2.554E+03
f12	1.893E+00	1.658E+00	1.703E+00	1.593E+00	1.719E+00	1.777E+00	1.772E+00	1.670E+00	1.918E+00	1.748E+00	1.708E+00	1.614E+00
f13	4.086E-01	4.446E-01	4.464E-01	4.280E-01	4.024E-01	4.411E-01	4.345E-01	4.181E-01	4.040E-01	4.282E-01	4.216E-01	3.923E-01
f14	2.850E-01	3.454E-01	2.827E-01	3.193E-01	3.067E-01	3.071E-01	2.731E-01	3.224E-01	**2.676E-01**	3.090E-01	3.049E-01	2.900E-01
f15	7.404E+00	7.254E+00	6.656E+00	7.757E+00	7.755E+00	7.036E+00	6.874E+00	7.526E+00	6.728E+00	6.991E+00	**6.142E+00**	8.069E+00
f16	1.126E+01	1.122E+01	1.131E+01	1.129E+01	1.148E+01	1.125E+01	1.137E+01	1.142E+01	1.126E+01	1.126E+01	1.122E+01	1.138E+01
f17	6.780E+05	6.340E+05	7.180E+05	7.010E+05	7.970E+05	6.510E+05	5.970E+05	7.190E+05	5.710E+05	**5.260E+05**	5.310E+05	6.680E+05
f18	7.474E+03	4.828E+03	4.808E+03	6.127E+03	5.415E+03	5.034E+03	6.346E+03	**4.545E+03**	5.932E+03	7.912E+03	6.046E+03	4.597E+03
f19	8.054E+00	7.416E+00	7.666E+00	7.729E+00	7.605E+00	7.719E+00	7.483E+00	8.176E+00	7.794E+00	8.196E+00	7.406E+00	7.604E+00
f20	6.018E+02	5.209E+02	5.813E+02	7.354E+02	6.364E+02	**4.664E+02**	6.093E+02	5.462E+02	5.766E+02	5.759E+02	5.627E+02	5.675E+02
f21	1.360E+05	1.660E+05	**1.030E+05**	1.190E+05	1.450E+05	1.160E+05	1.650E+05	1.360E+05	1.440E+05	1.090E+05	1.570E+05	1.470E+05
f22	2.559E+02	2.294E+02	2.358E+02	2.697E+02	2.565E+02	2.856E+02	2.732E+02	2.240E+02	2.841E+02	2.732E+02	3.044E+02	2.474E+02
f23	3.158E+02	3.159E+02	3.159E+02	3.159E+02	3.158E+02	3.159E+02	3.158E+02	**3.158E+02**	3.158E+02	3.159E+02	3.159E+02	3.158E+02
f24	2.329E+02	**2.293E+02**	2.310E+02	2.307E+02	2.299E+02	2.326E+02	2.304E+02	2.304E+02	2.307E+02	2.325E+02	2.307E+02	2.315E+02
f25	2.087E+02	2.091E+02	2.086E+02	2.087E+02	**2.081E+02**	2.084E+02	2.088E+02	2.082E+02	2.085E+02	2.089E+02	2.084E+02	2.085E+02
f26	1.071E+02	1.071E+02	1.104E+02	1.171E+02	1.105E+02	1.148E+02	1.138E+02	**1.038E+02**	1.071E+02	1.245E+02	1.114E+02	1.104E+02
f27	5.512E+02	5.556E+02	5.696E+02	5.844E+02	5.677E+02	5.706E+02	5.545E+02	5.529E+02	5.676E+02	5.525E+02	**5.438E+02**	5.722E+02
f28	1.103E+03	1.142E+03	1.074E+03	1.130E+03	**1.034E+03**	1.052E+03	1.189E+03	1.052E+03	1.214E+03	1.064E+03	1.079E+03	1.069E+03
f29	2.370E+06	1.600E+06	2.330E+06	3.610E+06	3.190E+06	1.520E+06	2.970E+06	1.640E+06	3.790E+06	7.390E+05	2.710E+06	1.530E+06
f30	3.970E+03	3.391E+03	3.681E+03	5.372E+03	3.823E+03	3.501E+03	3.918E+03	3.809E+03	3.667E+03	3.288E+03	3.838E+03	3.453E+03

Table 7.1: Average Error of ASw-PSO$_a^{rfit^*}$ (continued...)

Function ID	S-PSO	A-PSO	Δ								
			55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	6.670E+06	**5.200E+06**	5.810E+06	5.860E+06	5.920E+06	7.550E+06	6.340E+06	8.890E+06	8.070E+06	5.300E+06	5.300E+06
f2	2.879E+02	1.389E+02	1.696E+02	2.863E+02	2.583E+02	2.545E+02	2.424E+02	1.782E+02	2.109E+02	1.109E+02	2.308E+02
f3	3.663E+02	2.945E+02	3.502E+02	3.135E+02	2.978E+02	**2.016E+02**	4.036E+02	3.980E+02	7.504E+02	3.699E+02	3.819E+02
f4	1.746E+02	1.608E+02	1.647E+02	1.682E+02	1.806E+02	1.547E+02	1.689E+02	1.785E+02	1.862E+02	1.748E+02	1.745E+02
f5	2.085E+01	2.086E+01	2.085E+01	2.086E+01	2.084E+01	2.090E+01	2.086E+01	2.086E+01	2.087E+01	2.085E+01	2.088E+01
f6	1.033E+01	1.071E+01	1.061E+01	1.112E+01	1.066E+01	**9.923E+00**	1.082E+01	1.069E+01	1.094E+01	1.040E+01	1.148E+01
f7	1.058E-02	9.766E-03	9.191E-03	1.239E-02	1.190E-02	1.508E-02	1.059E-02	1.222E-02	1.058E-02	9.270E-03	1.360E-02
f8	1.917E+01	1.857E+01	1.882E+01	1.934E+01	1.914E+01	1.940E+01	1.877E+01	1.941E+01	1.887E+01	1.884E+01	2.015E+01
f9	5.871E+01	6.879E+01	6.457E+01	6.457E+01	5.884E+01	6.447E+01	7.037E+01	6.580E+01	6.828E+01	6.215E+01	6.544E+01
f10	5.584E+02	6.090E+02	5.886E+02	5.764E+02	5.250E+02	5.428E+02	5.484E+02	5.237E+02	6.392E+02	6.051E+02	5.633E+02
f11	2.639E+03	2.839E+03	2.606E+03	2.790E+03	2.855E+03	2.757E+03	3.139E+03	2.805E+03	2.831E+03	2.659E+03	2.881E+03
f12	1.893E+00	1.658E+00	1.897E+00	1.764E+00	1.729E+00	1.617E+00	1.793E+00	1.710E+00	1.925E+00	**1.561E+00**	1.634E+00
f13	4.086E-01	4.446E-01	4.295E-01	4.505E-01	4.560E-01	4.290E-01	**3.802E-01**	4.354E-01	4.117E-01	4.512E-01	4.096E-01
f14	2.850E-01	3.454E-01	3.062E-01	3.183E-01	3.485E-01	2.809E-01	2.790E-01	3.285E-01	3.182E-01	3.323E-01	2.990E-01
f15	7.404E+00	7.254E+00	7.386E+00	7.859E+00	6.958E+00	6.453E+00	7.290E+00	6.665E+00	7.177E+00	6.651E+00	6.601E+00
f16	1.126E+01	1.122E+01	1.144E+01	**1.108E+01**	1.130E+01	1.137E+01	1.157E+01	1.154E+01	1.132E+01	1.135E+01	1.148E+01
f17	6.780E+05	6.340E+05	6.970E+05	5.470E+05	7.330E+05	5.950E+05	6.210E+05	7.290E+05	5.650E+05	5.730E+05	6.440E+05
f18	7.474E+03	4.828E+03	1.618E+04	8.297E+03	6.477E+03	7.204E+03	2.910E+05	6.464E+04	8.012E+03	5.498E+03	1.470E+05
f19	8.054E+00	7.416E+00	7.707E+00	**7.306E+00**	7.743E+00	8.307E+00	7.983E+00	7.741E+00	7.796E+00	1.030E+01	8.221E+00
f20	6.018E+02	5.209E+02	5.510E+02	5.726E+02	5.502E+02	5.314E+02	5.906E+02	7.471E+02	6.004E+02	5.408E+02	6.034E+02
f21	1.360E+05	1.660E+05	1.540E+05	1.230E+05	1.670E+05	2.030E+05	1.150E+05	2.020E+05	1.890E+05	1.770E+05	1.090E+05
f22	2.559E+02	2.294E+02	2.624E+02	2.596E+02	**2.116E+02**	2.349E+02	2.717E+02	2.205E+02	2.175E+02	2.758E+02	2.367E+02
f23	3.158E+02	3.159E+02	3.158E+02	3.159E+02	3.159E+02	3.160E+02	3.159E+02	3.159E+02	3.159E+02	3.158E+02	3.159E+02
f24	2.329E+02	**2.293E+02**	2.296E+02	2.309E+02	2.313E+02	2.293E+02	2.294E+02	2.309E+02	2.336E+02	2.311E+02	2.301E+02
f25	2.087E+02	2.091E+02	2.083E+02	2.083E+02	2.093E+02	2.087E+02	2.087E+02	2.085E+02	2.090E+02	2.086E+02	2.083E+02
f26	1.071E+02	1.071E+02	1.071E+02	1.114E+02	1.187E+02	1.071E+02	1.189E+02	1.180E+02	1.146E+02	1.081E+02	1.071E+02
f27	5.512E+02	5.556E+02	5.688E+02	5.577E+02	5.477E+02	5.587E+02	5.703E+02	5.457E+02	5.650E+02	5.838E+02	5.543E+02
f28	1.103E+03	1.142E+03	1.084E+03	1.119E+03	1.171E+03	1.145E+03	1.103E+03	1.221E+03	1.072E+03	1.269E+03	1.123E+03
f29	2.370E+06	1.600E+06	1.340E+06	**7.170E+05**	3.340E+06	2.310E+06	1.120E+06	2.300E+06	7.190E+05	4.380E+06	3.790E+06
f30	3.970E+03	3.391E+03	4.603E+03	4.038E+03	4.272E+03	3.874E+03	3.668E+03	3.392E+03	3.400E+03	**3.245E+03**	3.354E+03

Table 7.2: Wilcoxon Signed Rank Test Statistical Values for ASw-PSO$_a^{rfit^*}$

S-PSO vs ASw-PSO$_a^{rfit^*}$			A-PSO vs ASw-PSO$_a^{rfit^*}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	197.5	267.5	5%	296	169
10%	291	174	10%	317	<u>148</u>
15%	283	<u>152</u>	15%	340	<u>125</u>
20%	153	312	20%	263	202
25%	287	178	25%	326	<u>139</u>
30%	160	275	30%	254	211
35%	270	195	35%	319	<u>146</u>
40%	196	269	40%	178	287
45%	208	257	45%	248	217
50%	171	294	50%	252	213
55%	220	245	55%	286	179
60%	227	238	60%	251	184
65%	243	222	65%	368	<u>97</u>
70%	188	277	70%	287	178
75%	187	278	75%	320	<u>145</u>
80%	288	177	80%	318	<u>117</u>
85%	279	186	85%	312	153
90%	251	214	90%	289	176
95%	258	207	95%	303	162

ASw-PSO$_s^{rfit^*}$ - The randomness increased the probability of switching. The results for the entire experiments for ASw-PSO$_s^{rfit^*}$ are taken for further analysis. The average fitness error values of the test are tabulated in Table 7.3. The **boldfaced** values show the best average fitness error value for the respective functions. It is seen that ASw-PSO$_s^{rfit^*}$ with $\Delta=\{85\%\}$ found more number of the best average errors.

Wilcoxon signed rank test was performed and the statistical values are shown in Table 7.4. From the results, it is observed that ASw-PSO$_s^{rfit^*}$ with $\Delta=\{85\%, 95\%\}$ are significantly better than S-PSO with significance level of 2% and 10% respectively. ASw-PSO$_s^{rfit^*}$ with $\Delta=\{85\%, 95\%\}$ are statistically as good as A-PSO. This shows switching towards the end of the search is able to improve PSO.

Table 7.3: Average Error of ASw-PSO$_s^{rfit^*}$

Function ID	S-PSO	A-PSO	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	6.670E+06	5.200E+06	7.620E+06	7.900E+06	9.210E+06	7.700E+06	**4.740E+06**	8.380E+06	7.280E+06	6.060E+06	7.250E+06	5.960E+06
f2	2.879E+02	1.389E+02	2.799E+02	1.512E+02	2.189E+02	2.093E+02	1.138E+02	1.237E+03	2.675E+02	4.821E+02	2.241E+02	1.661E+02
f3	3.663E+02	2.945E+02	3.678E+02	2.508E+02	**2.041E+02**	3.205E+02	3.970E+02	4.357E+02	2.864E+02	6.795E+02	3.350E+02	4.835E+02
f4	1.746E+02	1.608E+02	1.561E+02	1.771E+02	1.637E+02	1.518E+02	1.759E+02	1.607E+02	1.567E+02	1.703E+02	1.581E+02	1.787E+02
f5	2.085E+01	2.086E+01	**2.083E+01**	2.083E+01	2.085E+01	2.086E+01	2.086E+01	2.085E+01	2.089E+01	2.089E+01	2.085E+01	2.085E+01
f6	1.033E+01	1.071E+01	1.019E+01	1.092E+01	1.098E+01	1.137E+01	1.157E+01	1.145E+01	1.106E+01	**9.319E+00**	1.034E+01	1.198E+01
f7	1.058E-02	9.766E-03	9.849E-03	1.475E-02	1.263E-02	1.205E-02	1.271E-02	**8.865E-03**	1.500E-02	1.066E-02	1.018E-02	1.174E-02
f8	1.917E+01	1.857E+01	1.834E+01	1.891E+01	1.947E+01	1.970E+01	1.861E+01	1.897E+01	2.080E+01	1.861E+01	1.801E+01	1.990E+01
f9	5.871E+01	6.879E+01	6.374E+01	6.309E+01	5.638E+01	6.759E+01	6.218E+01	6.601E+01	6.567E+01	6.226E+01	6.448E+01	5.950E+01
f10	5.584E+02	6.090E+02	6.153E+02	5.902E+02	5.518E+02	5.911E+02	5.622E+02	6.793E+02	5.930E+02	5.303E+02	5.922E+02	5.818E+02
f11	2.639E+03	2.839E+03	2.697E+03	2.770E+03	2.838E+03	2.939E+03	2.679E+03	2.662E+03	2.709E+03	2.719E+03	3.035E+03	2.712E+03
f12	1.893E+00	1.658E+00	1.663E+00	1.855E+00	1.864E+00	1.829E+00	1.653E+00	1.695E+00	1.824E+00	1.606E+00	1.591E+00	**1.555E+00**
f13	4.086E-01	4.446E-01	4.344E-01	4.484E-01	4.067E-01	4.276E-01	4.312E-01	4.504E-01	4.237E-01	4.295E-01	4.228E-01	4.335E-01
f14	2.850E-01	3.454E-01	3.209E-01	3.615E-01	3.192E-01	3.491E-01	2.771E-01	3.078E-01	2.998E-01	3.293E-01	3.514E-01	2.763E-01
f15	7.404E+00	7.254E+00	7.349E+00	6.854E+00	7.054E+00	6.684E+00	6.858E+00	6.894E+00	7.245E+00	6.791E+00	7.292E+00	7.428E+00
f16	1.126E+01	1.122E+01	1.132E+01	1.134E+01	1.130E+01	1.146E+01	1.125E+01	1.145E+01	1.129E+01	1.139E+01	1.154E+01	1.120E+01
f17	6.780E+05	6.340E+05	7.740E+05	7.240E+05	6.920E+05	6.680E+05	6.370E+05	5.750E+05	6.260E+05	6.430E+05	8.150E+05	**5.630E+05**
f18	7.474E+03	**4.828E+03**	6.301E+03	7.096E+03	2.080E+04	8.546E+04	5.300E+03	5.577E+03	2.465E+04	6.595E+03	6.477E+03	7.436E+03
f19	8.054E+00	7.416E+00	8.940E+00	8.304E+00	8.028E+00	7.503E+00	7.909E+00	1.134E+01	7.513E+00	7.957E+00	1.021E+01	7.398E+00
f20	6.018E+02	**5.209E+02**	5.812E+02	5.739E+02	6.045E+02	5.527E+02	6.273E+02	6.040E+02	6.066E+02	6.239E+02	5.774E+02	5.998E+02
f21	1.360E+05	1.660E+05	1.310E+05	1.410E+05	2.010E+05	1.480E+05	1.860E+05	1.880E+05	1.630E+05	1.500E+05	1.400E+05	2.140E+05
f22	2.559E+02	2.294E+02	2.902E+02	2.588E+02	2.385E+02	2.225E+02	2.321E+02	3.227E+02	2.655E+02	**2.212E+02**	2.694E+02	2.907E+02
f23	3.158E+02	3.159E+02	3.159E+02	**3.158E+02**	3.158E+02	3.159E+02	3.159E+02	3.158E+02	3.159E+02	3.158E+02	3.158E+02	3.159E+02
f24	2.329E+02	**2.293E+02**	2.305E+02	2.304E+02	2.310E+02	2.300E+02	2.339E+02	2.304E+02	2.325E+02	2.301E+02	2.297E+02	2.310E+02
f25	2.087E+02	2.091E+02	2.089E+02	2.087E+02	2.086E+02	2.092E+02	2.084E+02	2.087E+02	2.083E+02	2.085E+02	**2.082E+02**	2.083E+02
f26	1.071E+02	1.071E+02	1.113E+02	1.038E+02	1.171E+02	1.071E+02	1.252E+02	1.071E+02	1.183E+02	1.104E+02	1.071E+02	1.071E+02
f27	5.512E+02	5.556E+02	5.133E+02	5.689E+02	5.702E+02	5.666E+02	5.489E+02	5.395E+02	5.320E+02	5.618E+02	5.289E+02	5.267E+02
f28	1.103E+03	1.142E+03	1.181E+03	1.133E+03	1.098E+03	1.203E+03	1.195E+03	1.076E+03	1.078E+03	1.114E+03	1.141E+03	1.085E+03
f29	2.370E+06	1.600E+06	4.460E+06	1.510E+06	1.341E+03	2.370E+06	1.650E+06	3.120E+06	3.260E+06	1.480E+06	2.110E+06	**1.253E+03**
f30	3.970E+03	3.391E+03	3.469E+03	3.434E+03	3.717E+03	3.677E+03	3.691E+03	3.856E+03	4.456E+03	3.898E+03	4.322E+03	3.800E+03

Table 7.3: Average Error of ASw-PSO$_s^{rfit^*}$ (continued...)

Function ID	S-PSO	A-PSO	Δ								
			55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	6.670E+06	5.200E+06	7.200E+06	6.200E+06	6.250E+06	6.830E+06	6.180E+06	6.930E+06	7.430E+06	7.050E+06	6.830E+06
f2	2.879E+02	1.389E+02	1.888E+02	2.251E+02	1.700E+02	2.796E+02	1.045E+02	2.709E+02	**8.648E+01**	2.164E+02	1.518E+02
f3	3.663E+02	2.945E+02	4.664E+02	4.051E+02	4.597E+02	2.144E+02	3.576E+02	4.136E+02	3.498E+02	5.312E+02	2.349E+02
f4	1.746E+02	1.608E+02	1.583E+02	1.737E+02	1.545E+02	1.737E+02	1.603E+02	1.602E+02	**1.490E+02**	1.568E+02	1.736E+02
f5	2.085E+01	2.086E+01	2.088E+01	2.088E+01	2.085E+01	2.088E+01	2.086E+01	2.087E+01	2.083E+01	2.089E+01	2.084E+01
f6	1.033E+01	1.071E+01	1.164E+01	1.087E+01	1.118E+01	1.140E+01	1.090E+01	1.102E+01	1.110E+01	1.004E+01	1.127E+01
f7	1.058E-02	9.766E-03	1.654E-02	1.377E-02	1.771E-02	1.108E-02	1.140E-02	1.182E-02	1.238E-02	1.329E-02	1.164E-02
f8	1.917E+01	1.857E+01	1.738E+01	1.910E+01	1.920E+01	2.073E+01	2.000E+01	1.871E+01	1.818E+01	1.781E+01	**1.737E+01**
f9	5.871E+01	6.879E+01	6.594E+01	6.520E+01	6.321E+01	6.454E+01	**5.585E+01**	5.844E+01	5.947E+01	6.215E+01	5.927E+01
f10	5.584E+02	6.090E+02	6.049E+02	5.696E+02	5.704E+02	**5.160E+02**	6.224E+02	5.656E+02	6.775E+02	5.507E+02	5.591E+02
f11	2.639E+03	2.839E+03	2.965E+03	2.917E+03	2.845E+03	2.764E+03	2.888E+03	2.901E+03	**2.593E+03**	2.628E+03	2.933E+03
f12	1.893E+00	1.658E+00	1.802E+00	1.588E+00	1.858E+00	1.747E+00	1.826E+00	1.789E+00	1.854E+00	1.653E+00	1.561E+00
f13	4.086E-01	4.446E-01	4.334E-01	**4.042E-01**	4.244E-01	4.274E-01	4.322E-01	4.562E-01	4.083E-01	4.065E-01	4.418E-01
f14	2.850E-01	3.454E-01	2.879E-01	3.006E-01	2.974E-01	3.286E-01	2.966E-01	3.499E-01	**2.661E-01**	3.273E-01	3.214E-01
f15	7.404E+00	7.254E+00	6.913E+00	7.187E+00	6.477E+00	6.649E+00	6.561E+00	7.010E+00	7.016E+00	7.104E+00	**6.218E+00**
f16	1.126E+01	1.122E+01	1.107E+01	1.155E+01	1.121E+01	1.128E+01	1.114E+01	1.127E+01	1.120E+01	**1.103E+01**	1.130E+01
f17	6.780E+05	6.340E+05	5.760E+05	8.330E+05	6.300E+05	5.730E+05	6.340E+05	6.730E+05	6.100E+05	7.880E+05	6.510E+05
f18	7.474E+03	**4.828E+03**	2.661E+04	8.384E+03	5.583E+03	5.963E+03	5.581E+03	6.820E+03	8.318E+03	6.630E+03	7.468E+03
f19	8.054E+00	7.416E+00	7.481E+00	7.727E+00	7.823E+00	1.017E+01	**7.231E+00**	7.370E+00	7.719E+00	9.696E+00	9.764E+00
f20	6.018E+02	**5.209E+02**	5.606E+02	6.683E+02	6.124E+02	5.774E+02	6.366E+02	6.643E+02	5.441E+02	5.493E+02	5.776E+02
f21	1.360E+05	1.660E+05	1.190E+05	1.950E+05	1.720E+05	1.440E+05	1.890E+05	1.370E+05	**1.110E+05**	1.600E+05	1.210E+05
f22	2.559E+02	2.294E+02	3.138E+02	2.683E+02	2.954E+02	2.602E+02	3.125E+02	2.497E+02	2.424E+02	2.492E+02	2.635E+02
f23	3.158E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	**3.158E+02**	3.158E+02	3.159E+02	3.159E+02
f24	2.329E+02	**2.293E+02**	2.312E+02	2.320E+02	2.318E+02	2.328E+02	2.308E+02	2.306E+02	2.296E+02	2.328E+02	2.303E+02
f25	2.087E+02	2.091E+02	2.082E+02	2.090E+02	2.084E+02	2.086E+02	2.093E+02	2.090E+02	2.086E+02	2.084E+02	2.087E+02
f26	1.071E+02	1.071E+02	1.076E+02	1.147E+02	1.171E+02	1.104E+02	1.037E+02	**1.004E+02**	1.104E+02	1.071E+02	1.071E+02
f27	5.512E+02	5.556E+02	5.732E+02	5.329E+02	5.677E+02	5.671E+02	5.389E+02	5.597E+02	5.412E+02	5.556E+02	**4.998E+02**
f28	1.103E+03	1.142E+03	1.153E+03	1.179E+03	1.227E+03	1.150E+03	1.095E+03	1.135E+03	**1.041E+03**	1.198E+03	1.085E+03
f29	2.370E+06	1.600E+06	3.430E+06	1.730E+06	6.080E+06	1.368E+03	1.290E+03	2.480E+06	1.460E+06	1.361E+03	1.810E+06
f30	3.970E+03	3.391E+03	3.674E+03	3.407E+03	**3.223E+03**	3.428E+03	3.539E+03	3.983E+03	3.489E+03	3.535E+03	3.648E+03

Table 7.4: Wilcoxon Signed Rank Test Statistical Values for ASw-PSO$_s^{rfit^*}$

S-PSO vs ASw-PSO$_s^{rfit^*}$			A-PSO vs ASw-PSO$_s^{rfit^*}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	260	205	5%	299	<u>136</u>
10%	254	211	10%	266.5	198.5
15%	231	234	15%	311	154
20%	243	192	20%	348	<u>117</u>
25%	220	245	25%	299	166
30%	280	185	30%	313	<u>152</u>
35%	290	175	35%	260	205
40%	202	263	40%	263	202
45%	251	214	45%	293	172
50%	196	269	50%	260	205
55%	264	201	55%	294	<u>141</u>
60%	280	185	60%	384	<u>81</u>
65%	259	206	65%	325	<u>140</u>
70%	228	237	70%	271	194
75%	158	307	75%	284.5	180.5
80%	272	193	80%	322	<u>143</u>
85%	<u>112</u>	323	85%	210	255
90%	196	269	90%	258	207
95%	<u>151</u>	314	95%	276	189

ASw-GSA$_a^{rfit^*}$ - With randomness, the ASw-GSA$_a^{rfit^*}$ worked almost like a periodical switch. Maximum number of switch for all value of Δ were recorded.

The average fitness error values are tabulated in Table 7.5. It can be seen that synchronous update is the better iteration strategy. S-GSA found the smallest average errors in 27 functions. However, the agents of S-GSA were not able to efficiently solve three functions; f16, f26 and f27.

Pairwise analysis using Wilcoxon signed rank test was conducted. The statistical values from the test are shown in Table 7.6. For all value of Δ, ASw-GSA$_a^{rfit^*}$ was not able to outperform S-GSA. Statistically, ASw-GSA$_a^{rfit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$ are significantly better than A-GSA with significance level of at least 5%.

Table 7.5: Average Error of ASw-GSA$_a^{rfit^*}$

Function ID	S-GSA	A-GSA	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	**1.300E+07**	7.110E+08	2.170E+08	3.580E+08	7.050E+08	7.590E+08	7.820E+08	7.550E+08	7.650E+08	7.640E+08	7.770E+08	7.760E+08
f2	**8.603E+03**	5.940E+10	7.330E+09	2.140E+10	4.370E+10	5.400E+10	5.640E+10	5.780E+10	5.860E+10	5.840E+10	6.040E+10	6.000E+10
f3	**5.784E+04**	9.770E+04	7.549E+04	5.803E+04	6.986E+04	8.456E+04	8.846E+04	8.738E+04	9.354E+04	8.870E+04	8.730E+04	9.392E+04
f4	**3.017E+02**	1.013E+04	2.231E+03	3.419E+03	6.711E+03	9.843E+03	1.020E+04	1.068E+04	1.045E+04	1.052E+04	1.071E+04	1.090E+04
f5	**2.000E+01**	2.095E+01	2.091E+01	2.005E+01	2.005E+01	2.096E+01	2.098E+01	2.095E+01	2.096E+01	2.094E+01	2.097E+01	2.097E+01
f6	**1.907E+01**	3.895E+01	3.499E+01	3.938E+01	3.935E+01	3.953E+01	3.933E+01	3.930E+01	3.914E+01	3.908E+01	3.942E+01	3.928E+01
f7	**0.000E+00**	5.439E+02	8.965E+01	2.373E+02	4.392E+02	5.220E+02	5.419E+02	5.399E+02	5.457E+02	5.451E+02	5.521E+02	5.596E+02
f8	**1.405E+02**	3.285E+02	1.535E+02	1.506E+02	2.334E+02	3.239E+02	3.173E+02	3.231E+02	3.194E+02	3.239E+02	3.269E+02	3.353E+02
f9	**1.624E+02**	3.781E+02	1.709E+02	1.696E+02	2.173E+02	3.443E+02	3.512E+02	3.536E+02	3.519E+02	3.552E+02	3.557E+02	3.683E+02
f10	**3.370E+03**	7.018E+03	4.288E+03	4.452E+03	5.648E+03	7.084E+03	7.088E+03	7.123E+03	7.015E+03	7.193E+03	7.219E+03	7.181E+03
f11	**4.058E+03**	7.155E+03	4.505E+03	4.636E+03	5.948E+03	7.249E+03	7.207E+03	7.267E+03	7.228E+03	7.197E+03	7.270E+03	7.297E+03
f12	**4.870E-04**	2.450E+00	1.566E-01	2.067E-01	2.114E+00	2.640E+00	2.677E+00	2.549E+00	2.590E+00	2.584E+00	2.603E+00	2.550E+00
f13	**3.017E-01**	6.146E+00	2.513E+00	4.241E+00	5.660E+00	6.251E+00	6.251E+00	6.256E+00	6.233E+00	6.380E+00	6.354E+00	6.302E+00
f14	**2.433E-01**	1.751E+02	3.578E+01	8.779E+01	1.587E+02	1.817E+02	1.852E+02	1.878E+02	1.843E+02	1.833E+02	1.798E+02	1.919E+02
f15	**3.659E+00**	3.470E+05	6.911E+01	1.902E+03	7.490E+04	2.040E+05	2.460E+05	2.660E+05	2.400E+05	2.540E+05	2.380E+05	3.350E+05
f16	1.363E+01	**1.309E+01**	1.309E+01	1.310E+01	1.313E+01	1.310E+01	1.310E+01	1.314E+01	1.315E+01	1.317E+01	1.317E+01	1.313E+01
f17	**5.310E+05**	1.840E+07	1.660E+07	1.360E+07	2.240E+07	2.290E+07	2.100E+07	2.520E+07	2.270E+07	2.130E+07	2.200E+07	2.350E+07
f18	**3.817E+02**	9.810E+08	2.649E+03	7.608E+02	9.000E+08	1.180E+09	1.160E+09	1.150E+09	1.020E+09	1.090E+09	1.190E+09	1.270E+09
f19	**1.153E+02**	2.924E+02	1.496E+02	1.715E+02	2.454E+02	2.701E+02	2.942E+02	2.792E+02	2.936E+02	2.841E+02	2.910E+02	2.942E+02
f20	**4.521E+04**	7.100E+04	7.560E+04	6.244E+04	6.579E+04	7.750E+04	8.659E+04	7.521E+04	8.367E+04	7.700E+04	8.467E+04	8.964E+04
f21	**1.550E+05**	4.760E+06	4.550E+06	2.590E+06	5.570E+06	6.330E+06	5.650E+06	4.560E+06	5.180E+06	5.040E+06	5.310E+06	5.170E+06
f22	**9.562E+02**	1.300E+03	1.121E+03	1.271E+03	1.396E+03	1.371E+03	1.406E+03	1.363E+03	1.364E+03	1.386E+03	1.466E+03	1.362E+03
f23	**2.130E+02**	6.697E+02	3.555E+02	3.703E+02	6.025E+02	6.850E+02	6.968E+02	6.796E+02	6.720E+02	6.945E+02	7.163E+02	6.972E+02
f24	**2.000E+02**	2.726E+02	2.083E+02	2.161E+02	2.282E+02	2.516E+02	2.645E+02	2.658E+02	2.691E+02	2.671E+02	2.688E+02	2.768E+02
f25	**2.000E+02**	2.249E+02	2.031E+02	2.017E+02	2.066E+02	2.150E+02	2.214E+02	2.217E+02	2.221E+02	2.224E+02	2.233E+02	2.246E+02
f26	1.868E+02	**1.064E+02**	1.069E+02	1.068E+02	1.069E+02	1.069E+02	1.070E+02	1.067E+02	1.066E+02	1.070E+02	1.066E+02	1.070E+02
f27	1.179E+03	8.293E+02	8.770E+02	**8.278E+02**	8.753E+02	8.831E+02	8.986E+02	8.960E+02	8.610E+02	8.793E+02	8.783E+02	8.917E+02
f28	**1.257E+03**	4.703E+03	1.680E+03	1.362E+03	2.953E+03	4.939E+03	5.029E+03	4.884E+03	4.938E+03	4.848E+03	4.992E+03	4.891E+03
f29	**2.001E+02**	1.170E+08	1.050E+08	1.090E+08	1.480E+08	1.430E+08	1.390E+08	1.430E+08	1.420E+08	1.540E+08	1.550E+08	1.580E+08
f30	**1.096E+04**	7.470E+05	9.310E+05	9.640E+05	1.010E+06	1.020E+06	9.710E+05	9.600E+05	9.220E+05	9.850E+05	9.370E+05	1.020E+06

Table 7.5: Average Error of ASw-GSA$_a^{rfit^*}$ (continued...)

Function ID	S-GSA	A-GSA	Δ								
			55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	**1.300E+07**	7.110E+08	7.810E+08	7.880E+08	7.850E+08	7.130E+08	7.570E+08	6.850E+08	6.850E+08	6.890E+08	7.020E+08
f2	**8.603E+03**	5.940E+10	6.180E+10	6.090E+10	6.000E+10	5.900E+10	5.910E+10	5.820E+10	5.820E+10	5.640E+10	6.030E+10
f3	**5.784E+04**	9.770E+04	9.478E+04	9.862E+04	9.461E+04	9.480E+04	9.091E+04	9.700E+04	9.700E+04	9.259E+04	9.884E+04
f4	**3.017E+02**	1.013E+04	1.091E+04	1.101E+04	1.056E+04	1.043E+04	1.027E+04	1.000E+04	1.000E+04	1.010E+04	1.065E+04
f5	**2.000E+01**	2.095E+01	2.098E+01	2.097E+01	2.095E+01	2.095E+01	2.096E+01	2.094E+01	2.094E+01	2.094E+01	2.095E+01
f6	**1.907E+01**	3.895E+01	3.962E+01	3.902E+01	3.923E+01	3.895E+01	3.920E+01	3.884E+01	3.884E+01	3.904E+01	3.877E+01
f7	**0.000E+00**	5.439E+02	5.575E+02	5.346E+02	5.431E+02	5.530E+02	5.178E+02	5.557E+02	5.557E+02	5.412E+02	5.538E+02
f8	**1.405E+02**	3.285E+02	3.353E+02	3.304E+02	3.361E+02	3.333E+02	3.342E+02	3.244E+02	3.244E+02	3.294E+02	3.341E+02
f9	**1.624E+02**	3.781E+02	3.651E+02	3.685E+02	3.645E+02	3.647E+02	3.704E+02	3.636E+02	3.636E+02	3.682E+02	3.672E+02
f10	**3.370E+03**	7.018E+03	7.230E+03	7.130E+03	7.114E+03	7.108E+03	7.067E+03	7.057E+03	7.057E+03	7.009E+03	7.048E+03
f11	**4.058E+03**	7.155E+03	7.222E+03	7.260E+03	7.288E+03	7.199E+03	7.207E+03	7.180E+03	7.180E+03	7.129E+03	7.157E+03
f12	**4.870E-04**	2.450E+00	2.642E+00	2.524E+00	2.606E+00	2.535E+00	2.517E+00	2.609E+00	2.609E+00	2.489E+00	2.521E+00
f13	**3.017E-01**	6.146E+00	6.349E+00	6.306E+00	6.298E+00	6.273E+00	6.346E+00	6.310E+00	6.310E+00	6.271E+00	6.169E+00
f14	**2.433E-01**	1.751E+02	1.906E+02	1.921E+02	1.869E+02	1.802E+02	1.928E+02	1.806E+02	1.806E+02	1.882E+02	1.742E+02
f15	**3.659E+00**	3.470E+05	3.230E+05	2.890E+05	3.240E+05	3.520E+05	3.270E+05	3.370E+05	3.370E+05	3.830E+05	3.170E+05
f16	1.363E+01	**1.309E+01**	1.315E+01	1.312E+01	1.314E+01	1.313E+01	1.313E+01	1.310E+01	1.310E+01	1.309E+01	1.310E+01
f17	**5.310E+05**	1.840E+07	2.500E+07	2.460E+07	2.210E+07	2.040E+07	2.070E+07	2.120E+07	2.120E+07	2.020E+07	1.850E+07
f18	**3.817E+02**	9.810E+08	1.070E+09	1.110E+09	1.120E+09	1.110E+09	1.100E+09	1.160E+09	1.160E+09	1.020E+09	8.770E+08
f19	**1.153E+02**	2.924E+02	2.763E+02	2.859E+02	2.829E+02	2.804E+02	2.913E+02	2.901E+02	2.901E+02	2.724E+02	2.709E+02
f20	**4.521E+04**	7.100E+04	8.141E+04	7.492E+04	7.283E+04	7.240E+04	7.547E+04	6.527E+04	6.527E+04	6.313E+04	7.087E+04
f21	**1.550E+05**	4.760E+06	5.100E+06	4.850E+06	4.970E+06	4.700E+06	4.150E+06	4.360E+06	4.360E+06	4.450E+06	4.120E+06
f22	**9.562E+02**	1.300E+03	1.402E+03	1.411E+03	1.362E+03	1.378E+03	1.349E+03	1.354E+03	1.354E+03	1.304E+03	1.365E+03
f23	**2.130E+02**	6.697E+02	7.078E+02	7.034E+02	7.080E+02	6.891E+02	6.898E+02	6.844E+02	6.844E+02	6.700E+02	6.702E+02
f24	**2.000E+02**	2.726E+02	2.707E+02	2.744E+02	2.770E+02	2.752E+02	2.744E+02	2.752E+02	2.752E+02	2.728E+02	2.761E+02
f25	**2.000E+02**	2.249E+02	2.247E+02	2.257E+02	2.254E+02	2.252E+02	2.275E+02	2.251E+02	2.251E+02	2.246E+02	2.257E+02
f26	1.868E+02	**1.064E+02**	1.070E+02	1.070E+02	1.066E+02	1.065E+02	1.067E+02	1.064E+02	1.064E+02	1.066E+02	1.067E+02
f27	1.179E+03	8.293E+02	8.780E+02	8.860E+02	8.404E+02	8.794E+02	8.555E+02	8.696E+02	8.696E+02	8.518E+02	8.546E+02
f28	**1.257E+03**	4.703E+03	4.917E+03	4.925E+03	5.024E+03	4.764E+03	4.880E+03	4.868E+03	4.868E+03	4.806E+03	4.790E+03
f29	**2.001E+02**	1.170E+08	1.510E+08	1.510E+08	1.380E+08	1.430E+08	1.290E+08	1.470E+08	1.470E+08	1.250E+08	1.300E+08
f30	**1.096E+04**	7.470E+05	1.010E+06	9.830E+05	9.140E+05	9.860E+05	8.570E+05	7.830E+05	7.830E+05	8.050E+05	7.930E+05

Table 7.6: Wilcoxon Signed Rank Test Statistical Values for ASw-GSA$_a^{rfit^*}$

S-GSA vs ASw-GSA$_a^{rfit^*}$			A-GSA vs ASw-GSA$_a^{rfit^*}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	433	32	5%	55	410
10%	433	32	10%	29	436
15%	441	24	15%	135	330
20%	443	22	20%	304	161
25%	443	22	25%	339	126
30%	443	22	30%	305	160
35%	443	22	35%	330	135
40%	443	22	40%	337	128
45%	443	22	45%	374	91
50%	443	22	50%	405	60
55%	443	22	55%	385	80
60%	443	22	60%	409	56
65%	443	22	65%	387	78
70%	443	22	70%	364	101
75%	443	22	75%	333	132
80%	443	22	80%	262	203
85%	443	22	85%	262	203
90%	443	22	90%	237	228
95%	443	22	95%	276	159

ASw-GSA$_s^{rfit^*}$ - The average number of switching from all experiments with exception of $\Delta = \{55\%\}$ are more than 50%. There was no switching by ASw-GSA$_s^{rfit^*}$ with $\Delta = \{55\%\}$ for all functions. Hence, the result from this test is omitted.

The average fitness error for the functions are listed in Table 7.7. It can be seen that unlike ASw-GSA$_s^{rfit^*}$, in this test, the smallest average fitness errors were not dominated by S-GSA. The smallest values were distributed among the tested algorithms.

The results of pairwise statistical analysis using Wilcoxon sign ranked test are presented in Table 7.8. It is found that ASw-GSA$_s^{rfit^*}$ with $\Delta = \{40\%, 60\%, 80\%\}$ are significantly better than S-GSA with significance level of 10% and ASw-GSA$_s^{rfit^*}$ with $\Delta = \{65\%, 70\%, 90\%\}$ are better than S-GSA with level of significance of 5%. On the other hand, ASw-GSA$_s^{rfit^*}$ with $\Delta = \{5\%, 10\%\}$ are worse than S-GSA with significance level of 1% and 2% respectively. The results of Wilcoxon sign rank test against A-GSA show that ASw-GSA$_s^{rfit^*}$ with all values of Δ performed significantly better with 1% significance level.

The findings show that, even though synchronous update is a good iteration strategy for GSA, integration of asynchronous update as part of the iteration strategy towards the later stage of the search provides disturbance to the population diversity of GSA and improves the GSA's overall performance. On the other hand, as observed for $\Delta = \{5\%, 10\%\}$ too many switching is bad for the performance.

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Table 7.7: Average Error of ASw-GSA$_{s}^{rfit^{*}}$

Function ID	S-GSA	A-GSA	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	1.300E+07	7.110E+08	2.170E+08	5.510E+07	2.200E+07	1.630E+07	1.360E+07	1.170E+07	1.140E+07	1.130E+07	1.170E+07	1.180E+07
f2	8.603E+03	5.940E+10	7.330E+09	8.593E+04	1.243E+04	8.513E+03	8.928E+03	8.439E+03	9.358E+03	8.511E+03	8.114E+03	8.698E+03
f3	5.784E+04	9.770E+04	7.549E+04	7.777E+04	7.048E+04	5.707E+04	5.157E+04	5.309E+04	5.166E+04	5.270E+04	5.211E+04	5.408E+04
f4	3.017E+02	1.013E+04	2.231E+03	3.216E+02	2.916E+02	2.863E+02	2.752E+02	2.777E+02	2.719E+02	2.578E+02	**2.525E+02**	2.617E+02
f5	**2.000E+01**	2.095E+01	2.091E+01	2.089E+01	2.018E+01	2.003E+01	2.000E+01	2.000E+01	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**
f6	1.907E+01	3.895E+01	3.499E+01	2.115E+01	2.026E+01	2.017E+01	1.967E+01	1.987E+01	1.979E+01	1.906E+01	1.961E+01	1.957E+01
f7	**0.000E+00**	5.439E+02	8.965E+01	8.282E-01	1.996E-01	3.041E-02	3.798E-03	4.327E-04	5.070E-05	5.880E-06	6.270E-07	7.540E-08
f8	1.405E+02	3.285E+02	1.535E+02	1.407E+02	1.398E+02	1.418E+02	1.397E+02	1.384E+02	1.387E+02	1.405E+02	1.457E+02	1.391E+02
f9	1.624E+02	3.781E+02	1.709E+02	1.647E+02	1.584E+02	1.625E+02	1.617E+02	**1.581E+02**	1.630E+02	1.636E+02	1.636E+02	1.642E+02
f10	3.370E+03	7.018E+03	4.288E+03	3.268E+03	3.359E+03	3.203E+03	3.320E+03	3.315E+03	3.229E+03	3.218E+03	3.245E+03	3.172E+03
f11	4.058E+03	7.155E+03	4.505E+03	**3.830E+03**	4.009E+03	4.153E+03	4.040E+03	4.072E+03	4.038E+03	3.929E+03	4.194E+03	4.145E+03
f12	**4.870E-04**	2.450E+00	1.566E-01	9.275E-02	2.608E-02	8.690E-03	3.925E-03	1.692E-03	1.726E-03	1.113E-03	8.768E-04	6.079E-04
f13	3.017E-01	6.146E+00	2.513E+00	3.754E-01	3.343E-01	3.336E-01	3.220E-01	3.242E-01	3.056E-01	3.113E-01	3.067E-01	3.003E-01
f14	2.433E-01	1.751E+02	3.578E+01	2.605E-01	2.497E-01	2.446E-01	2.428E-01	2.401E-01	2.471E-01	2.499E-01	2.466E-01	2.306E-01
f15	3.659E+00	3.470E+05	6.911E+01	9.076E+00	4.616E+00	3.586E+00	4.124E+00	3.891E+00	**3.575E+00**	3.745E+00	3.786E+00	3.813E+00
f16	1.363E+01	1.309E+01	1.309E+01	1.316E+01	1.313E+01	1.314E+01	1.313E+01	**1.308E+01**	1.323E+01	1.322E+01	1.317E+01	1.316E+01
f17	**5.310E+05**	1.840E+07	1.660E+07	5.290E+06	1.940E+06	1.050E+06	8.150E+05	6.300E+05	5.880E+05	5.820E+05	5.680E+05	5.390E+05
f18	3.817E+02	9.810E+08	2.649E+03	2.325E+03	4.971E+02	4.405E+02	4.147E+02	3.669E+02	4.114E+02	3.463E+02	3.502E+02	4.089E+02
f19	1.153E+02	2.924E+02	1.496E+02	1.087E+02	1.033E+02	1.143E+02	1.020E+02	9.266E+01	8.703E+01	9.522E+01	9.338E+01	9.554E+01
f20	4.521E+04	7.100E+04	7.560E+04	7.731E+04	7.843E+04	5.460E+04	4.247E+04	4.359E+04	3.953E+04	3.793E+04	3.945E+04	3.898E+04
f21	1.550E+05	4.760E+06	4.550E+06	1.750E+06	3.770E+05	2.060E+05	1.830E+05	1.740E+05	1.680E+05	1.590E+05	1.570E+05	**1.500E+05**
f22	9.562E+02	1.300E+03	1.121E+03	8.981E+02	8.655E+02	9.080E+02	9.101E+02	8.740E+02	9.463E+02	9.100E+02	9.451E+02	8.665E+02
f23	2.130E+02	6.697E+02	3.555E+02	2.195E+02	2.074E+02	2.073E+02	2.099E+02	2.003E+02	2.001E+02	2.092E+02	2.000E+02	2.125E+02
f24	**2.000E+02**	2.726E+02	2.083E+02	2.018E+02	2.007E+02	2.003E+02	2.001E+02	2.001E+02	2.000E+02	2.000E+02	2.000E+02	**2.000E+02**
f25	**2.000E+02**	2.249E+02	2.031E+02	2.003E+02	2.001E+02	2.000E+02	2.000E+02	2.000E+02	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**
f26	1.868E+02	**1.064E+02**	1.069E+02	1.070E+02	1.070E+02	1.072E+02	1.071E+02	1.073E+02	1.072E+02	1.078E+02	1.077E+02	1.073E+02
f27	1.179E+03	8.293E+02	8.770E+02	7.972E+02	8.326E+02	7.932E+02	**7.299E+02**	7.784E+02	7.558E+02	7.782E+02	8.795E+02	8.224E+02
f28	1.257E+03	4.703E+03	1.680E+03	1.328E+03	1.409E+03	1.217E+03	1.226E+03	**1.105E+03**	1.132E+03	1.390E+03	1.330E+03	1.262E+03
f29	**2.001E+02**	1.170E+08	1.050E+08	2.050E+07	6.820E+06	1.680E+06	4.170E+05	6.402E+04	1.047E+04	3.183E+03	2.923E+02	2.353E+02
f30	1.096E+04	7.470E+05	9.310E+05	1.830E+05	1.070E+05	5.139E+04	2.115E+04	1.408E+04	1.267E+04	1.080E+04	1.320E+04	1.410E+04

Table 7.7: Average Error of ASw-GSA$_s^{rfit^*}$ (continued...)

Function ID	S-GSA	A-GSA	Δ 60%	65%	70%	75%	80%	85%	90%	95%
f1	1.300E+07	7.110E+08	1.170E+07	1.160E+07	1.180E+07	1.140E+07	**1.070E+07**	1.180E+07	1.180E+07	1.210E+07
f2	8.603E+03	5.940E+10	8.174E+03	**7.923E+03**	8.394E+03	8.219E+03	8.077E+03	8.471E+03	8.200E+03	8.157E+03
f3	5.784E+04	9.770E+04	5.442E+04	5.183E+04	5.092E+04	4.951E+04	5.106E+04	5.076E+04	**4.852E+04**	5.238E+04
f4	3.017E+02	1.013E+04	2.659E+02	2.722E+02	2.632E+02	2.637E+02	2.673E+02	2.540E+02	2.663E+02	2.742E+02
f5	**2.000E+01**	2.095E+01	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**
f6	1.907E+01	3.895E+01	1.916E+01	2.000E+01	1.956E+01	1.923E+01	1.940E+01	1.980E+01	**1.891E+01**	1.963E+01
f7	**0.000E+00**	5.439E+02	1.060E-09	1.190E-10	1.300E-11	1.480E-12	1.140E-13	**0.000E+00**	**0.000E+00**	**0.000E+00**
f8	1.405E+02	3.285E+02	1.389E+02	1.422E+02	1.393E+02	1.404E+02	**1.383E+02**	1.424E+02	1.388E+02	1.421E+02
f9	1.624E+02	3.781E+02	1.634E+02	1.616E+02	1.631E+02	1.626E+02	1.651E+02	1.625E+02	1.585E+02	1.638E+02
f10	3.370E+03	7.018E+03	3.356E+03	3.342E+03	3.342E+03	3.298E+03	3.245E+03	3.280E+03	3.253E+03	**3.143E+03**
f11	4.058E+03	7.155E+03	3.909E+03	3.964E+03	4.022E+03	4.056E+03	4.024E+03	4.055E+03	3.945E+03	4.102E+03
f12	**4.870E-04**	2.450E+00	6.720E-04	6.267E-04	1.129E-03	1.169E-03	1.072E-03	1.066E-03	6.769E-04	5.333E-04
f13	3.017E-01	6.146E+00	**2.881E-01**	2.951E-01	3.000E-01	2.966E-01	3.096E-01	2.910E-01	3.004E-01	2.931E-01
f14	2.433E-01	1.751E+02	2.473E-01	**2.302E-01**	2.473E-01	2.452E-01	2.447E-01	2.353E-01	2.350E-01	2.385E-01
f15	3.659E+00	3.470E+05	3.639E+00	3.674E+00	3.772E+00	3.803E+00	3.634E+00	3.677E+00	3.803E+00	3.667E+00
f16	1.363E+01	1.309E+01	1.316E+01	1.315E+01	1.319E+01	1.325E+01	1.325E+01	1.327E+01	1.325E+01	1.333E+01
f17	**5.310E+05**	1.840E+07	6.100E+05	5.420E+05	5.580E+05	5.700E+05	5.610E+05	5.650E+05	5.530E+05	6.060E+05
f18	3.817E+02	9.810E+08	**3.429E+02**	4.345E+02	4.637E+02	4.499E+02	3.870E+02	4.023E+02	4.356E+02	3.693E+02
f19	1.153E+02	2.924E+02	8.790E+01	9.500E+01	8.738E+01	9.512E+01	**8.383E+01**	8.588E+01	9.642E+01	8.546E+01
f20	4.521E+04	7.100E+04	4.005E+04	3.619E+04	3.926E+04	3.540E+04	3.793E+04	3.691E+04	**3.397E+04**	3.570E+04
f21	1.550E+05	4.760E+06	1.590E+05	**1.500E+05**	1.650E+05	1.630E+05	1.600E+05	1.680E+05	1.630E+05	1.660E+05
f22	9.562E+02	1.300E+03	9.001E+02	8.710E+02	8.806E+02	8.586E+02	9.219E+02	9.105E+02	8.841E+02	**8.227E+02**
f23	2.130E+02	6.697E+02	2.041E+02	2.085E+02	**2.000E+02**	**2.000E+02**	2.089E+02	2.083E+02	2.043E+02	2.084E+02
f24	**2.000E+02**	2.726E+02	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**
f25	**2.000E+02**	2.249E+02	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**
f26	1.868E+02	**1.064E+02**	1.078E+02	1.078E+02	1.084E+02	1.087E+02	1.088E+02	1.084E+02	1.087E+02	1.165E+02
f27	1.179E+03	8.293E+02	8.548E+02	7.756E+02	8.177E+02	8.705E+02	8.708E+02	8.646E+02	9.316E+02	8.977E+02
f28	1.257E+03	4.703E+03	1.349E+03	1.299E+03	1.134E+03	1.282E+03	1.171E+03	1.165E+03	1.182E+03	1.141E+03
f29	**2.001E+02**	1.170E+08	2.100E+02	2.005E+02	2.013E+02	2.002E+02	2.001E+02	2.001E+02	2.001E+02	2.001E+02
f30	1.096E+04	7.470E+05	1.122E+04	1.293E+04	**1.031E+04**	1.169E+04	1.122E+04	1.218E+04	1.266E+04	1.109E+04

Table 7.8: Wilcoxon Signed Rank Test Statistical Values for ASw-GSA$_s^{rfit^*}$

S-GSA vs ASw-GSA$_s^{rfit^*}$			A-GSA vs ASw-GSA$_s^{rfit^*}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	433	32	5%	55	410
10%	351	114	10%	25	440
15%	303	162	15%	28	437
20%	266	199	20%	3	462
25%	225	240	25%	3	462
30%	165	300	30%	2	463
35%	195.5	269.5	35%	3	462
40%	151.5	313.5	40%	4	461
45%	207.5	257.5	45%	12	453
50%	172.5	262.5	50%	3	462
60%	140.5	294.5	60%	12	453
65%	133.5	301.5	65%	4	461
70%	127.5	307.5	70%	4	461
75%	162.5	272.5	75%	12	453
80%	137.5	297.5	80%	12	453
85%	159	306	85%	12	453
90%	124	341	90%	13	452
95%	158	307	95%	14	451

ASw-SKF$_a^{rfit^*}$ - Randomness increased probability of switching, the average number of switching was significantly higher than via the adaptive switching SKF which was implemented without the randomness.

Table 7.9 shows the average fitness value from the experiments conducted. The values highlighted with **boldface** are the smallest average error value for the respective functions. The smallest values are distributed among ASw-SKF$_a^{rfit^*}$ tested.

Wilcoxon signed rank test was conducted and the statistical values are shown in Table 7.10. The statistic values show that ASw-SKF$_a^{rfit^*}$ with all value of Δ is significantly better than S-SKF. The value of Δ that allows more number of switching gave better significance level. ASw-SKF$_a^{rfit^*}$ with $\Delta = \{80\%. 85\%, 90\%, 95\%\}$ had 10% significance level while others' significance level is 1%.

Comparison of ASw-SKF$_a^{rfit^*}$ with A-SKF found that $\Delta = \{50\%, 55\%, 65\%, 70\%, 80\%, 85\%, 90\%\}$ performed on par with A-SKF. ASw-SKF$_a^{rfit^*}$ with other values of Δ has better performance than A-SKF with significance level of at least 10%.

Table 7.9: Average Error of ASw-SKF$_a^{rfit^*}$

Function ID	S-SKF	A-SKF	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	4.860E+05	1.100E+07	2.630E+05	2.960E+05	3.790E+05	2.920E+05	2.750E+05	4.460E+05	4.650E+05	4.370E+05	3.160E+05	**2.040E+05**
f2	2.450E+08	1.290E+06	7.990E+05	**2.764E+04**	3.050E+06	3.540E+05	1.340E+06	1.700E+06	3.410E+05	3.130E+06	7.150E+06	3.830E+07
f3	1.841E+04	9.901E+03	**5.589E+03**	7.212E+03	6.778E+03	9.718E+03	7.842E+03	8.553E+03	9.284E+03	7.695E+03	9.962E+03	9.413E+03
f4	3.646E+01	1.177E+02	1.376E+01	**7.984E+00**	1.811E+01	2.175E+01	1.526E+01	2.922E+01	2.745E+01	3.276E+01	2.250E+01	2.249E+01
f5	2.002E+01	2.001E+01	**2.000E+01**	2.000E+01	**2.000E+01**	**2.000E+01**	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01
f6	2.195E+01	1.817E+01	1.588E+01	**1.462E+01**	1.499E+01	1.708E+01	1.546E+01	1.600E+01	1.720E+01	1.816E+01	1.531E+01	1.618E+01
f7	1.635E-01	8.444E-02	7.778E-02	8.983E-02	7.148E-02	**5.172E-02**	6.199E-02	5.773E-02	6.738E-02	7.675E-02	1.704E-01	1.336E-01
f8	5.878E+00	5.473E+00	**6.966E-01**	1.858E+00	1.980E+00	3.282E+00	3.647E+00	3.825E+00	4.037E+00	4.090E+00	4.813E+00	7.340E+00
f9	9.087E+01	7.526E+01	7.582E+01	7.499E+01	6.866E+01	7.469E+01	7.607E+01	**6.656E+01**	7.076E+01	8.060E+01	7.060E+01	7.061E+01
f10	2.263E+02	1.620E+02	**3.781E+01**	8.057E+01	1.235E+02	1.232E+02	1.276E+02	1.478E+02	1.329E+02	1.436E+02	1.514E+02	1.986E+02
f11	2.640E+03	2.585E+03	2.580E+03	2.486E+03	2.406E+03	**2.393E+03**	2.447E+03	2.480E+03	2.509E+03	2.539E+03	2.483E+03	2.563E+03
f12	3.592E-01	2.099E-01	1.997E-01	**1.825E-01**	2.197E-01	2.184E-01	1.860E-01	1.991E-01	2.184E-01	2.055E-01	2.361E-01	2.153E-01
f13	4.443E-01	3.567E-01	3.458E-01	3.612E-01	3.629E-01	3.442E-01	3.507E-01	3.480E-01	3.545E-01	3.457E-01	3.353E-01	3.550E-01
f14	2.593E-01	2.273E-01	2.336E-01	2.319E-01	2.224E-01	2.239E-01	2.358E-01	2.299E-01	2.287E-01	2.225E-01	2.220E-01	2.321E-01
f15	2.192E+01	1.640E+01	1.757E+01	1.669E+01	1.436E+01	1.556E+01	1.304E+01	1.400E+01	1.326E+01	1.327E+01	1.674E+01	1.501E+01
f16	1.060E+01	1.067E+01	**1.021E+01**	1.028E+01	1.045E+01	1.046E+01	1.051E+01	1.040E+01	1.047E+01	1.034E+01	1.056E+01	1.058E+01
f17	1.050E+05	1.170E+06	1.070E+05	**1.030E+05**	1.410E+05	1.250E+05	1.150E+05	1.590E+05	1.540E+05	1.370E+05	1.240E+05	1.620E+05
f18	1.150E+07	8.560E+06	1.510E+03	1.903E+03	**1.265E+03**	1.806E+03	6.698E+03	1.884E+03	4.921E+03	1.377E+03	5.028E+04	2.370E+06
f19	2.050E+01	1.985E+01	1.234E+01	1.212E+01	**8.928E+00**	1.453E+01	1.237E+01	1.092E+01	2.280E+01	2.024E+01	1.525E+01	1.305E+01
f20	2.984E+04	2.415E+04	**6.607E+03**	7.957E+03	1.206E+04	1.332E+04	1.761E+04	1.434E+04	1.784E+04	1.645E+04	1.821E+04	2.226E+04
f21	2.610E+05	5.550E+05	1.570E+05	1.640E+05	1.740E+05	1.900E+05	**1.350E+05**	2.130E+05	2.420E+05	1.800E+05	2.080E+05	2.040E+05
f22	6.217E+02	4.973E+02	4.800E+02	5.429E+02	5.071E+02	5.256E+02	5.523E+02	5.581E+02	5.276E+02	5.074E+02	5.190E+02	5.292E+02
f23	3.181E+02	3.161E+02	**3.159E+02**	3.164E+02	3.160E+02	3.161E+02	3.162E+02	3.160E+02	3.161E+02	3.163E+02	3.163E+02	3.166E+02
f24	2.310E+02	2.292E+02	**2.269E+02**	2.278E+02	2.273E+02	2.280E+02	2.282E+02	2.277E+02	2.280E+02	2.288E+02	2.275E+02	2.295E+02
f25	2.151E+02	2.143E+02	2.141E+02	2.152E+02	**2.138E+02**	2.143E+02	2.145E+02	2.145E+02	2.143E+02	2.141E+02	2.149E+02	2.138E+02
f26	1.204E+02	1.204E+02	**1.004E+02**	1.037E+02	1.070E+02	1.037E+02	1.103E+02	1.038E+02	1.137E+02	1.137E+02	1.204E+02	1.303E+02
f27	5.985E+02	5.476E+02	5.682E+02	6.004E+02	6.059E+02	5.855E+02	6.140E+02	**4.954E+02**	5.201E+02	6.145E+02	6.127E+02	6.109E+02
f28	1.574E+03	1.610E+03	1.698E+03	1.700E+03	1.580E+03	1.630E+03	1.516E+03	1.545E+03	1.713E+03	1.595E+03	1.543E+03	1.635E+03
f29	2.477E+03	1.189E+03	1.006E+03	9.544E+02	1.009E+03	1.035E+03	1.002E+03	**9.412E+02**	1.123E+03	1.013E+03	1.036E+03	1.003E+03
f30	5.438E+03	3.848E+03	**2.490E+03**	2.820E+03	2.994E+03	3.009E+03	2.926E+03	3.050E+03	3.278E+03	3.122E+03	3.197E+03	3.165E+03

Table 7.9: Average Error of ASw-SKF$_{a}^{rfit^{*}}$ (continued...)

Function ID	S-SKF	A-SKF	55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	4.860E+05	1.100E+07	2.840E+05	4.680E+05	4.570E+05	3.720E+05	3.270E+05	4.480E+05	9.480E+05	1.130E+06	3.440E+06
f2	2.450E+08	1.290E+06	7.920E+06	5.110E+06	2.650E+07	2.140E+07	2.380E+06	9.050E+06	1.170E+07	1.800E+07	9.270E+06
f3	1.841E+04	9.901E+03	1.152E+04	1.143E+04	1.057E+04	9.612E+03	1.233E+04	9.341E+03	9.408E+03	1.194E+04	9.489E+03
f4	3.646E+01	1.177E+02	3.094E+01	1.708E+01	2.040E+01	2.909E+01	2.680E+01	5.726E+01	5.254E+01	4.898E+01	6.502E+01
f5	2.002E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01
f6	2.195E+01	1.817E+01	2.416E+01	1.704E+01	1.699E+01	1.851E+01	1.724E+01	1.761E+01	1.627E+01	1.678E+01	1.772E+01
f7	1.635E-01	8.444E-02	1.167E-01	1.108E-01	7.989E-02	7.960E-02	8.257E-02	9.816E-02	1.223E-01	7.812E-02	8.342E-02
f8	5.878E+00	5.473E+00	6.689E+00	5.921E+00	6.635E+00	7.318E+00	5.754E+00	4.682E+00	5.738E+00	5.395E+00	4.853E+00
f9	9.087E+01	7.526E+01	7.173E+01	7.466E+01	7.642E+01	7.807E+01	7.496E+01	7.573E+01	7.708E+01	7.714E+01	6.785E+01
f10	2.263E+02	1.620E+02	2.279E+02	2.615E+02	1.966E+02	2.661E+02	1.670E+02	2.406E+02	2.033E+02	1.945E+02	1.918E+02
f11	2.640E+03	2.585E+03	2.434E+03	2.497E+03	2.710E+03	2.512E+03	2.717E+03	2.659E+03	2.664E+03	2.610E+03	2.734E+03
f12	3.592E-01	2.099E-01	2.387E-01	2.085E-01	2.413E-01	2.069E-01	2.231E-01	2.128E-01	2.324E-01	2.047E-01	1.997E-01
f13	4.443E-01	3.567E-01	3.695E-01	3.667E-01	3.384E-01	**3.296E-01**	3.536E-01	3.604E-01	3.701E-01	3.515E-01	3.836E-01
f14	2.593E-01	2.273E-01	2.133E-01	**2.122E-01**	2.275E-01	2.239E-01	2.197E-01	2.265E-01	2.343E-01	2.184E-01	2.371E-01
f15	2.192E+01	1.640E+01	1.506E+01	**1.296E+01**	1.553E+01	1.550E+01	1.416E+01	1.324E+01	1.783E+01	1.494E+01	1.514E+01
f16	1.060E+01	1.067E+01	1.053E+01	1.050E+01	1.050E+01	1.062E+01	1.034E+01	1.059E+01	1.060E+01	1.030E+01	1.064E+01
f17	1.050E+05	1.170E+06	1.180E+05	1.540E+05	1.450E+05	1.840E+05	2.030E+05	2.340E+05	2.730E+05	3.710E+05	5.780E+05
f18	1.150E+07	8.560E+06	8.620E+05	6.870E+05	1.720E+05	7.930E+05	2.630E+06	6.060E+05	4.040E+05	7.959E+04	7.810E+06
f19	2.050E+01	1.985E+01	1.350E+01	1.524E+01	2.554E+01	1.477E+01	9.771E+00	1.966E+01	2.310E+01	2.263E+01	1.333E+01
f20	2.984E+04	2.415E+04	2.357E+04	2.240E+04	1.927E+04	2.481E+04	2.092E+04	2.320E+04	2.310E+04	1.876E+04	2.254E+04
f21	2.610E+05	5.550E+05	1.670E+05	2.100E+05	2.870E+05	2.450E+05	2.280E+05	3.170E+05	3.730E+05	3.800E+05	3.920E+05
f22	6.217E+02	4.973E+02	5.113E+02	5.585E+02	5.473E+02	5.175E+02	5.325E+02	5.232E+02	5.259E+02	**4.769E+02**	5.100E+02
f23	3.181E+02	3.161E+02	3.164E+02	3.164E+02	3.164E+02	3.166E+02	3.165E+02	3.168E+02	3.161E+02	3.167E+02	3.162E+02
f24	2.310E+02	2.292E+02	2.294E+02	2.294E+02	2.297E+02	2.291E+02	2.298E+02	2.294E+02	2.287E+02	2.288E+02	2.282E+02
f25	2.151E+02	2.143E+02	2.144E+02	2.159E+02	2.145E+02	2.144E+02	2.143E+02	2.148E+02	2.144E+02	2.144E+02	2.152E+02
f26	1.204E+02	1.204E+02	1.237E+02	1.171E+02	1.137E+02	1.170E+02	1.170E+02	1.270E+02	1.071E+02	1.270E+02	1.137E+02
f27	5.985E+02	5.476E+02	5.567E+02	5.467E+02	5.810E+02	5.934E+02	5.531E+02	5.705E+02	5.994E+02	5.819E+02	5.571E+02
f28	1.574E+03	1.610E+03	1.815E+03	1.601E+03	1.511E+03	1.839E+03	1.514E+03	1.821E+03	1.524E+03	1.723E+03	**1.504E+03**
f29	2.477E+03	1.189E+03	9.830E+02	9.545E+02	1.074E+03	1.009E+03	1.013E+03	1.402E+03	1.080E+03	1.290E+03	2.990E+03
f30	5.438E+03	3.848E+03	3.006E+03	2.886E+03	2.809E+03	3.481E+03	3.172E+03	3.546E+03	3.342E+03	3.428E+03	3.591E+03

Table 7.10: Wilcoxon Signed Rank Test Statistical Values for ASw-SKF$_a^{rfit^*}$

S-SKF vs ASw-SKF$_a^{rfit^*}$			A-SKF vs ASw-SKF$_a^{rfit^*}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	42	423	5%	57	408
10%	33	432	10%	83	382
15%	50	415	15%	66	399
20%	44	421	20%	56	409
25%	40	425	25%	86	379
30%	28	437	30%	53	412
35%	59	406	35%	68	397
40%	61	404	40%	97	368
45%	43	422	45%	111	324
50%	83.5	381.5	50%	156	309
55%	90	375	55%	197	268
60%	72	393	60%	137	328
65%	94	371	65%	192	273
70%	75	390	70%	182	283
75%	47	418	75%	146	289
80%	139	326	80%	207	258
85%	138	327	85%	174	291
90%	140	325	90%	211	254
95%	147	318	95%	152	313

ASw-SKF$_s^{rfit^*}$ - Maximum number of switching occurred for all functions in almost all value of Δ. The average fitness error values are tabulated in Table 7.11. It is observed that more number of the best average fitness error was found by ASw-SKF$_s^{rfit^*}$ with $\Delta = \{5\%\}$.

The results of Wilcoxon signed rank test are shown in Table 7.12. ASw-SKF$_s^{rfit^*}$ outperformed S-SKF with significance level ranging from 10% to 1%. The statistically better performance is observed for all value of Δ. ASw-SKF$_s^{rfit^*}$ with $\Delta = \{5\%, 10\%, 15\%, 25\%\}$ are significantly better than A-SKF with significance level of 1% for $\Delta = \{5\%, 10\%\}$ and significance level of 10% for $\Delta = \{15\%, 25\%\}$.

Table 7.11: Average Error of ASw-SKF$_s^{rfit^*}$

Function ID	S-SKF	A-SKF	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	4.860E+05	1.100E+07	3.220E+05	5.320E+05	3.760E+05	3.540E+05	5.110E+05	4.250E+05	4.060E+05	6.490E+05	5.210E+05	3.870E+05
f2	2.450E+08	1.290E+06	3.803E+04	**2.995E+04**	5.140E+06	1.460E+06	6.360E+05	4.850E+06	1.590E+07	4.520E+06	1.340E+07	1.260E+07
f3	1.841E+04	9.901E+03	**4.222E+03**	8.604E+03	9.132E+03	1.000E+04	1.192E+04	1.205E+04	1.388E+04	1.378E+04	1.592E+04	1.059E+04
f4	3.646E+01	1.177E+02	1.901E+01	**9.802E+00**	2.132E+01	2.216E+01	2.873E+01	3.659E+01	1.177E+01	2.749E+01	1.337E+01	2.922E+01
f5	2.002E+01	2.001E+01	**2.000E+01**	**2.000E+01**	2.000E+01	2.000E+01	2.000E+01	**2.000E+01**	2.000E+01	2.000E+01	2.000E+01	2.001E+01
f6	2.195E+01	1.817E+01	1.764E+01	1.788E+01	1.764E+01	1.889E+01	**1.739E+01**	1.878E+01	1.984E+01	1.832E+01	1.889E+01	1.845E+01
f7	1.635E-01	**8.444E-02**	1.240E-01	1.129E-01	1.369E-01	1.312E-01	1.835E-01	2.432E-01	1.227E-01	2.715E-01	9.788E-02	2.957E-01
f8	5.878E+00	5.473E+00	**4.803E-01**	1.070E+00	1.372E+00	1.928E+00	2.389E+00	2.496E+00	2.790E+00	2.699E+00	2.907E+00	2.940E+00
f9	9.087E+01	**7.526E+01**	8.482E+01	8.932E+01	8.733E+01	9.847E+01	8.161E+01	8.790E+01	9.201E+01	8.759E+01	8.913E+01	9.228E+01
f10	2.263E+02	1.620E+02	3.652E+01	**2.938E+01**	8.092E+01	1.188E+02	1.089E+02	1.203E+02	1.540E+02	1.515E+02	1.420E+02	1.496E+02
f11	2.640E+03	2.585E+03	2.818E+03	2.745E+03	2.769E+03	2.827E+03	2.668E+03	**2.585E+03**	2.730E+03	2.737E+03	2.758E+03	2.649E+03
f12	3.592E-01	2.099E-01	**1.979E-01**	2.292E-01	2.680E-01	2.442E-01	2.847E-01	2.769E-01	3.093E-01	2.660E-01	2.823E-01	3.005E-01
f13	4.443E-01	**3.567E-01**	4.398E-01	4.339E-01	4.162E-01	4.191E-01	4.423E-01	4.390E-01	4.279E-01	4.414E-01	4.511E-01	4.159E-01
f14	2.593E-01	**2.273E-01**	2.462E-01	2.700E-01	2.479E-01	2.694E-01	2.622E-01	2.541E-01	2.674E-01	2.636E-01	2.736E-01	2.648E-01
f15	2.192E+01	**1.640E+01**	1.884E+01	2.247E+01	2.071E+01	2.037E+01	2.457E+01	2.126E+01	2.318E+01	2.397E+01	2.376E+01	1.728E+01
f16	1.060E+01	1.067E+01	**1.025E+01**	1.080E+01	1.054E+01	1.055E+01	1.050E+01	1.074E+01	1.039E+01	1.065E+01	1.059E+01	1.077E+01
f17	1.050E+05	1.170E+06	1.270E+05	1.440E+05	1.890E+05	1.290E+05	1.630E+05	1.180E+05	1.730E+05	1.310E+05	1.450E+05	1.430E+05
f18	1.150E+07	8.560E+06	**1.914E+03**	1.958E+03	2.560E+03	2.674E+03	2.629E+03	3.197E+03	5.923E+04	1.913E+04	1.600E+05	1.290E+05
f19	2.050E+01	1.985E+01	**7.894E+00**	1.395E+01	1.038E+01	1.459E+01	1.699E+01	2.387E+01	1.543E+01	1.757E+01	1.748E+01	1.832E+01
f20	2.984E+04	2.415E+04	**4.906E+03**	1.007E+04	1.267E+04	1.479E+04	1.429E+04	1.543E+04	2.056E+04	1.943E+04	1.972E+04	2.190E+04
f21	2.610E+05	5.550E+05	**1.270E+05**	2.880E+05	2.550E+05	2.040E+05	2.130E+05	2.020E+05	2.280E+05	2.150E+05	2.750E+05	2.090E+05
f22	6.217E+02	**4.973E+02**	5.370E+02	5.384E+02	5.353E+02	5.648E+02	5.381E+02	5.976E+02	6.209E+02	6.075E+02	5.736E+02	6.261E+02
f23	3.181E+02	3.161E+02	**3.158E+02**	3.161E+02	3.165E+02	3.163E+02	3.161E+02	3.165E+02	3.164E+02	3.168E+02	3.166E+02	3.167E+02
f24	2.310E+02	**2.292E+02**	2.304E+02	**2.292E+02**	2.323E+02	2.304E+02	2.316E+02	2.320E+02	2.303E+02	2.323E+02	2.319E+02	2.313E+02
f25	2.151E+02	2.143E+02	**2.128E+02**	2.129E+02	2.140E+02	2.145E+02	2.150E+02	2.146E+02	2.164E+02	2.140E+02	2.129E+02	2.140E+02
f26	1.204E+02	1.204E+02	**1.005E+02**	1.038E+02	1.104E+02	1.138E+02	1.071E+02	1.105E+02	1.038E+02	1.105E+02	1.038E+02	1.337E+02
f27	5.985E+02	**5.476E+02**	6.432E+02	6.788E+02	6.444E+02	6.482E+02	7.190E+02	6.310E+02	6.089E+02	6.611E+02	7.083E+02	6.697E+02
f28	1.574E+03	1.610E+03	1.538E+03	1.515E+03	1.560E+03	1.507E+03	**1.356E+03**	1.521E+03	1.649E+03	1.670E+03	1.485E+03	1.721E+03
f29	2.477E+03	1.189E+03	1.085E+03	1.115E+03	1.172E+03	1.114E+03	1.107E+03	1.102E+03	1.128E+03	1.122E+03	1.509E+03	1.108E+03
f30	5.438E+03	3.848E+03	3.326E+03	3.464E+03	**3.110E+03**	3.362E+03	3.690E+03	3.879E+03	4.128E+03	3.862E+03	3.646E+03	3.709E+03

Table 7.11: Average Error of ASw-SKF$_s^{rfit^*}$ (continued...)

Function ID	S-SKF	A-SKF	Δ								
			55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	4.860E+05	1.100E+07	4.280E+05	4.300E+05	4.350E+05	4.050E+05	3.070E+05	2.960E+05	3.340E+05	3.140E+05	**2.750E+05**
f2	2.450E+08	1.290E+06	6.820E+06	1.660E+06	7.260E+06	2.050E+06	5.470E+06	3.580E+07	1.760E+07	2.270E+07	2.170E+07
f3	1.841E+04	9.901E+03	1.340E+04	1.350E+04	1.473E+04	1.519E+04	1.308E+04	1.274E+04	1.429E+04	1.234E+04	1.479E+04
f4	3.646E+01	1.177E+02	1.964E+01	3.386E+01	2.324E+01	2.816E+01	1.748E+01	4.365E+01	4.030E+01	1.561E+01	2.824E+01
f5	2.002E+01	2.001E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.001E+01
f6	2.195E+01	1.817E+01	1.845E+01	1.896E+01	1.791E+01	1.853E+01	1.802E+01	1.903E+01	1.864E+01	1.816E+01	1.898E+01
f7	1.635E-01	**8.444E-02**	1.344E-01	1.760E-01	1.554E-01	1.555E-01	1.624E-01	4.045E-01	2.018E-01	2.991E-01	2.080E-01
f8	5.878E+00	5.473E+00	3.751E+00	3.730E+00	2.863E+00	2.564E+00	3.749E+00	2.659E+00	3.369E+00	4.205E+00	4.763E+00
f9	9.087E+01	**7.526E+01**	8.579E+01	8.592E+01	8.268E+01	8.129E+01	8.504E+01	8.945E+01	8.582E+01	8.814E+01	8.692E+01
f10	2.263E+02	1.620E+02	1.109E+02	1.065E+02	1.122E+02	1.294E+02	1.195E+02	1.269E+02	1.360E+02	1.672E+02	1.454E+02
f11	2.640E+03	2.585E+03	2.677E+03	2.676E+03	2.801E+03	2.783E+03	2.676E+03	2.816E+03	2.852E+03	2.758E+03	2.709E+03
f12	3.592E-01	2.099E-01	2.930E-01	2.792E-01	2.677E-01	3.069E-01	3.162E-01	2.694E-01	2.535E-01	2.940E-01	3.396E-01
f13	4.443E-01	**3.567E-01**	4.725E-01	4.340E-01	4.082E-01	4.394E-01	4.166E-01	4.570E-01	4.788E-01	4.452E-01	4.252E-01
f14	2.593E-01	**2.273E-01**	2.629E-01	2.813E-01	2.759E-01	2.683E-01	2.861E-01	2.757E-01	2.786E-01	2.811E-01	2.807E-01
f15	2.192E+01	**1.640E+01**	2.148E+01	2.328E+01	2.517E+01	1.945E+01	2.339E+01	2.658E+01	2.039E+01	2.354E+01	2.379E+01
f16	1.060E+01	1.067E+01	1.062E+01	1.092E+01	1.071E+01	1.069E+01	1.041E+01	1.078E+01	1.044E+01	1.072E+01	1.058E+01
f17	1.050E+05	1.170E+06	1.850E+05	9.815E+04	1.700E+05	1.200E+05	1.050E+05	1.140E+05	1.110E+05	1.440E+05	**8.112E+04**
f18	1.150E+07	8.560E+06	9.437E+03	1.848E+04	1.167E+04	5.750E+05	4.992E+03	1.590E+05	4.685E+04	2.060E+05	4.970E+05
f19	2.050E+01	1.985E+01	1.949E+01	1.504E+01	1.711E+01	2.809E+01	1.948E+01	2.794E+01	2.034E+01	1.668E+01	1.241E+01
f20	2.984E+04	2.415E+04	2.455E+04	1.993E+04	2.073E+04	1.868E+04	2.153E+04	2.396E+04	2.317E+04	2.390E+04	1.825E+04
f21	2.610E+05	5.550E+05	2.860E+05	1.650E+05	2.160E+05	2.040E+05	2.260E+05	2.220E+05	2.160E+05	2.150E+05	1.850E+05
f22	6.217E+02	**4.973E+02**	5.921E+02	6.431E+02	5.961E+02	6.152E+02	6.099E+02	6.376E+02	7.206E+02	5.924E+02	5.893E+02
f23	3.181E+02	3.161E+02	3.171E+02	3.165E+02	3.165E+02	3.166E+02	3.163E+02	3.163E+02	3.167E+02	3.172E+02	3.167E+02
f24	2.310E+02	**2.292E+02**	2.315E+02	2.312E+02	2.305E+02	2.319E+02	2.319E+02	2.324E+02	2.296E+02	2.340E+02	2.308E+02
f25	2.151E+02	2.143E+02	2.140E+02	2.134E+02	2.142E+02	2.145E+02	2.152E+02	2.129E+02	2.147E+02	2.149E+02	2.150E+02
f26	1.204E+02	1.204E+02	1.171E+02	1.104E+02	1.171E+02	1.104E+02	1.105E+02	1.138E+02	1.038E+02	1.071E+02	1.171E+02
f27	5.985E+02	**5.476E+02**	6.467E+02	6.676E+02	6.648E+02	6.649E+02	6.720E+02	5.641E+02	7.228E+02	6.624E+02	7.000E+02
f28	1.574E+03	1.610E+03	1.569E+03	1.642E+03	1.435E+03	1.466E+03	1.495E+03	1.475E+03	1.501E+03	1.441E+03	1.770E+03
f29	2.477E+03	1.189E+03	1.716E+03	1.200E+03	1.215E+03	**1.069E+03**	1.369E+03	1.213E+03	1.194E+03	1.820E+03	1.241E+03
f30	5.438E+03	3.848E+03	3.712E+03	3.972E+03	4.832E+03	4.607E+03	4.615E+03	4.139E+03	6.576E+03	4.577E+03	4.239E+03

Table 7.12: Wilcoxon Signed Rank Test Statistical Values for ASw-SKF$_s^{rfit^*}$

S-SKF vs ASw-SKF$_s^{rfit^*}$			A-SKF vs ASw-SKF$_s^{rfit^*}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	63	402	5%	101	364
10%	134	331	10%	102	333
15%	74	391	15%	147	318
20%	80	385	20%	182	283
25%	115	350	25%	143	322
30%	84	381	30%	210	255
35%	115	350	35%	233	232
40%	144	321	40%	228	237
45%	143	322	45%	205	260
50%	144	321	50%	224	241
55%	108	357	55%	232	233
60%	101	364	60%	243	222
65%	87	378	65%	221	244
70%	96	369	70%	226	239
75%	64	371	75%	225	240
80%	130	335	80%	244	221
85%	130	335	85%	243	222
90%	101	364	90%	255	210
95%	80	385	95%	257	208

7.4.2.2 D^p as the Switching Indicator

ASw-PSO$_a^{rD^p}$ - Based on the average number of switching, the results from the entire experiment are accepted. The average fitness error values of the experiments are tabulated in Table 7.13. The values in **boldface** indicate the best average fitness error value for the respective function.

Comparison of ASw-PSO$_a^{rD^p}$ with S-PSO and A-PSO using Wilcoxon signed rank test gives the statistical values in Table 7.14. ASw-PSO$_a^{rD^p}$ is as good as S-PSO with exception for ASw-PSO$_a^{rD^p}$ with $\Delta = \{5\%\}$ where S-PSO is significantly better with significance level of 10%. ASw-PSO$_a^{rD^p}$ with $\Delta = \{5\%, 35\%, 55\%, 65\%, 75\%, 85\%, 95\%\}$ are statistically worse than A-PSO.

Table 7.13: Average Error of ASw-PSO$_a^{rD^p}$

Function ID	S-PSO	A-PSO	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	6.670E+06	**5.200E+06**	8.280E+06	7.150E+06	7.820E+06	6.310E+06	7.930E+06	8.350E+06	6.380E+06	6.210E+06	6.000E+06	6.470E+06
f2	2.879E+02	1.389E+02	3.175E+02	3.410E+02	1.508E+02	1.744E+02	2.652E+02	2.110E+02	2.485E+02	**1.378E+02**	2.140E+02	3.568E+02
f3	3.663E+02	2.945E+02	4.911E+02	**2.033E+02**	5.072E+02	3.551E+02	3.049E+02	4.501E+02	3.208E+02	2.205E+02	3.291E+02	2.412E+02
f4	1.746E+02	1.608E+02	1.690E+02	1.702E+02	1.714E+02	1.669E+02	1.652E+02	1.716E+02	1.619E+02	1.647E+02	1.567E+02	**1.491E+02**
f5	2.085E+01	2.086E+01	2.085E+01	2.085E+01	2.086E+01	2.087E+01	2.089E+01	2.087E+01	2.085E+01	2.088E+01	2.085E+01	2.085E+01
f6	1.033E+01	1.071E+01	1.107E+01	1.042E+01	1.004E+01	1.124E+01	1.087E+01	1.097E+01	1.128E+01	1.144E+01	1.107E+01	1.092E+01
f7	1.058E-02	9.766E-03	1.279E-02	1.386E-02	1.247E-02	9.593E-03	9.523E-03	1.713E-02	9.594E-03	1.083E-02	1.435E-02	1.343E-02
f8	1.917E+01	1.857E+01	**1.718E+01**	1.827E+01	2.113E+01	2.070E+01	1.917E+01	1.980E+01	2.023E+01	1.791E+01	1.887E+01	1.918E+01
f9	5.871E+01	6.879E+01	6.690E+01	6.516E+01	6.243E+01	6.451E+01	5.967E+01	6.587E+01	6.156E+01	6.609E+01	6.275E+01	6.232E+01
f10	5.584E+02	6.090E+02	6.881E+02	5.662E+02	6.210E+02	5.406E+02	5.228E+02	5.870E+02	6.165E+02	5.374E+02	5.150E+02	5.707E+02
f11	**2.639E+03**	2.839E+03	2.792E+03	2.816E+03	2.811E+03	2.773E+03	3.048E+03	2.719E+03	2.642E+03	2.872E+03	2.647E+03	2.775E+03
f12	1.893E+00	1.658E+00	1.721E+00	1.815E+00	1.837E+00	1.560E+00	1.631E+00	**1.458E+00**	1.949E+00	1.869E+00	1.806E+00	1.638E+00
f13	4.086E-01	4.446E-01	4.242E-01	4.383E-01	4.437E-01	4.339E-01	4.091E-01	4.071E-01	4.357E-01	4.488E-01	4.252E-01	4.303E-01
f14	2.850E-01	3.454E-01	3.259E-01	3.129E-01	3.217E-01	3.002E-01	3.324E-01	2.873E-01	3.415E-01	3.467E-01	3.160E-01	3.169E-01
f15	7.404E+00	7.254E+00	7.528E+00	6.911E+00	7.111E+00	**6.106E+00**	6.712E+00	7.466E+00	7.493E+00	6.883E+00	7.099E+00	6.173E+00
f16	1.126E+01	1.122E+01	1.130E+01	1.137E+01	1.139E+01	1.130E+01	1.123E+01	1.128E+01	1.137E+01	**1.105E+01**	1.133E+01	1.147E+01
f17	6.780E+05	6.340E+05	7.250E+05	7.230E+05	**5.270E+05**	5.360E+05	5.950E+05	6.040E+05	6.920E+05	5.700E+05	6.930E+05	6.890E+05
f18	7.474E+03	4.828E+03	5.416E+03	8.273E+04	4.484E+03	9.482E+03	9.828E+03	7.933E+03	6.193E+03	5.109E+03	**4.472E+03**	4.942E+04
f19	8.054E+00	7.416E+00	8.439E+00	7.157E+00	8.452E+00	7.834E+00	8.110E+00	1.014E+01	1.028E+01	1.024E+01	7.866E+00	**7.127E+00**
f20	6.018E+02	5.209E+02	7.030E+02	5.474E+02	6.633E+02	5.759E+02	4.978E+02	**4.801E+02**	5.713E+02	5.952E+02	6.664E+02	5.391E+02
f21	1.360E+05	1.660E+05	1.510E+05	1.400E+05	1.350E+05	1.250E+05	1.420E+05	1.650E+05	1.500E+05	2.070E+05	1.390E+05	1.560E+05
f22	2.559E+02	2.294E+02	2.792E+02	2.897E+02	2.122E+02	2.754E+02	2.778E+02	2.517E+02	2.648E+02	2.601E+02	2.006E+02	2.365E+02
f23	3.158E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.158E+02
f24	2.329E+02	**2.293E+02**	2.301E+02	2.314E+02	2.317E+02	2.310E+02	2.295E+02	2.324E+02	2.298E+02	2.305E+02	2.328E+02	2.316E+02
f25	2.087E+02	2.091E+02	2.084E+02	2.087E+02	2.084E+02	2.084E+02	2.086E+02	2.085E+02	2.084E+02	2.082E+02	2.090E+02	2.086E+02
f26	1.071E+02	1.071E+02	1.038E+02	1.071E+02	1.037E+02	1.138E+02	1.104E+02	1.143E+02	1.004E+02	1.171E+02	1.104E+02	1.138E+02
f27	5.512E+02	5.556E+02	5.578E+02	5.508E+02	5.446E+02	5.661E+02	5.320E+02	5.458E+02	5.620E+02	5.465E+02	5.335E+02	5.787E+02
f28	1.103E+03	1.142E+03	1.069E+03	1.054E+03	1.042E+03	1.068E+03	1.077E+03	1.120E+03	1.103E+03	1.123E+03	**1.024E+03**	1.221E+03
f29	2.370E+06	1.600E+06	3.350E+06	1.520E+06	**1.267E+03**	8.040E+05	8.500E+05	1.750E+06	2.520E+06	1.880E+06	1.540E+06	9.390E+05
f30	3.970E+03	3.391E+03	3.748E+03	4.068E+03	3.870E+03	3.997E+03	3.556E+03	4.016E+03	3.367E+03	3.425E+03	3.326E+03	3.741E+03

Table 7.13: Average Error of ASw-PSO$_a^{rD^p}$ (continued...)

Function ID	S-PSO	A-PSO	Δ								
			55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	6.670E+06	**5.200E+06**	6.230E+06	7.400E+06	5.210E+06	7.220E+06	1.010E+07	6.480E+06	6.880E+06	6.820E+06	6.330E+06
f2	2.879E+02	1.389E+02	3.045E+02	2.580E+02	2.098E+02	3.210E+02	2.283E+02	5.129E+02	1.835E+02	2.188E+02	2.324E+02
f3	3.663E+02	2.945E+02	5.379E+02	3.008E+02	3.341E+02	3.304E+02	3.525E+02	4.473E+02	4.330E+02	3.808E+02	2.812E+02
f4	1.746E+02	1.608E+02	1.670E+02	1.683E+02	1.628E+02	1.643E+02	1.651E+02	1.613E+02	1.671E+02	1.767E+02	1.551E+02
f5	2.085E+01	2.086E+01	2.086E+01	2.087E+01	2.088E+01	2.085E+01	2.087E+01	**2.084E+01**	2.088E+01	2.089E+01	2.086E+01
f6	1.033E+01	1.071E+01	1.138E+01	1.115E+01	1.020E+01	1.103E+01	1.088E+01	9.946E+00	1.018E+01	**9.078E+00**	1.101E+01
f7	1.058E-02	9.766E-03	1.165E-02	8.612E-03	1.711E-02	1.484E-02	1.469E-02	**8.536E-03**	1.255E-02	9.602E-03	1.247E-02
f8	1.917E+01	1.857E+01	1.940E+01	1.987E+01	1.963E+01	1.990E+01	1.900E+01	1.865E+01	1.890E+01	1.841E+01	1.921E+01
f9	5.871E+01	6.879E+01	6.097E+01	6.159E+01	6.627E+01	5.821E+01	6.265E+01	5.686E+01	6.132E+01	6.663E+01	**5.617E+01**
f10	5.584E+02	6.090E+02	6.773E+02	6.177E+02	5.695E+02	**4.958E+02**	5.625E+02	6.729E+02	6.347E+02	5.278E+02	6.264E+02
f11	**2.639E+03**	2.839E+03	2.650E+03	2.840E+03	2.782E+03	2.818E+03	2.755E+03	2.818E+03	2.909E+03	2.732E+03	2.860E+03
f12	1.893E+00	1.658E+00	1.605E+00	1.692E+00	1.761E+00	1.549E+00	1.661E+00	1.592E+00	1.647E+00	1.940E+00	1.693E+00
f13	4.086E-01	4.446E-01	4.705E-01	**4.006E-01**	4.321E-01	4.316E-01	4.260E-01	4.538E-01	4.206E-01	4.309E-01	4.302E-01
f14	2.850E-01	3.454E-01	2.978E-01	2.875E-01	2.945E-01	2.883E-01	3.451E-01	2.918E-01	**2.557E-01**	3.208E-01	2.731E-01
f15	7.404E+00	7.254E+00	6.829E+00	7.204E+00	6.880E+00	6.523E+00	7.770E+00	6.745E+00	6.763E+00	6.989E+00	6.782E+00
f16	1.126E+01	1.122E+01	1.149E+01	1.132E+01	1.115E+01	1.142E+01	1.131E+01	1.148E+01	1.145E+01	1.146E+01	1.148E+01
f17	6.780E+05	6.340E+05	6.840E+05	6.320E+05	7.990E+05	7.430E+05	7.390E+05	6.840E+05	5.550E+05	6.140E+05	6.910E+05
f18	7.474E+03	4.828E+03	6.020E+03	2.760E+05	7.734E+03	6.143E+03	8.304E+03	5.718E+03	1.225E+04	7.587E+03	5.687E+03
f19	8.054E+00	7.416E+00	7.857E+00	7.468E+00	1.192E+01	7.565E+00	7.507E+00	7.611E+00	7.813E+00	1.003E+01	8.543E+00
f20	6.018E+02	5.209E+02	5.220E+02	7.179E+02	6.134E+02	5.596E+02	5.441E+02	5.358E+02	6.357E+02	6.497E+02	6.325E+02
f21	1.360E+05	1.660E+05	1.090E+05	1.170E+05	1.860E+05	1.590E+05	1.730E+05	1.260E+05	1.660E+05	**1.040E+05**	1.690E+05
f22	2.559E+02	2.294E+02	2.479E+02	2.442E+02	**1.886E+02**	2.889E+02	2.713E+02	2.309E+02	2.573E+02	2.803E+02	2.189E+02
f23	3.158E+02	3.159E+02	3.158E+02	3.159E+02	3.158E+02	**3.158E+02**	3.159E+02	3.158E+02	3.159E+02	3.158E+02	3.159E+02
f24	2.329E+02	**2.293E+02**	2.319E+02	2.311E+02	2.318E+02	2.318E+02	2.302E+02	2.302E+02	2.302E+02	2.311E+02	2.317E+02
f25	2.087E+02	2.091E+02	2.090E+02	2.087E+02	2.086E+02	2.089E+02	2.084E+02	2.084E+02	2.086E+02	**2.082E+02**	2.094E+02
f26	1.071E+02	1.071E+02	1.105E+02	**1.004E+02**	1.071E+02	1.071E+02	1.038E+02	1.211E+02	1.146E+02	1.104E+02	1.138E+02
f27	5.512E+02	5.556E+02	5.879E+02	5.397E+02	6.090E+02	5.477E+02	5.596E+02	**5.320E+02**	5.906E+02	5.542E+02	5.479E+02
f28	1.103E+03	1.142E+03	1.074E+03	1.123E+03	1.070E+03	1.107E+03	1.103E+03	1.116E+03	1.115E+03	1.091E+03	1.144E+03
f29	2.370E+06	1.600E+06	3.030E+06	3.240E+06	4.620E+06	1.450E+06	1.630E+06	7.300E+05	2.480E+06	1.780E+06	2.250E+06
f30	3.970E+03	3.391E+03	3.528E+03	4.056E+03	3.925E+03	**3.248E+03**	3.636E+03	3.705E+03	3.451E+03	3.480E+03	4.269E+03

Table 7.14: Wilcoxon Signed Rank Test Statistical Values for ASw-PSO$_{a}^{rD^{p}}$

S-PSO vs ASw-PSO$_{a}^{rD^{p}}$			A-PSO vs ASw-PSO$_{a}^{rD^{p}}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	323	<u>142</u>	5%	333	<u>132</u>
10%	256	179	10%	226	239
15%	181	284	15%	209	256
20%	182	283	20%	261	204
25%	193	272	25%	248	217
30%	288	177	30%	272	193
35%	245	220	35%	318	<u>147</u>
40%	162	303	40%	290	175
45%	165	300	45%	197	268
50%	233	202	50%	265	200
55%	226	209	55%	323	<u>112</u>
60%	254	211	60%	302	163
65%	235	200	65%	317	<u>148</u>
70%	204	231	70%	243	222
75%	241	194	75%	347	<u>118</u>
80%	154	281	80%	278	187
85%	298	167	85%	326	<u>109</u>
90%	223	212	90%	273	192
95%	239	226	95%	352	<u>113</u>

ASw-PSO$_{s}^{rD^{p}}$ - The average fitness error values of the experiments are tabulated in Table 7.15. The best average fitness errors are distributed among the tested algorithms.

The statistical values of the Wilcoxon test for pairwise comparison of ASw-PSO$_{s}^{rD^{p}}$ with S-PSO and A-PSO are tabulated in Table 7.16. The values show that ASw-PSO$_{s}^{rD^{p}}$ is on par with S-PSO. A-PSO is significantly better than ASw-PSO$_{s}^{rD^{p}}$ with $\Delta = \{25\%, 40\%, 45\%, 55\%, 65\%, 70\%, 80\%, 90\%\}$ and the significance level is between 10% to 2%.

Table 7.15: Average Error of ASw-PSO$_s^{rD^p}$

Function ID	S-PSO	A-PSO	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	6.670E+06	5.200E+06	**4.880E+06**	8.370E+06	8.650E+06	9.560E+06	7.520E+06	6.130E+06	8.260E+06	6.800E+06	1.050E+07	6.170E+06
f2	2.879E+02	1.389E+02	1.204E+02	2.708E+02	2.534E+02	1.325E+02	1.551E+02	**6.871E+01**	2.869E+02	1.518E+02	1.740E+02	1.363E+02
f3	3.663E+02	2.945E+02	4.532E+02	4.426E+02	4.459E+02	3.169E+02	3.529E+02	3.441E+02	3.525E+02	3.648E+02	3.943E+02	4.990E+02
f4	1.746E+02	1.608E+02	1.608E+02	1.578E+02	**1.459E+02**	1.828E+02	1.654E+02	1.688E+02	1.671E+02	1.762E+02	1.646E+02	1.636E+02
f5	2.085E+01	2.086E+01	2.085E+01	2.087E+01	**2.082E+01**	2.086E+01	2.087E+01	2.090E+01	2.086E+01	2.086E+01	2.087E+01	2.085E+01
f6	1.033E+01	1.071E+01	1.124E+01	1.124E+01	1.072E+01	1.230E+01	1.152E+01	1.025E+01	1.025E+01	1.235E+01	1.157E+01	1.161E+01
f7	1.058E-02	9.766E-03	1.107E-02	8.530E-03	1.312E-02	9.021E-03	1.255E-02	1.206E-02	1.377E-02	9.033E-03	8.531E-03	1.115E-02
f8	1.917E+01	1.857E+01	1.970E+01	1.844E+01	1.937E+01	1.988E+01	1.900E+01	1.834E+01	1.871E+01	1.851E+01	1.977E+01	1.877E+01
f9	**5.871E+01**	6.879E+01	6.392E+01	6.160E+01	6.374E+01	6.394E+01	6.016E+01	6.122E+01	6.277E+01	7.462E+01	6.384E+01	6.424E+01
f10	5.584E+02	6.090E+02	6.554E+02	5.739E+02	6.023E+02	5.923E+02	6.074E+02	6.051E+02	5.629E+02	6.500E+02	6.397E+02	5.604E+02
f11	2.639E+03	2.839E+03	2.686E+03	2.841E+03	**2.572E+03**	2.857E+03	2.728E+03	2.755E+03	2.820E+03	2.651E+03	2.957E+03	2.744E+03
f12	1.893E+00	1.658E+00	1.678E+00	1.790E+00	1.689E+00	1.659E+00	1.823E+00	1.761E+00	1.856E+00	1.705E+00	1.666E+00	1.647E+00
f13	4.086E-01	4.446E-01	4.169E-01	4.502E-01	**3.998E-01**	4.236E-01	4.161E-01	4.198E-01	4.302E-01	4.193E-01	4.421E-01	4.070E-01
f14	2.850E-01	3.454E-01	3.411E-01	2.996E-01	3.124E-01	2.820E-01	2.805E-01	2.888E-01	2.984E-01	3.091E-01	3.094E-01	2.866E-01
f15	7.404E+00	7.254E+00	7.265E+00	6.754E+00	7.374E+00	7.093E+00	7.142E+00	6.646E+00	7.069E+00	7.576E+00	7.484E+00	6.729E+00
f16	1.126E+01	1.122E+01	1.142E+01	1.111E+01	1.140E+01	1.124E+01	1.134E+01	**1.105E+01**	1.127E+01	1.126E+01	1.112E+01	1.140E+01
f17	6.780E+05	6.340E+05	5.670E+05	6.170E+05	7.950E+05	**4.570E+05**	7.200E+05	6.240E+05	7.560E+05	7.780E+05	5.650E+05	5.790E+05
f18	7.474E+03	4.828E+03	6.765E+03	1.029E+04	4.610E+03	4.548E+03	7.352E+03	6.698E+03	**4.547E+03**	5.608E+03	3.149E+04	6.595E+03
f19	8.054E+00	7.416E+00	7.357E+00	1.017E+01	8.004E+00	7.415E+00	9.066E+00	7.826E+00	9.865E+00	7.580E+00	7.338E+00	8.098E+00
f20	6.018E+02	5.209E+02	**5.200E+02**	6.218E+02	6.082E+02	5.651E+02	5.712E+02	6.177E+02	5.572E+02	5.929E+02	6.021E+02	5.340E+02
f21	1.360E+05	1.660E+05	1.940E+05	2.180E+05	1.950E+05	**1.220E+05**	1.520E+05	1.440E+05	1.620E+05	2.230E+05	1.580E+05	1.570E+05
f22	2.559E+02	2.294E+02	2.571E+02	2.491E+02	3.297E+02	2.505E+02	2.653E+02	2.346E+02	2.726E+02	2.693E+02	2.875E+02	2.724E+02
f23	3.158E+02	3.159E+02	3.158E+02	3.159E+02	3.159E+02	3.159E+02	3.160E+02	3.159E+02	3.158E+02	3.158E+02	3.158E+02	3.159E+02
f24	2.329E+02	**2.293E+02**	2.304E+02	2.333E+02	2.334E+02	2.317E+02	2.317E+02	2.307E+02	2.314E+02	2.323E+02	2.305E+02	2.318E+02
f25	2.087E+02	2.091E+02	2.085E+02	2.083E+02	2.088E+02	2.090E+02	2.090E+02	2.088E+02	2.088E+02	2.093E+02	2.085E+02	**2.081E+02**
f26	1.071E+02	1.071E+02	1.104E+02	1.038E+02	1.104E+02	1.004E+02	1.004E+02	1.105E+02	1.138E+02	1.071E+02	1.004E+02	1.071E+02
f27	5.512E+02	5.556E+02	5.356E+02	5.451E+02	5.155E+02	5.510E+02	5.592E+02	5.704E+02	5.949E+02	**5.133E+02**	5.664E+02	5.805E+02
f28	1.103E+03	1.142E+03	1.132E+03	1.128E+03	1.143E+03	1.194E+03	1.126E+03	1.117E+03	1.132E+03	1.125E+03	1.194E+03	**1.095E+03**
f29	2.370E+06	1.600E+06	2.440E+06	2.730E+06	1.080E+06	**1.218E+03**	2.370E+06	6.590E+05	2.380E+06	1.488E+03	3.810E+06	8.800E+05
f30	3.970E+03	3.391E+03	4.154E+03	3.384E+03	4.026E+03	3.478E+03	3.399E+03	3.580E+03	3.598E+03	4.088E+03	3.207E+03	3.581E+03

Table 7.15: Average Error of ASw-PSO$_s^{rD^p}$ (continued...)

Function ID	S-PSO	A-PSO	55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	6.670E+06	5.200E+06	8.820E+06	8.480E+06	7.150E+06	6.810E+06	6.460E+06	8.550E+06	9.340E+06	7.880E+06	7.060E+06
f2	2.879E+02	1.389E+02	2.380E+02	1.241E+02	8.528E+02	3.181E+02	1.201E+02	1.437E+02	4.354E+02	3.220E+02	1.634E+02
f3	3.663E+02	2.945E+02	4.278E+02	3.554E+02	4.927E+02	3.641E+02	3.305E+02	4.520E+02	**2.444E+02**	3.425E+02	4.169E+02
f4	1.746E+02	1.608E+02	1.846E+02	1.597E+02	1.664E+02	1.702E+02	1.568E+02	1.464E+02	1.590E+02	1.810E+02	1.639E+02
f5	2.085E+01	2.086E+01	2.085E+01	2.085E+01	2.086E+01	2.085E+01	2.084E+01	2.088E+01	**2.082E+01**	2.086E+01	2.087E+01
f6	1.033E+01	1.071E+01	1.217E+01	1.129E+01	1.118E+01	1.112E+01	1.060E+01	1.093E+01	**9.721E+00**	1.127E+01	1.056E+01
f7	1.058E-02	9.766E-03	**7.056E-03**	1.198E-02	1.139E-02	1.106E-02	1.549E-02	1.075E-02	1.665E-02	7.951E-03	9.924E-03
f8	1.917E+01	1.857E+01	1.983E+01	1.920E+01	1.877E+01	**1.642E+01**	2.013E+01	1.970E+01	2.000E+01	1.834E+01	2.089E+01
f9	**5.871E+01**	6.879E+01	6.593E+01	6.557E+01	6.293E+01	6.165E+01	6.686E+01	6.530E+01	6.199E+01	6.292E+01	6.650E+01
f10	5.584E+02	6.090E+02	6.428E+02	6.159E+02	**5.346E+02**	5.712E+02	6.636E+02	6.540E+02	5.466E+02	6.529E+02	5.993E+02
f11	2.639E+03	2.839E+03	2.840E+03	2.631E+03	2.624E+03	2.602E+03	2.681E+03	3.112E+03	2.586E+03	2.900E+03	2.825E+03
f12	1.893E+00	1.658E+00	**1.582E+00**	1.716E+00	1.675E+00	1.750E+00	1.652E+00	1.847E+00	1.634E+00	1.856E+00	1.782E+00
f13	4.086E-01	4.446E-01	4.307E-01	4.412E-01	4.079E-01	4.197E-01	4.365E-01	4.194E-01	4.023E-01	4.378E-01	4.383E-01
f14	2.850E-01	3.454E-01	3.197E-01	3.036E-01	2.754E-01	2.931E-01	**2.730E-01**	3.092E-01	3.070E-01	2.779E-01	3.187E-01
f15	7.404E+00	7.254E+00	7.273E+00	6.823E+00	6.843E+00	6.611E+00	6.892E+00	6.914E+00	7.231E+00	**6.319E+00**	7.105E+00
f16	1.126E+01	1.122E+01	1.112E+01	1.130E+01	1.110E+01	1.133E+01	1.154E+01	1.126E+01	1.141E+01	1.133E+01	1.134E+01
f17	6.780E+05	6.340E+05	6.350E+05	6.610E+05	5.840E+05	6.830E+05	7.650E+05	5.830E+05	6.770E+05	6.340E+05	5.970E+05
f18	7.474E+03	4.828E+03	5.419E+03	6.121E+03	9.613E+03	5.918E+03	1.082E+04	2.901E+04	6.000E+03	2.450E+05	4.661E+03
f19	8.054E+00	7.416E+00	7.606E+00	7.575E+00	8.154E+00	7.744E+00	**7.296E+00**	8.117E+00	7.878E+00	7.681E+00	7.674E+00
f20	6.018E+02	5.209E+02	6.630E+02	5.306E+02	5.881E+02	6.106E+02	6.242E+02	7.318E+02	6.266E+02	6.362E+02	6.695E+02
f21	1.360E+05	1.660E+05	1.270E+05	1.690E+05	1.900E+05	1.730E+05	1.440E+05	2.010E+05	2.010E+05	1.230E+05	1.780E+05
f22	2.559E+02	2.294E+02	2.660E+02	**2.180E+02**	2.814E+02	2.718E+02	2.837E+02	2.747E+02	2.958E+02	2.542E+02	2.370E+02
f23	3.158E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	3.159E+02	**3.158E+02**
f24	2.329E+02	**2.293E+02**	2.315E+02	2.298E+02	2.317E+02	2.326E+02	2.309E+02	2.293E+02	2.302E+02	2.326E+02	2.332E+02
f25	2.087E+02	2.091E+02	2.085E+02	2.084E+02	2.093E+02	2.084E+02	2.086E+02	2.083E+02	2.088E+02	2.086E+02	2.085E+02
f26	1.071E+02	1.071E+02	1.104E+02	1.176E+02	**1.004E+02**	1.137E+02	1.104E+02	1.104E+02	1.104E+02	1.137E+02	1.071E+02
f27	5.512E+02	5.556E+02	5.962E+02	5.541E+02	5.772E+02	5.662E+02	5.529E+02	5.871E+02	5.491E+02	5.517E+02	5.461E+02
f28	1.103E+03	1.142E+03	1.104E+03	1.111E+03	1.138E+03	1.128E+03	1.096E+03	1.126E+03	1.148E+03	1.113E+03	1.096E+03
f29	2.370E+06	1.600E+06	1.297E+03	2.280E+06	2.980E+06	1.490E+06	1.460E+06	1.840E+06	7.740E+05	1.600E+06	4.210E+06
f30	3.970E+03	3.391E+03	4.104E+03	3.532E+03	3.439E+03	3.653E+03	3.636E+03	3.453E+03	**3.194E+03**	3.953E+03	3.812E+03

Table 7.16: Wilcoxon Signed Rank Test Statistical Values for ASw-PSO$_s^{rD^p}$

S-PSO vs ASw-PSO$_s^{rD^p}$			A-PSO vs ASw-PSO$_s^{rD^p}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	248	217	5%	250	215
10%	283	182	10%	270	195
15%	306	159	15%	289	176
20%	176	289	20%	227	238
25%	252	183	25%	324	<u>141</u>
30%	177	288	30%	226	239
35%	287	178	35%	298	167
40%	280	185	40%	341	<u>124</u>
45%	303	162	45%	316	<u>149</u>
50%	189	276	50%	222	213
55%	261	174	55%	332	<u>133</u>
60%	182	283	60%	296	169
65%	276	189	65%	315	<u>150</u>
70%	271	194	70%	313	<u>152</u>
75%	249	216	75%	245	220
80%	302	163	80%	353	<u>112</u>
85%	207	258	85%	255	210
90%	266	199	90%	326.5	<u>138.5</u>
95%	248.5	216.5	95%	276	189

ASw-GSA$_a^{rD^p}$ - Table 7.17 presents the average fitness error value for each fitness function. It is observed that purely synchronous update is the better iteration strategy with more number of the best average fitness error.

Wilcoxon signed rank test was conducted on ASw-GSA$_a^{rD^p}$ against S-GSA and A-GSA. ASw-GSA$_a^{rD^p}$ was not able to perform as good as S-GSA. All statistic values are below 109. On the other hand, ASw-GSA$_a^{rD^p}$ with $\Delta = \{5\%, 10\%\}$ are better than A-GSA with 1% significance level. The statistical values of Wilcoxon signed rank test are shown in Table 7.18.

Table 7.17: Average Error of ASw-GSA$_a^{rD^p}$

Function ID	S-GSA	A-GSA	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	**1.300E+07**	7.110E+08	1.930E+08	3.190E+08	7.460E+08	7.230E+08	8.170E+08	7.480E+08	7.270E+08	8.040E+08	7.930E+08	7.940E+08
f2	**8.603E+03**	5.940E+10	5.790E+09	2.060E+10	4.380E+10	5.310E+10	5.670E+10	5.700E+10	5.620E+10	5.830E+10	5.760E+10	5.940E+10
f3	5.784E+04	9.770E+04	7.187E+04	**5.571E+04**	6.987E+04	8.463E+04	9.039E+04	8.860E+04	8.741E+04	8.968E+04	9.078E+04	9.761E+04
f4	**3.017E+02**	1.013E+04	1.955E+03	3.096E+03	6.408E+03	1.024E+04	1.059E+04	1.005E+04	1.017E+04	1.074E+04	1.044E+04	1.112E+04
f5	**2.000E+01**	2.095E+01	2.082E+01	2.003E+01	2.003E+01	2.096E+01	2.097E+01	2.097E+01	2.095E+01	2.097E+01	2.099E+01	2.097E+01
f6	**1.907E+01**	3.895E+01	3.442E+01	3.918E+01	3.946E+01	3.928E+01	3.942E+01	3.931E+01	3.929E+01	3.926E+01	3.919E+01	3.952E+01
f7	**0.000E+00**	5.439E+02	8.439E+01	2.052E+02	4.186E+02	5.412E+02	5.415E+02	5.249E+02	5.440E+02	5.399E+02	5.390E+02	5.772E+02
f8	**1.405E+02**	3.285E+02	1.491E+02	1.518E+02	2.240E+02	3.156E+02	3.217E+02	3.249E+02	3.286E+02	3.206E+02	3.250E+02	3.305E+02
f9	**1.624E+02**	3.781E+02	1.735E+02	1.738E+02	1.998E+02	3.465E+02	3.560E+02	3.502E+02	3.498E+02	3.580E+02	3.569E+02	3.655E+02
f10	**3.370E+03**	7.018E+03	4.398E+03	4.469E+03	5.064E+03	7.084E+03	7.184E+03	7.034E+03	7.116E+03	7.231E+03	7.142E+03	7.161E+03
f11	**4.058E+03**	7.155E+03	4.707E+03	4.784E+03	5.860E+03	7.194E+03	7.239E+03	7.189E+03	7.211E+03	7.242E+03	7.244E+03	7.226E+03
f12	**4.870E-04**	2.450E+00	1.412E-01	2.748E-01	2.172E+00	2.630E+00	2.610E+00	2.677E+00	2.614E+00	2.654E+00	2.606E+00	2.650E+00
f13	**3.017E-01**	6.146E+00	2.262E+00	4.083E+00	5.488E+00	6.272E+00	6.263E+00	6.296E+00	6.314E+00	6.211E+00	6.229E+00	6.428E+00
f14	**2.433E-01**	1.751E+02	3.013E+01	8.403E+01	1.532E+02	1.799E+02	1.892E+02	1.845E+02	1.817E+02	1.882E+02	1.826E+02	1.981E+02
f15	**3.659E+00**	3.470E+05	9.977E+01	2.153E+03	6.869E+04	1.930E+05	2.330E+05	2.290E+05	2.290E+05	2.520E+05	2.290E+05	3.460E+05
f16	1.363E+01	1.309E+01	1.308E+01	1.313E+01	1.319E+01	1.310E+01	1.318E+01	1.311E+01	1.312E+01	1.314E+01	1.317E+01	1.318E+01
f17	**5.310E+05**	1.840E+07	1.570E+07	1.020E+07	2.380E+07	2.370E+07	2.290E+07	2.270E+07	2.250E+07	2.440E+07	2.340E+07	2.610E+07
f18	**3.817E+02**	9.810E+08	1.193E+04	8.313E+02	8.280E+08	1.030E+09	1.140E+09	1.240E+09	1.090E+09	1.080E+09	1.180E+09	1.170E+09
f19	**1.153E+02**	2.924E+02	1.389E+02	1.482E+02	2.495E+02	2.837E+02	3.022E+02	2.994E+02	2.819E+02	2.938E+02	2.912E+02	2.802E+02
f20	**4.521E+04**	7.100E+04	6.513E+04	5.833E+04	7.000E+04	7.103E+04	8.671E+04	7.340E+04	7.768E+04	8.432E+04	8.580E+04	8.623E+04
f21	**1.550E+05**	4.760E+06	4.430E+06	1.750E+06	5.260E+06	5.230E+06	5.700E+06	5.140E+06	4.680E+06	4.690E+06	5.480E+06	5.440E+06
f22	**9.562E+02**	1.300E+03	1.088E+03	1.168E+03	1.408E+03	1.302E+03	1.394E+03	1.385E+03	1.399E+03	1.392E+03	1.402E+03	1.443E+03
f23	**2.130E+02**	6.697E+02	3.425E+02	3.337E+02	5.623E+02	7.124E+02	7.141E+02	7.213E+02	7.003E+02	7.163E+02	7.009E+02	7.204E+02
f24	**2.000E+02**	2.726E+02	2.077E+02	2.150E+02	2.277E+02	2.502E+02	2.656E+02	2.660E+02	2.679E+02	2.689E+02	2.671E+02	2.708E+02
f25	**2.000E+02**	2.249E+02	2.019E+02	2.020E+02	2.053E+02	2.144E+02	2.208E+02	2.225E+02	2.216E+02	2.222E+02	2.234E+02	2.251E+02
f26	1.868E+02	1.064E+02	1.069E+02	1.070E+02	1.069E+02	1.067E+02	1.066E+02	1.067E+02	1.066E+02	1.065E+02	1.066E+02	1.067E+02
f27	1.179E+03	**8.293E+02**	8.806E+02	8.837E+02	8.941E+02	9.015E+02	8.880E+02	8.785E+02	8.745E+02	8.687E+02	8.815E+02	8.817E+02
f28	**1.257E+03**	4.703E+03	1.649E+03	1.457E+03	2.134E+03	5.057E+03	4.767E+03	4.713E+03	4.968E+03	4.841E+03	5.061E+03	4.885E+03
f29	**2.001E+02**	1.170E+08	1.290E+08	1.190E+08	1.730E+08	1.420E+08	1.480E+08	1.390E+08	1.360E+08	1.480E+08	1.610E+08	1.520E+08
f30	**1.096E+04**	7.470E+05	9.760E+05	8.360E+05	1.050E+06	1.020E+06	9.670E+05	9.250E+05	8.470E+05	1.010E+06	9.450E+05	9.810E+05

Table 7.17: Average Error of ASw-GSA$_a^{rD^p}$ (continued...)

Function ID	S-GSA	A-GSA	Δ								
			55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	**1.300E+07**	7.110E+08	8.020E+08	7.710E+08	7.180E+08	7.410E+08	6.730E+08	7.520E+08	6.990E+08	7.410E+08	7.280E+08
f2	**8.603E+03**	5.940E+10	5.980E+10	6.020E+10	5.970E+10	6.020E+10	5.920E+10	5.920E+10	5.770E+10	5.780E+10	5.950E+10
f3	5.784E+04	9.770E+04	9.480E+04	9.507E+04	9.479E+04	9.361E+04	9.836E+04	9.490E+04	9.582E+04	9.231E+04	9.424E+04
f4	**3.017E+02**	1.013E+04	1.129E+04	1.085E+04	1.076E+04	1.060E+04	9.952E+03	1.005E+04	9.977E+03	1.041E+04	1.010E+04
f5	**2.000E+01**	2.095E+01	2.096E+01	2.096E+01	2.097E+01	2.094E+01	2.098E+01	2.096E+01	2.094E+01	2.094E+01	2.095E+01
f6	**1.907E+01**	3.895E+01	3.908E+01	3.921E+01	3.942E+01	3.911E+01	3.906E+01	3.916E+01	3.914E+01	3.883E+01	3.896E+01
f7	**0.000E+00**	5.439E+02	5.428E+02	5.515E+02	5.430E+02	5.365E+02	5.510E+02	5.318E+02	5.302E+02	5.314E+02	5.344E+02
f8	**1.405E+02**	3.285E+02	3.265E+02	3.342E+02	3.345E+02	3.333E+02	3.325E+02	3.293E+02	3.326E+02	3.288E+02	3.259E+02
f9	**1.624E+02**	3.781E+02	3.673E+02	3.673E+02	3.652E+02	3.650E+02	3.664E+02	3.675E+02	3.627E+02	3.625E+02	3.662E+02
f10	**3.370E+03**	7.018E+03	7.239E+03	7.012E+03	7.115E+03	7.101E+03	7.049E+03	7.056E+03	7.016E+03	6.995E+03	7.122E+03
f11	**4.058E+03**	7.155E+03	7.249E+03	7.223E+03	7.248E+03	7.180E+03	7.138E+03	7.199E+03	7.158E+03	7.148E+03	7.129E+03
f12	**4.870E-04**	2.450E+00	2.686E+00	2.630E+00	2.521E+00	2.596E+00	2.610E+00	2.483E+00	2.474E+00	2.476E+00	2.513E+00
f13	**3.017E-01**	6.146E+00	6.263E+00	6.160E+00	6.366E+00	6.213E+00	6.234E+00	6.202E+00	6.286E+00	6.201E+00	6.085E+00
f14	**2.433E-01**	1.751E+02	1.856E+02	1.934E+02	1.851E+02	1.895E+02	1.813E+02	1.824E+02	1.906E+02	1.801E+02	1.764E+02
f15	**3.659E+00**	3.470E+05	3.900E+05	2.780E+05	3.280E+05	3.620E+05	3.470E+05	3.710E+05	3.260E+05	3.290E+05	3.390E+05
f16	1.363E+01	1.309E+01	1.319E+01	1.322E+01	1.314E+01	1.315E+01	1.308E+01	**1.306E+01**	1.313E+01	1.309E+01	1.307E+01
f17	**5.310E+05**	1.840E+07	2.300E+07	2.090E+07	2.130E+07	2.170E+07	1.940E+07	2.130E+07	2.060E+07	1.920E+07	1.890E+07
f18	**3.817E+02**	9.810E+08	1.180E+09	1.150E+09	1.090E+09	1.120E+09	1.090E+09	1.080E+09	1.010E+09	1.060E+09	1.110E+09
f19	**1.153E+02**	2.924E+02	2.916E+02	2.908E+02	2.718E+02	2.833E+02	2.772E+02	2.860E+02	2.853E+02	2.618E+02	2.803E+02
f20	**4.521E+04**	7.100E+04	9.445E+04	7.837E+04	8.302E+04	7.172E+04	6.883E+04	6.981E+04	7.007E+04	6.375E+04	6.470E+04
f21	**1.550E+05**	4.760E+06	5.090E+06	5.760E+06	4.400E+06	4.660E+06	4.450E+06	4.620E+06	4.320E+06	4.490E+06	4.040E+06
f22	**9.562E+02**	1.300E+03	1.409E+03	1.379E+03	1.375E+03	1.377E+03	1.400E+03	1.366E+03	1.362E+03	1.275E+03	1.259E+03
f23	**2.130E+02**	6.697E+02	7.132E+02	7.099E+02	7.144E+02	7.113E+02	6.904E+02	6.898E+02	7.000E+02	6.774E+02	6.770E+02
f24	**2.000E+02**	2.726E+02	2.749E+02	2.735E+02	2.759E+02	2.762E+02	2.738E+02	2.753E+02	2.750E+02	2.739E+02	2.757E+02
f25	**2.000E+02**	2.249E+02	2.254E+02	2.254E+02	2.258E+02	2.247E+02	2.256E+02	2.257E+02	2.263E+02	2.258E+02	2.257E+02
f26	1.868E+02	1.064E+02	1.065E+02	1.068E+02	1.068E+02	1.067E+02	1.067E+02	1.065E+02	**1.063E+02**	1.067E+02	1.064E+02
f27	1.179E+03	**8.293E+02**	9.214E+02	9.209E+02	8.815E+02	8.793E+02	8.691E+02	8.534E+02	8.561E+02	8.356E+02	8.618E+02
f28	**1.257E+03**	4.703E+03	4.907E+03	4.810E+03	4.987E+03	4.888E+03	4.716E+03	4.821E+03	4.794E+03	4.718E+03	4.740E+03
f29	**2.001E+02**	1.170E+08	1.580E+08	1.470E+08	1.500E+08	1.460E+08	1.450E+08	1.320E+08	1.400E+08	1.370E+08	1.330E+08
f30	**1.096E+04**	7.470E+05	1.000E+06	8.870E+05	9.850E+05	9.310E+05	9.160E+05	8.200E+05	8.780E+05	8.680E+05	8.900E+05

Table 7.18: Wilcoxon Signed Rank Test Statistical Values for ASw-GSA$_{a}^{rD^p}$

S-GSA vs ASw-GSA$_{a}^{rD^p}$			A-GSA vs ASw-GSA$_{a}^{rD^p}$		
Δ	*R*+	*R*−	Δ	*R*+	*R*−
5%	434	<u>31</u>	5%	<u>61</u>	404
10%	413	<u>52</u>	10%	<u>62</u>	403
15%	441	<u>24</u>	15%	163	302
20%	443	<u>22</u>	20%	321	<u>144</u>
25%	443	<u>22</u>	25%	344	<u>121</u>
30%	443	<u>22</u>	30%	318	<u>147</u>
35%	443	<u>22</u>	35%	321	<u>144</u>
40%	443	<u>22</u>	40%	316	<u>149</u>
45%	443	<u>22</u>	45%	333	<u>132</u>
50%	443	<u>22</u>	50%	367	<u>68</u>
55%	443	<u>22</u>	55%	404	<u>61</u>
60%	443	<u>22</u>	60%	388	<u>77</u>
65%	443	<u>22</u>	65%	363	<u>102</u>
70%	443	<u>22</u>	70%	379	<u>86</u>
75%	443	<u>22</u>	75%	271	164
80%	443	<u>22</u>	80%	311	154
85%	443	<u>22</u>	85%	245	220
90%	442	<u>23</u>	90%	240	225
95%	442	<u>23</u>	95%	266	199

ASw-GSA$_s^{rD^p}$ - Unlike ASw-GSA$_s^{rD^p}$, where switching occurred with all value of Δ tested, very few number of switching or none occurred when $\Delta \geq 55\%$ for ASw-GSA$_s^{rD^p}$. This show the adaptiveness of the proposed iteration strategy. As observed in chapter 4, the agents in S-GSA lose their diversity rapidly, hence, the longer ASw-GSA$_s^{rD^p}$ adopts the synchronous update, the higher the chance for the population to lose its diversity and remains stagnant. This lower the chance for the condition, $\frac{D^p(t+1)}{D^p(t)} \leq rand$ to be true.

The average fitness error values are shown in Table 7.19. The best fitness errors are distributed among the algorithms tested, S-GSA does not monopolize the best fitness errors.

Wilcoxon pairwise comparison of ASw-GSA$_s^{rD^p}$ with $\Delta = \{5\%, 10\%, 15\%, 20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 50\%\}$ against S-GSA found that ASw-GSA$_s^{rD^p}$ with $\Delta = \{20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 50\%\}$ perform as good as S-PSO. Comparison with A-GSA shows that ASw-GSA$_s^{rD^p}$ with $\Delta = \{5\%, 10\%, 15\%, 20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 50\%\}$ are significantly better with 1% significance level. Table 7.20 shows the statistic values of the Wilcoxon signed rank test.

Table 7.19: Average Error of ASw-GSA$_s^{rD^p}$

Function ID	S-GSA	A-GSA	Δ									
			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	1.300E+07	7.110E+08	1.740E+08	5.730E+07	2.500E+07	1.550E+07	1.410E+07	1.220E+07	1.160E+07	**1.120E+07**	1.210E+07	1.170E+07
f2	8.603E+03	5.940E+10	1.390E+07	8.750E+05	9.351E+04	1.559E+04	8.631E+03	**8.191E+03**	8.646E+03	8.379E+03	8.255E+03	8.750E+03
f3	5.784E+04	9.770E+04	6.971E+04	7.645E+04	6.861E+04	5.294E+04	**5.075E+04**	5.230E+04	5.118E+04	5.570E+04	5.449E+04	5.624E+04
f4	3.017E+02	1.013E+04	4.336E+02	3.218E+02	3.048E+02	2.852E+02	2.798E+02	2.637E+02	**2.568E+02**	2.612E+02	2.621E+02	2.609E+02
f5	**2.000E+01**	2.095E+01	2.012E+01	2.088E+01	2.015E+01	2.001E+01	2.000E+01	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**
f6	**1.907E+01**	3.895E+01	2.293E+01	2.058E+01	1.982E+01	2.006E+01	1.972E+01	2.005E+01	2.000E+01	1.934E+01	2.008E+01	1.911E+01
f7	**0.000E+00**	5.439E+02	1.118E+00	7.518E-01	1.560E-01	1.510E-02	9.990E-04	5.810E-05	1.460E-06	1.120E-07	**0.000E+00**	2.130E-10
f8	1.405E+02	3.285E+02	1.399E+02	1.409E+02	1.395E+02	1.411E+02	1.414E+02	1.391E+02	1.383E+02	1.427E+02	1.451E+02	**1.379E+02**
f9	1.624E+02	3.781E+02	1.652E+02	1.642E+02	1.607E+02	1.633E+02	1.610E+02	1.616E+02	1.666E+02	1.641E+02	**1.579E+02**	1.677E+02
f10	3.370E+03	7.018E+03	3.238E+03	3.363E+03	3.314E+03	3.233E+03	3.251E+03	**3.176E+03**	3.375E+03	3.299E+03	3.433E+03	3.374E+03
f11	4.058E+03	7.155E+03	4.138E+03	4.078E+03	4.150E+03	**3.912E+03**	4.118E+03	4.168E+03	4.042E+03	4.215E+03	4.048E+03	4.136E+03
f12	**4.870E-04**	2.450E+00	3.510E-01	8.257E-02	2.170E-02	6.267E-03	1.808E-03	9.231E-04	9.656E-04	9.798E-04	6.446E-04	6.378E-04
f13	3.017E-01	6.146E+00	4.446E-01	3.627E-01	3.414E-01	3.240E-01	3.156E-01	3.056E-01	3.056E-01	2.892E-01	3.085E-01	**2.871E-01**
f14	2.433E-01	1.751E+02	2.793E-01	2.577E-01	2.515E-01	2.464E-01	2.345E-01	2.371E-01	2.368E-01	2.406E-01	2.518E-01	**2.250E-01**
f15	3.659E+00	3.470E+05	2.896E+01	1.024E+01	4.510E+00	3.615E+00	3.669E+00	3.841E+00	3.745E+00	3.753E+00	3.748E+00	**3.403E+00**
f16	1.363E+01	**1.309E+01**	1.312E+01	1.315E+01	1.319E+01	1.316E+01	1.320E+01	1.324E+01	1.335E+01	1.359E+01	1.360E+01	1.365E+01
f17	5.310E+05	1.840E+07	1.810E+07	6.290E+06	1.960E+06	1.140E+06	6.800E+05	5.920E+05	5.710E+05	5.490E+05	5.520E+05	**4.980E+05**
f18	3.817E+02	9.810E+08	1.230E+05	1.109E+04	9.954E+02	4.706E+02	4.119E+02	**3.703E+02**	3.885E+02	4.001E+02	4.082E+02	4.273E+02
f19	1.153E+02	2.924E+02	1.330E+02	1.111E+02	9.133E+01	9.506E+01	8.477E+01	8.921E+01	9.440E+01	9.072E+01	**8.319E+01**	9.417E+01
f20	4.521E+04	7.100E+04	6.666E+04	6.645E+04	7.303E+04	5.070E+04	4.192E+04	**3.799E+04**	3.943E+04	4.070E+04	4.382E+04	4.318E+04
f21	1.550E+05	4.760E+06	4.610E+06	2.100E+06	4.390E+05	2.040E+05	1.840E+05	**1.480E+05**	1.570E+05	1.750E+05	1.630E+05	1.520E+05
f22	9.562E+02	1.300E+03	9.477E+02	8.809E+02	9.050E+02	**8.409E+02**	8.800E+02	8.965E+02	8.757E+02	8.964E+02	9.247E+02	9.636E+02
f23	2.130E+02	6.697E+02	2.163E+02	2.194E+02	2.105E+02	2.184E+02	2.005E+02	2.092E+02	2.000E+02	2.173E+02	**2.000E+02**	2.087E+02
f24	**2.000E+02**	2.726E+02	2.052E+02	2.017E+02	2.006E+02	2.002E+02	2.001E+02	2.000E+02	2.000E+02	**2.000E+02**	**2.000E+02**	**2.000E+02**
f25	**2.000E+02**	2.249E+02	2.010E+02	2.003E+02	2.001E+02	2.000E+02	2.000E+02	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**
f26	1.868E+02	**1.064E+02**	1.070E+02	1.069E+02	1.073E+02	1.070E+02	1.071E+02	1.081E+02	1.241E+02	1.650E+02	1.725E+02	1.791E+02
f27	1.179E+03	8.293E+02	8.103E+02	**7.648E+02**	8.155E+02	8.269E+02	8.559E+02	8.207E+02	9.814E+02	1.047E+03	1.209E+03	1.168E+03
f28	1.257E+03	4.703E+03	1.213E+03	1.204E+03	1.106E+03	1.467E+03	1.317E+03	1.306E+03	**1.006E+03**	1.227E+03	1.164E+03	1.370E+03
f29	**2.001E+02**	1.170E+08	2.540E+07	2.210E+07	7.310E+06	2.020E+06	4.560E+05	8.153E+04	1.666E+04	1.295E+03	2.693E+02	2.021E+02
f30	**1.096E+04**	7.470E+05	6.900E+05	1.700E+05	1.630E+05	3.308E+04	1.962E+04	1.341E+04	1.313E+04	1.375E+04	1.200E+04	1.136E+04

Table 7.20: Wilcoxon Signed Rank Test Statistical Values for ASw-GSA$_s^{rD^p}$

S-GSA vs ASw-GSA$_s^{rD^p}$			A-GSA vs ASw-GSA$_s^{rD^p}$		
Δ	*R*+	*R*−	Δ	*R*+	*R*−
5%	367	98	5%	3	462
10%	354	111	10%	4	461
15%	315	150	15%	21	444
20%	302	163	20%	3	462
25%	261	204	25%	11	454
30%	157.5	307.5	30%	4	461
35%	201.5	263.5	35%	15	450
40%	195.5	239.5	40%	21	444
45%	212	253	45%	22	443
50%	183.5	251.5	50%	23	442

ASw-SKF$_a^{rD^p}$ - The average number of switching shows that in the tests, almost maximum number of switching permissible was carried out by ASw-SKF$_a^{rD^p}$. This is contributed by the initial iteration strategy; asynchronous update. Diversity of asynchronously updated SKF oscillates at a low value, this contributes to fulfilment of the switching condition.

The average fitness error values are presented in Table 7.21. It is observed that more number of the best solution was found by ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$. This indicates that ASw-SKF$_a^{rD^p}$ benefited from higher number of switching.

Table 7.22 shows the results of Wilcoxon signed rank test for ASw-SKF$_a^{rD^p}$ againts S-SKF and A-SKF. The result of the test shows that ASw-SKF$_a^{rD^p}$ is significantly better than S-SKF for all value of Δ. The range of the level of significance is from 1% to 10%. As for comparison of ASw-SKF$_a^{rD^p}$ with A-SKF, it is seen that ASw-SKF$_a^{rD^p}$ is significantly better except when $\Delta = \{50\%, 60\%, 65\%, 70\%, 75\%, 85\%, 95\%\}$.

Table 7.21: Average Error of ASw-SKF$_a^{rD^p}$

Function ID	S-SKF	A-SKF	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	4.860E+05	1.100E+07	2.290E+05	2.320E+05	2.880E+05	3.660E+05	2.630E+05	2.950E+05	4.250E+05	2.940E+05	2.870E+05	3.090E+05
f2	2.450E+08	1.290E+06	**1.050E+04**	5.130E+04	9.180E+05	2.890E+06	2.330E+06	7.110E+06	2.100E+06	2.290E+06	3.580E+06	1.530E+07
f3	1.841E+04	9.901E+03	**4.840E+03**	6.184E+03	8.074E+03	8.952E+03	1.077E+04	1.125E+04	1.205E+04	7.602E+03	8.323E+03	1.119E+04
f4	3.646E+01	1.177E+02	1.659E+01	**1.476E+01**	1.729E+01	3.535E+01	1.974E+01	1.570E+01	1.643E+01	2.838E+01	3.413E+01	2.383E+01
f5	2.002E+01	2.001E+01	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	**2.000E+01**	2.001E+01	2.001E+01
f6	2.195E+01	1.817E+01	1.667E+01	1.574E+01	1.583E+01	1.532E+01	1.612E+01	1.621E+01	1.541E+01	1.572E+01	1.631E+01	1.536E+01
f7	1.635E-01	**8.444E-02**	1.986E-01	2.558E-01	9.148E-02	1.163E-01	1.240E-01	1.395E-01	1.135E-01	1.108E-01	1.821E-01	3.513E-01
f8	5.878E+00	5.473E+00	**3.768E-01**	1.891E+00	2.004E+00	2.694E+00	4.437E+00	4.129E+00	3.913E+00	4.732E+00	4.760E+00	7.125E+00
f9	9.087E+01	7.526E+01	6.978E+01	7.190E+01	7.798E+01	**6.598E+01**	7.529E+01	6.970E+01	7.038E+01	6.622E+01	6.736E+01	7.831E+01
f10	2.263E+02	1.620E+02	**2.648E+01**	6.735E+01	1.085E+02	1.397E+02	1.518E+02	1.426E+02	1.513E+02	1.725E+02	1.709E+02	1.768E+02
f11	2.640E+03	2.585E+03	2.439E+03	2.481E+03	2.596E+03	2.677E+03	2.622E+03	2.514E+03	2.540E+03	2.477E+03	2.595E+03	2.703E+03
f12	3.592E-01	2.099E-01	2.043E-01	1.851E-01	1.899E-01	**1.811E-01**	2.086E-01	2.214E-01	1.994E-01	2.023E-01	2.183E-01	2.021E-01
f13	4.443E-01	3.567E-01	3.580E-01	3.426E-01	3.375E-01	3.506E-01	3.869E-01	3.303E-01	3.680E-01	3.420E-01	3.502E-01	3.467E-01
f14	2.593E-01	2.273E-01	2.372E-01	2.311E-01	2.271E-01	2.343E-01	**2.128E-01**	2.333E-01	2.280E-01	2.221E-01	2.279E-01	2.220E-01
f15	2.192E+01	1.640E+01	1.730E+01	1.768E+01	1.586E+01	1.538E+01	1.365E+01	1.485E+01	1.472E+01	1.514E+01	1.764E+01	1.430E+01
f16	1.060E+01	1.067E+01	1.022E+01	1.041E+01	**1.017E+01**	1.046E+01	1.038E+01	1.062E+01	1.043E+01	1.051E+01	1.056E+01	1.058E+01
f17	1.050E+05	1.170E+06	**1.030E+05**	1.300E+05	1.380E+05	1.130E+05	1.440E+05	1.570E+05	1.470E+05	1.230E+05	1.370E+05	1.160E+05
f18	1.150E+07	8.560E+06	1.682E+03	1.619E+03	2.129E+03	5.347E+03	1.385E+04	2.530E+05	1.291E+04	**1.035E+03**	3.400E+05	1.470E+05
f19	2.050E+01	1.985E+01	1.578E+01	1.203E+01	1.467E+01	1.261E+01	1.392E+01	1.654E+01	1.433E+01	1.641E+01	1.110E+01	1.416E+01
f20	2.984E+04	2.415E+04	**6.680E+03**	9.934E+03	1.095E+04	1.443E+04	1.775E+04	1.723E+04	1.789E+04	1.751E+04	1.612E+04	2.200E+04
f21	2.610E+05	5.550E+05	**1.320E+05**	1.870E+05	1.540E+05	2.310E+05	1.710E+05	1.810E+05	2.110E+05	2.340E+05	1.790E+05	2.480E+05
f22	6.217E+02	4.973E+02	**4.797E+02**	5.259E+02	4.914E+02	5.236E+02	5.459E+02	5.152E+02	5.554E+02	5.043E+02	5.478E+02	5.228E+02
f23	3.181E+02	3.161E+02	**3.158E+02**	3.160E+02	3.163E+02	3.162E+02	3.160E+02	3.162E+02	3.162E+02	3.163E+02	3.162E+02	3.163E+02
f24	2.310E+02	2.292E+02	**2.268E+02**	2.277E+02	2.283E+02	2.290E+02	2.281E+02	2.299E+02	2.284E+02	2.286E+02	2.295E+02	2.280E+02
f25	2.151E+02	2.143E+02	2.141E+02	2.144E+02	2.144E+02	2.143E+02	2.142E+02	2.140E+02	2.146E+02	**2.137E+02**	2.143E+02	2.143E+02
f26	1.204E+02	1.204E+02	**1.004E+02**	1.071E+02	1.071E+02	1.137E+02	1.137E+02	1.104E+02	1.071E+02	1.137E+02	1.137E+02	1.104E+02
f27	5.985E+02	5.476E+02	5.559E+02	5.795E+02	**5.083E+02**	5.715E+02	5.870E+02	6.066E+02	5.851E+02	5.735E+02	6.043E+02	5.871E+02
f28	1.574E+03	1.610E+03	1.767E+03	1.631E+03	1.587E+03	**1.389E+03**	1.662E+03	1.651E+03	1.571E+03	1.670E+03	1.599E+03	1.774E+03
f29	2.477E+03	1.189E+03	1.061E+03	9.765E+02	9.054E+02	1.084E+03	1.012E+03	**8.810E+02**	1.055E+03	1.046E+03	9.940E+02	1.221E+03
f30	5.438E+03	3.848E+03	**2.531E+03**	2.897E+03	2.847E+03	3.005E+03	2.864E+03	2.974E+03	3.273E+03	3.079E+03	3.796E+03	3.419E+03

Table 7.21: Average Error of ASw-SKF$_a^{rD^p}$ (continued...)

Function ID	S-SKF	A-SKF	55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	4.860E+05	1.100E+07	**2.090E+05**	5.380E+05	2.960E+05	4.120E+05	4.900E+05	5.550E+05	5.370E+05	1.070E+06	1.820E+06
f2	2.450E+08	1.290E+06	6.070E+06	2.720E+07	6.100E+06	4.060E+06	2.340E+07	1.810E+07	3.290E+06	5.220E+06	2.670E+07
f3	1.841E+04	9.901E+03	9.143E+03	1.121E+04	1.092E+04	9.508E+03	7.976E+03	1.065E+04	1.386E+04	8.319E+03	1.271E+04
f4	3.646E+01	1.177E+02	2.064E+01	3.315E+01	2.823E+01	3.763E+01	3.422E+01	3.459E+01	3.555E+01	4.257E+01	7.854E+01
f5	2.002E+01	2.001E+01	2.001E+01	2.000E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01	2.001E+01
f6	2.195E+01	1.817E+01	1.577E+01	1.559E+01	1.590E+01	1.547E+01	1.559E+01	**1.495E+01**	1.615E+01	1.607E+01	1.580E+01
f7	1.635E-01	**8.444E-02**	1.918E-01	1.460E-01	1.884E-01	1.272E-01	1.229E-01	1.638E-01	1.779E-01	1.288E-01	1.565E-01
f8	5.878E+00	5.473E+00	6.286E+00	5.899E+00	5.096E+00	6.348E+00	6.158E+00	5.942E+00	5.654E+00	6.393E+00	5.564E+00
f9	9.087E+01	7.526E+01	7.247E+01	7.422E+01	6.868E+01	7.150E+01	7.373E+01	7.423E+01	7.299E+01	7.446E+01	7.291E+01
f10	2.263E+02	1.620E+02	1.486E+02	1.707E+02	1.586E+02	2.528E+02	1.896E+02	1.805E+02	2.180E+02	2.253E+02	1.708E+02
f11	2.640E+03	2.585E+03	2.553E+03	2.621E+03	2.601E+03	2.479E+03	2.652E+03	2.408E+03	2.650E+03	**2.377E+03**	2.671E+03
f12	3.592E-01	2.099E-01	2.301E-01	2.573E-01	2.359E-01	2.089E-01	2.501E-01	2.317E-01	2.147E-01	2.578E-01	2.346E-01
f13	4.443E-01	3.567E-01	3.518E-01	3.708E-01	3.747E-01	3.477E-01	3.603E-01	3.421E-01	3.688E-01	**3.284E-01**	3.324E-01
f14	2.593E-01	2.273E-01	2.309E-01	2.176E-01	2.278E-01	2.421E-01	2.261E-01	2.300E-01	2.331E-01	2.373E-01	2.207E-01
f15	2.192E+01	1.640E+01	1.396E+01	**1.262E+01**	1.657E+01	1.654E+01	1.357E+01	1.282E+01	1.465E+01	1.641E+01	1.556E+01
f16	1.060E+01	1.067E+01	1.059E+01	1.040E+01	1.047E+01	1.056E+01	1.053E+01	1.050E+01	1.064E+01	1.056E+01	1.065E+01
f17	1.050E+05	1.170E+06	1.280E+05	1.660E+05	1.710E+05	1.640E+05	1.850E+05	2.510E+05	3.160E+05	4.250E+05	5.520E+05
f18	1.150E+07	8.560E+06	2.850E+05	3.460E+05	4.300E+05	8.740E+05	1.950E+06	4.440E+06	1.660E+05	1.420E+06	2.340E+06
f19	2.050E+01	1.985E+01	9.915E+00	1.140E+01	1.614E+01	1.479E+01	**9.038E+00**	1.576E+01	1.528E+01	1.502E+01	2.081E+01
f20	2.984E+04	2.415E+04	2.373E+04	2.304E+04	2.109E+04	2.548E+04	2.271E+04	2.363E+04	2.458E+04	2.698E+04	2.219E+04
f21	2.610E+05	5.550E+05	1.710E+05	2.280E+05	2.140E+05	2.560E+05	1.990E+05	2.500E+05	3.390E+05	4.510E+05	3.930E+05
f22	6.217E+02	4.973E+02	5.433E+02	4.798E+02	5.553E+02	6.023E+02	5.996E+02	5.273E+02	5.209E+02	5.245E+02	5.354E+02
f23	3.181E+02	3.161E+02	3.165E+02	3.166E+02	3.169E+02	3.162E+02	3.161E+02	3.163E+02	3.164E+02	3.160E+02	3.166E+02
f24	2.310E+02	2.292E+02	2.285E+02	2.295E+02	2.286E+02	2.289E+02	2.297E+02	2.293E+02	2.294E+02	2.291E+02	2.290E+02
f25	2.151E+02	2.143E+02	2.139E+02	2.144E+02	2.142E+02	2.150E+02	2.149E+02	2.140E+02	2.150E+02	2.142E+02	2.148E+02
f26	1.204E+02	1.204E+02	1.104E+02	1.137E+02	1.171E+02	1.104E+02	1.170E+02	1.038E+02	1.170E+02	1.237E+02	1.171E+02
f27	5.985E+02	5.476E+02	5.974E+02	5.902E+02	5.641E+02	5.253E+02	5.316E+02	5.447E+02	5.793E+02	5.805E+02	5.893E+02
f28	1.574E+03	1.610E+03	1.639E+03	1.677E+03	1.808E+03	1.612E+03	1.682E+03	1.537E+03	1.804E+03	1.573E+03	1.615E+03
f29	2.477E+03	1.189E+03	1.162E+03	1.144E+03	9.595E+02	1.437E+03	1.328E+03	9.383E+02	1.051E+03	1.026E+03	1.070E+03
f30	5.438E+03	3.848E+03	3.219E+03	3.389E+03	2.969E+03	3.114E+03	3.141E+03	3.153E+03	3.144E+03	3.137E+03	3.892E+03

Table 7.22: Wilcoxon Signed Rank Test Statistical Values for ASw-SKF$_a^{rD^p}$

S-SKF vs ASw-SKF$_a^{rD^p}$			A-SKF vs ASw-SKF$_a^{rD^p}$		
Δ	*R*+	*R*−	Δ	*R*+	*R*−
5%	22	443	5%	55	410
10%	47	418	10%	73	392
15%	41	424	15%	43	422
20%	42	423	20%	98	367
25%	47	418	25%	140	325
30%	60	405	30%	131	334
35%	26	439	35%	112	353
40%	45	420	40%	108	357
45%	60	405	45%	134	301
50%	80	385	50%	210	255
55%	54	411	55%	107	328
60%	78	387	60%	174	291
65%	50	415	65%	160	305
70%	81	384	70%	170	295
75%	97	368	75%	168	297
80%	60	405	80%	123	342
85%	125	340	85%	217	248
90%	118	347	90%	150	315
95%	147	318	95%	218	247

ASw-SKF$_s^{rD^p}$ – Adaptiveness of the proposed strategy is observed through the average number of switches. Less than maximum number of permissible switch is seen for majority of the tests. This is because a population that adopts the initial iteration strategy of this variant, which is the synchronous update, is more prone to lose its diversity rapidly and stagnated. The stagnant diversity prevents switches.

Table 7.23 presents the average fitness error values. Based on these values, the Wilcoxon signed rank test is performed.

The statistical values of Wilcoxon test are listed in Table 7.24. It is found that other than $\Delta = \{55\%\}$, ASw-SKF$_s^{rD^p}$ is significantly better than S-SKF, with level of significance ranging from 1% to 10%. For comparison with A-SKF, it is seen that ASw-SKF$_s^{rD^p}$ with more number of switch $\Delta = \{5\%, 10\%\}$ is better than A-SKF.

Table 7.23: Average Error of ASw-SKF$_s^{rD^p}$

Function ID	S-SKF	A-SKF	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
f1	4.860E+05	1.100E+07	4.030E+05	4.830E+05	4.230E+05	4.510E+05	4.170E+05	4.930E+05	3.800E+05	5.470E+05	3.920E+05	6.840E+05
f2	2.450E+08	1.290E+06	**1.006E+04**	1.125E+04	5.010E+05	1.610E+06	1.610E+06	6.450E+05	9.720E+06	1.980E+07	3.730E+06	2.460E+06
f3	1.841E+04	9.901E+03	**4.491E+03**	1.149E+04	1.006E+04	9.436E+03	9.759E+03	1.181E+04	1.268E+04	8.880E+03	1.095E+04	1.158E+04
f4	3.646E+01	1.177E+02	**6.055E+00**	1.517E+01	1.514E+01	1.325E+01	1.890E+01	2.090E+01	2.571E+01	2.371E+01	2.010E+01	2.423E+01
f5	2.002E+01	2.001E+01	2.000E+01	**2.000E+01**	**2.000E+01**	**2.000E+01**	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01	2.000E+01
f6	2.195E+01	1.817E+01	1.873E+01	1.806E+01	1.788E+01	1.850E+01	1.775E+01	1.810E+01	1.811E+01	1.878E+01	1.968E+01	1.846E+01
f7	1.635E-01	**8.444E-02**	1.996E-01	1.028E-01	1.102E-01	1.834E-01	1.583E-01	1.684E-01	1.765E-01	2.849E-01	1.545E-01	1.543E-01
f8	5.878E+00	5.473E+00	**6.888E-01**	1.214E+00	1.532E+00	2.133E+00	1.510E+00	2.677E+00	3.488E+00	3.741E+00	3.226E+00	3.018E+00
f9	9.087E+01	**7.526E+01**	8.384E+01	8.039E+01	9.494E+01	9.273E+01	8.432E+01	8.644E+01	8.527E+01	9.230E+01	9.410E+01	9.322E+01
f10	2.263E+02	1.620E+02	**2.102E+01**	7.626E+01	1.011E+02	9.606E+01	9.965E+01	1.420E+02	1.511E+02	1.242E+02	1.522E+02	1.130E+02
f11	2.640E+03	**2.585E+03**	2.785E+03	2.648E+03	2.608E+03	2.749E+03	2.624E+03	2.728E+03	2.712E+03	2.773E+03	2.814E+03	2.673E+03
f12	3.592E-01	**2.099E-01**	2.485E-01	2.517E-01	2.823E-01	2.613E-01	2.729E-01	2.834E-01	2.892E-01	2.845E-01	2.905E-01	2.853E-01
f13	4.443E-01	**3.567E-01**	4.257E-01	4.442E-01	4.293E-01	4.528E-01	4.178E-01	4.466E-01	3.964E-01	4.564E-01	4.231E-01	4.345E-01
f14	2.593E-01	**2.273E-01**	2.678E-01	2.693E-01	2.722E-01	2.726E-01	2.746E-01	2.797E-01	2.588E-01	2.711E-01	2.642E-01	2.643E-01
f15	2.192E+01	**1.640E+01**	2.030E+01	2.582E+01	2.153E+01	2.105E+01	2.255E+01	2.196E+01	2.307E+01	1.993E+01	2.286E+01	2.254E+01
f16	1.060E+01	1.067E+01	1.039E+01	1.072E+01	1.058E+01	1.045E+01	1.056E+01	1.056E+01	1.058E+01	1.076E+01	1.064E+01	1.071E+01
f17	1.050E+05	1.170E+06	1.410E+05	1.440E+05	1.910E+05	1.990E+05	1.550E+05	1.430E+05	1.100E+05	1.710E+05	1.430E+05	1.420E+05
f18	1.150E+07	8.560E+06	**1.328E+03**	2.861E+03	2.614E+03	4.507E+04	5.677E+03	2.799E+03	1.443E+04	2.131E+04	5.988E+03	6.585E+03
f19	2.050E+01	1.985E+01	1.435E+01	**1.316E+01**	1.573E+01	1.898E+01	2.377E+01	1.702E+01	2.016E+01	1.722E+01	1.964E+01	1.508E+01
f20	2.984E+04	2.415E+04	**5.709E+03**	1.155E+04	1.439E+04	1.367E+04	1.706E+04	1.448E+04	2.201E+04	2.144E+04	2.068E+04	1.835E+04
f21	2.610E+05	5.550E+05	2.120E+05	1.710E+05	2.180E+05	**1.530E+05**	1.910E+05	2.010E+05	1.960E+05	2.140E+05	1.910E+05	2.220E+05
f22	6.217E+02	**4.973E+02**	5.434E+02	6.214E+02	5.810E+02	5.931E+02	6.220E+02	5.834E+02	5.580E+02	6.000E+02	6.241E+02	5.220E+02
f23	3.181E+02	3.161E+02	3.161E+02	3.164E+02	**3.160E+02**	3.164E+02	3.166E+02	3.165E+02	3.167E+02	3.164E+02	3.166E+02	3.168E+02
f24	2.310E+02	**2.292E+02**	2.305E+02	2.313E+02	2.316E+02	2.319E+02	2.310E+02	2.311E+02	2.331E+02	2.303E+02	2.313E+02	2.310E+02
f25	2.151E+02	2.143E+02	2.133E+02	2.137E+02	2.138E+02	2.156E+02	2.150E+02	2.148E+02	2.140E+02	**2.127E+02**	2.130E+02	2.160E+02
f26	1.204E+02	1.204E+02	**1.005E+02**	1.038E+02	1.071E+02	1.171E+02	1.104E+02	1.204E+02	1.137E+02	1.171E+02	1.138E+02	1.171E+02
f27	5.985E+02	**5.476E+02**	6.781E+02	6.835E+02	6.409E+02	6.784E+02	7.108E+02	6.769E+02	7.601E+02	6.908E+02	6.232E+02	6.408E+02
f28	1.574E+03	1.610E+03	1.598E+03	1.529E+03	1.738E+03	1.409E+03	1.651E+03	1.619E+03	1.673E+03	1.577E+03	1.487E+03	1.637E+03
f29	2.477E+03	1.189E+03	1.175E+03	1.187E+03	1.202E+03	1.262E+03	**1.083E+03**	1.207E+03	1.100E+03	1.229E+03	1.216E+03	1.239E+03
f30	5.438E+03	3.848E+03	**2.976E+03**	3.351E+03	3.912E+03	3.518E+03	3.910E+03	3.510E+03	4.129E+03	3.679E+03	4.495E+03	4.088E+03

Table 7.23: Average Error of ASw-SKF$_s^{rD^p}$ (continued...)

Function ID	S-SKF	A-SKF	55%	60%	65%	70%	75%	80%	85%	90%	95%
f1	4.860E+05	1.100E+07	5.640E+05	5.160E+05	3.440E+05	4.890E+05	2.580E+05	3.950E+05	4.060E+05	3.320E+05	**2.030E+05**
f2	2.450E+08	1.290E+06	4.370E+06	3.330E+07	4.180E+06	6.020E+06	3.830E+06	5.010E+07	9.200E+07	2.800E+07	2.180E+07
f3	1.841E+04	9.901E+03	1.061E+04	1.567E+04	1.273E+04	1.759E+04	1.526E+04	1.689E+04	1.771E+04	1.467E+04	1.031E+04
f4	3.646E+01	1.177E+02	3.938E+01	4.360E+01	2.548E+01	3.453E+01	3.859E+01	3.402E+01	3.539E+01	3.990E+01	2.341E+01
f5	2.002E+01	2.001E+01	2.000E+01	2.000E+01	2.000E+01	2.001E+01	2.000E+01	2.000E+01	2.001E+01	2.000E+01	2.002E+01
f6	2.195E+01	1.817E+01	1.786E+01	1.755E+01	1.860E+01	1.877E+01	**1.720E+01**	1.866E+01	1.904E+01	1.869E+01	1.844E+01
f7	1.635E-01	**8.444E-02**	1.280E-01	9.611E-02	3.069E-01	1.371E-01	1.041E-01	1.957E-01	2.282E-01	1.757E-01	2.975E-01
f8	5.878E+00	5.473E+00	2.924E+00	3.600E+00	3.568E+00	3.092E+00	3.159E+00	3.002E+00	3.300E+00	4.421E+00	5.429E+00
f9	9.087E+01	**7.526E+01**	8.533E+01	8.417E+01	9.112E+01	8.621E+01	8.544E+01	9.078E+01	8.555E+01	9.163E+01	8.560E+01
f10	2.263E+02	1.620E+02	1.670E+02	1.247E+02	1.108E+02	1.132E+02	1.531E+02	1.747E+02	1.457E+02	1.747E+02	1.975E+02
f11	2.640E+03	**2.585E+03**	2.812E+03	2.814E+03	2.900E+03	2.955E+03	2.787E+03	2.604E+03	2.638E+03	2.682E+03	2.917E+03
f12	3.592E-01	**2.099E-01**	2.974E-01	2.567E-01	2.883E-01	2.923E-01	2.643E-01	2.908E-01	2.977E-01	2.747E-01	3.163E-01
f13	4.443E-01	**3.567E-01**	4.636E-01	4.290E-01	4.200E-01	4.156E-01	4.272E-01	4.081E-01	4.206E-01	3.991E-01	4.427E-01
f14	2.593E-01	**2.273E-01**	2.614E-01	2.701E-01	2.571E-01	2.469E-01	2.742E-01	2.634E-01	2.778E-01	2.657E-01	2.633E-01
f15	2.192E+01	**1.640E+01**	2.194E+01	2.129E+01	2.139E+01	2.113E+01	1.995E+01	1.978E+01	1.868E+01	2.044E+01	2.282E+01
f16	1.060E+01	1.067E+01	1.063E+01	1.065E+01	1.064E+01	1.069E+01	**1.028E+01**	1.062E+01	1.071E+01	1.075E+01	1.090E+01
f17	1.050E+05	1.170E+06	1.650E+05	1.830E+05	1.340E+05	1.550E+05	1.450E+05	1.120E+05	1.390E+05	1.460E+05	**1.010E+05**
f18	1.150E+07	8.560E+06	3.270E+05	7.859E+04	2.811E+04	2.320E+05	8.750E+04	1.520E+05	6.610E+05	1.210E+05	1.030E+06
f19	2.050E+01	1.985E+01	1.838E+01	1.632E+01	3.609E+01	2.002E+01	2.131E+01	2.105E+01	2.634E+01	2.055E+01	2.091E+01
f20	2.984E+04	2.415E+04	1.969E+04	2.365E+04	2.123E+04	2.378E+04	1.962E+04	2.209E+04	1.975E+04	1.951E+04	1.995E+04
f21	2.610E+05	5.550E+05	2.310E+05	2.520E+05	2.480E+05	2.130E+05	2.270E+05	2.560E+05	1.980E+05	2.150E+05	1.920E+05
f22	6.217E+02	**4.973E+02**	6.104E+02	5.579E+02	6.182E+02	5.957E+02	6.024E+02	5.928E+02	6.356E+02	6.443E+02	6.938E+02
f23	3.181E+02	3.161E+02	3.169E+02	3.164E+02	3.173E+02	3.166E+02	3.177E+02	3.171E+02	3.170E+02	3.174E+02	3.176E+02
f24	2.310E+02	**2.292E+02**	2.319E+02	2.320E+02	2.318E+02	2.309E+02	2.317E+02	2.312E+02	2.295E+02	2.332E+02	2.325E+02
f25	2.151E+02	2.143E+02	2.157E+02	2.138E+02	2.143E+02	2.134E+02	2.149E+02	2.149E+02	2.144E+02	2.143E+02	2.143E+02
f26	1.204E+02	1.204E+02	1.138E+02	1.237E+02	1.137E+02	1.104E+02	1.171E+02	1.204E+02	1.071E+02	1.171E+02	1.171E+02
f27	5.985E+02	**5.476E+02**	7.066E+02	6.984E+02	6.817E+02	6.843E+02	6.700E+02	6.812E+02	6.270E+02	7.214E+02	7.095E+02
f28	1.574E+03	1.610E+03	1.531E+03	**1.373E+03**	1.672E+03	1.612E+03	1.483E+03	1.773E+03	1.688E+03	1.548E+03	1.531E+03
f29	2.477E+03	1.189E+03	3.309E+03	1.127E+03	2.005E+03	1.271E+03	1.195E+03	1.459E+03	1.171E+03	2.077E+03	1.490E+03
f30	5.438E+03	3.848E+03	4.194E+03	3.756E+03	3.911E+03	5.855E+03	3.890E+03	4.217E+03	4.043E+03	4.111E+03	5.907E+03

Table 7.24: Wilcoxon Signed Rank Test Statistical Values for ASw-SKF$_s^{rD^p}$

S-SKF vs ASw-SKF$_s^{rD^p}$			A-SKF vs ASw-SKF$_s^{rD^p}$		
Δ	$R+$	$R-$	Δ	$R+$	$R-$
5%	<u>86</u>	379	5%	<u>119</u>	346
10%	<u>89</u>	376	10%	<u>151</u>	314
15%	<u>89</u>	376	15%	192	273
20%	<u>98</u>	367	20%	181	284
25%	<u>99</u>	366	25%	203	262
30%	<u>130</u>	335	30%	201	264
35%	<u>105</u>	360	35%	230	235
40%	<u>130</u>	335	40%	185	280
45%	<u>115</u>	350	45%	229	236
50%	<u>144</u>	321	50%	251	214
55%	159	306	55%	246	219
60%	<u>134</u>	331	60%	191	274
65%	<u>129</u>	336	65%	267	198
70%	<u>139</u>	326	70%	256	209
75%	<u>98</u>	367	75%	233	232
80%	<u>95</u>	370	80%	294	171
85%	<u>113</u>	352	85%	250	215
90%	<u>136</u>	329	90%	270	195
95%	<u>126</u>	339	95%	279	186

7.4.2.3 Multiple Comparisons Among Algorithms

The results of ASw-PSO$_s^{rfit^*}$ with Δ= {85%}, ASw-GSA$_s^{rfit^*}$ with Δ= {90%}, and ASw-SKF$_a^{rfit^\wedge}$ with Δ= {30%} are compared with S-PSO, A-PSO, S-GSA, A-GSA, S-SKF and A-SKF. The Friedman ranks in Table 7.25 show that for all parent algorithms the adaptive switching with randomness implementations are ranked better than the synchronous or asynchronous implementation. The statistics of Holm test in Table 7.26 show that the algorithms are significantly as good as each other with exception to poorly performing A-GSA.

Table 7.25: Average Rankings of Friedman Test for Adaptive Switching with Randomness

Algorithm	Ranking
ASw-SKF$_a^{rfit*}$	3.4
ASw-PSO$_s^{rfit*}$	3.8
A-PSO	4.2333
S-PSO	4.6333
ASw-GSA$_s^{rfit*}$	4.7
A-SKF	4.7833
S-GSA	5.2
S-SKF	5.6167
A-GSA	8.6333

Table 7.26: Statistics of Holm Test for Adaptive Switching with Randomness

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
36	A-GSA vs. ASw-SKF$_a^{rfit*}$	7.401051	0	0.001389
35	ASw-PSO$_s^{rfit*}$ vs. A-GSA	6.835366	0	0.001429
34	A-PSO vs. A-GSA	6.22254	0	0.001471
33	S-PSO vs. A-GSA	5.656854	0	0.001515
32	A-GSA vs. ASw-GSA$_s^{rfit*}$	5.562573	0	0.001563
31	A-GSA vs. A-SKF	5.444722	0	0.001613
30	S-GSA vs. A-GSA	4.855467	0.000001	0.001667
29	A-GSA vs. S-SKF	4.266211	0.00002	0.001724
28	S-SKF vs. ASw-SKF$_a^{rfit*}$	3.13484	0.001719	0.001786
27	ASw-PSO$_s^{rfit*}$ vs. S-SKF	2.569155	0.010195	0.001852
26	S-GSA vs. ASw-SKF$_a^{rfit*}$	2.545584	0.010909	0.001923
25	ASw-PSO$_s^{rfit*}$ vs. S-GSA	1.979899	0.047715	0.002
24	A-PSO vs. S-SKF	1.956329	0.050426	0.002083
23	A-SKF vs. ASw-SKF$_a^{rfit*}$	1.956329	0.050426	0.002174
22	ASw-GSA$_s^{rfit*}$ vs. ASw-SKF$_a^{rfit*}$	1.838478	0.065992	0.002273
21	S-PSO vs. ASw-SKF$_a^{rfit*}$	1.744197	0.081125	0.002381
20	S-PSO vs. S-SKF	1.390643	0.164334	0.0025
19	ASw-PSO$_s^{rfit*}$ vs. A-SKF	1.390643	0.164334	0.002632
18	A-PSO vs. S-GSA	1.367073	0.171602	0.002778
17	ASw-GSA$_s^{rfit*}$ vs. S-SKF	1.296362	0.194851	0.002941
16	ASw-PSO$_s^{rfit*}$ vs. ASw-GSA$_s^{rfit*}$	1.272792	0.203092	0.003125
15	S-SKF vs. A-SKF	1.178511	0.238593	0.003333
14	S-PSO vs. ASw-PSO$_s^{rfit*}$	1.178511	0.238593	0.003571
13	A-PSO vs. ASw-SKF$_a^{rfit*}$	1.178511	0.238593	0.003846
12	S-PSO vs. S-GSA	0.801388	0.422907	0.004167
11	A-PSO vs. A-SKF	0.777817	0.436677	0.004545
10	S-GSA vs. ASw-GSA$_s^{rfit*}$	0.707107	0.4795	0.005
9	A-PSO vs. ASw-GSA$_s^{rfit*}$	0.659966	0.509275	0.005556
8	A-PSO vs. ASw-PSO$_s^{rfit*}$	0.612826	0.539991	0.00625
7	S-GSA vs. S-SKF	0.589256	0.55569	0.007143
6	S-GSA vs. A-SKF	0.589256	0.55569	0.008333
5	S-PSO vs. A-PSO	0.565685	0.571608	0.01
4	ASw-PSO$_s^{rfit*}$ vs. ASw-SKF$_a^{rfit*}$	0.565685	0.571608	0.0125
3	S-PSO vs. A-SKF	0.212132	0.832004	0.016667
2	ASw-GSA$_s^{rfit*}$ vs. A-SKF	0.117851	0.906186	0.025
1	S-PSO vs. ASw-GSA$_s^{rfit*}$	0.094281	0.924886	0.05

7.4.3 Fitness Error and Population's Diversity

The results of ASw-PSO $_s^{rfit^*}$ with Δ= {85%}, ASw-GSA $_s^{rfit^*}$ with Δ= {65%}, and ASw-SKF$_a^{Dp}$ with Δ= {5%} are analysed here. These setting are chosen from the group of settings that provide good performance from the experiments conducted. All other settings that provide good performance exhibit similar trend.

7.4.3.1 PSO using Adaptive Switching Iteration Strategy with Randomness

The boxplots in Figure 7.14 to Figure 7.17 show the error distribution of ASw-PSO $_s^{rfit^*}$ with Δ= {85%}, S-PSO and A-PSO. The boxplots of the three PSO algorithms highlight their performance. The boxes are located at the same level, however, ASw-PSO $_s^{rfit^*}$'s has boxes with shorter whisker.

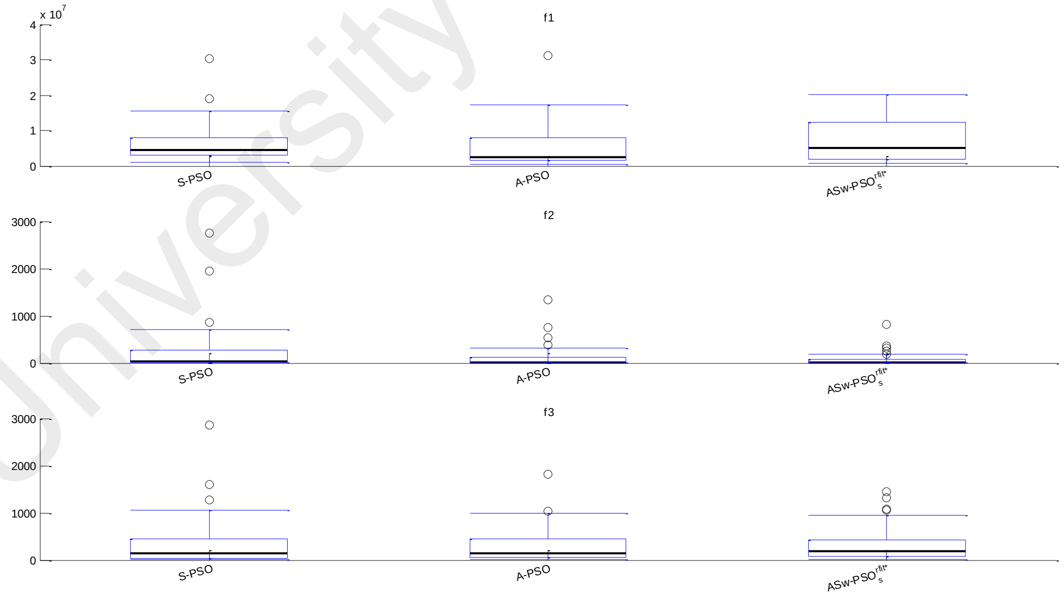

Figure 7.14: Fitness Error Distribution of Unimodal Functions for ASw-PSO $_s^{rfit^*}$ with Δ= {85%}

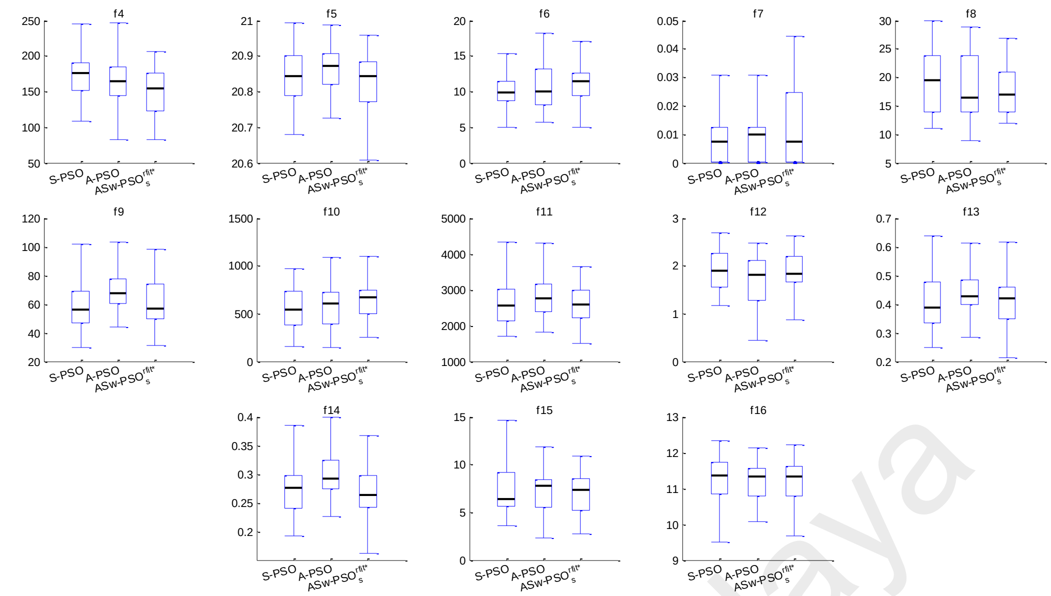

Figure 7.15: Fitness Error Distribution of Simple Multimodal Functions for ASw-PSO $_s^{rfit^*}$ with $\Delta = \{85\%\}$

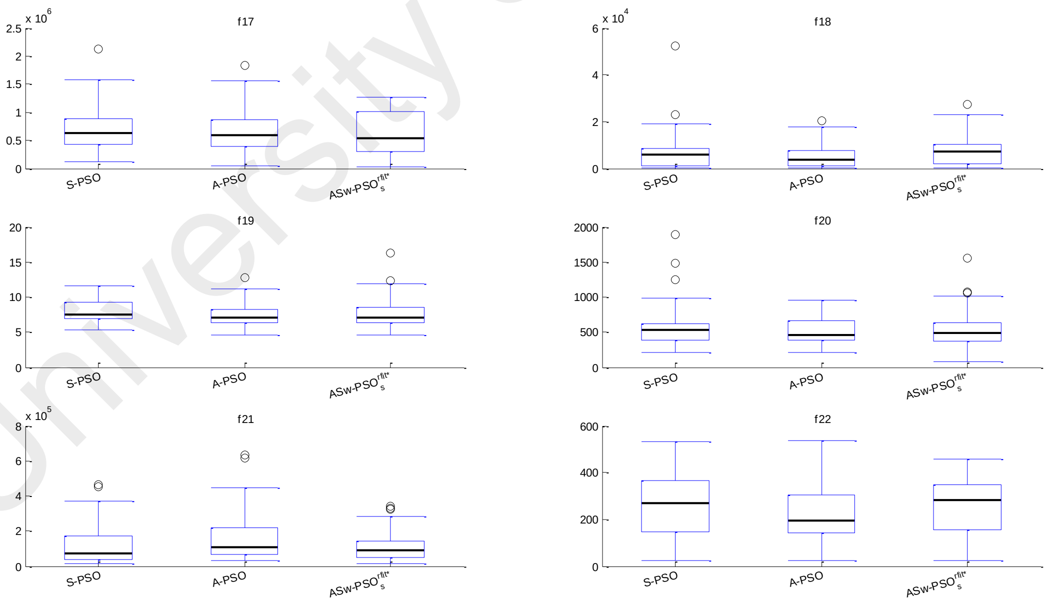

Figure 7.16: Fitness Error Distribution of Hybrid Functions for ASw-PSO $_s^{rfit^*}$ with $\Delta = \{85\%\}$

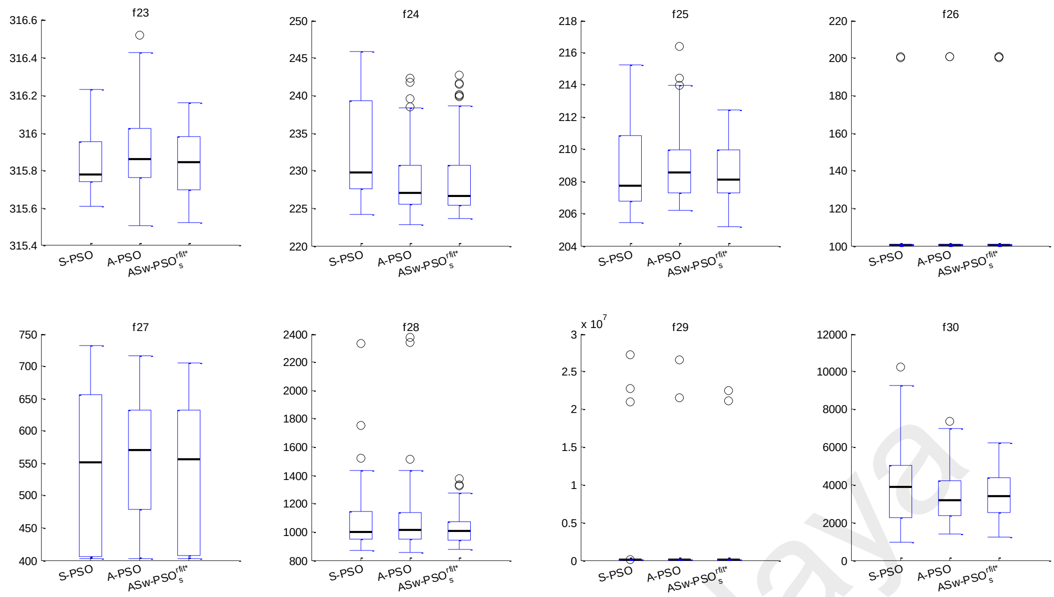

Figure 7.17: Fitness Error Distribution of Composite Functions for ASw-PSO $_s^{rfit^*}$ with Δ= {85%}

Figure 7.18 shows the rate of fitness error with respect to iteration for ASw-PSO $_s^{rfit^*}$ with Δ= {85%}, S-PSO and A-PSO for selected functions. Since the population of ASw-PSO $_s^{rfit^*}$ with Δ= {85%} switch from S-PSO to A-PSO, and both S-PSO and A-PSO has similar error rate trend, therefore, ASw-PSO $_s^{rfit^*}$ with Δ= {85%} exhibits similar behavior.

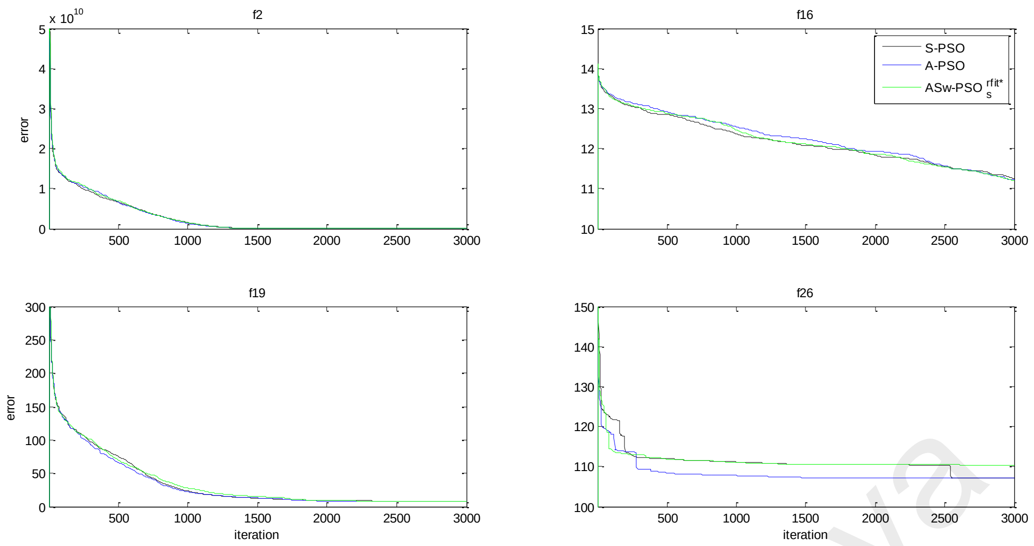

Figure 7.18: Fitness Error Rate of ASw-PSO $_{s}^{rfit^*}$ with Δ= {85%}

The position diversity of the three PSO variants are shown in Figure 7.19 to Figure 7.22. Like the fitness error rate, the position diversity of the algorithms also shares similar trend.

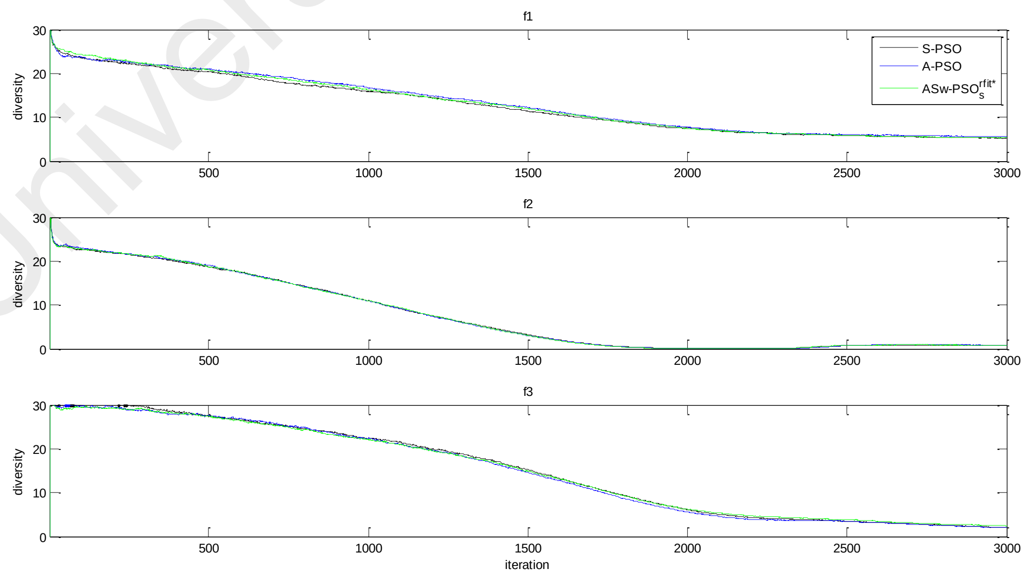

Figure 7.19: Rate of Position Diversity of Unimodal Functions for ASw-PSO $_{s}^{rfit^*}$ with Δ= {85%}

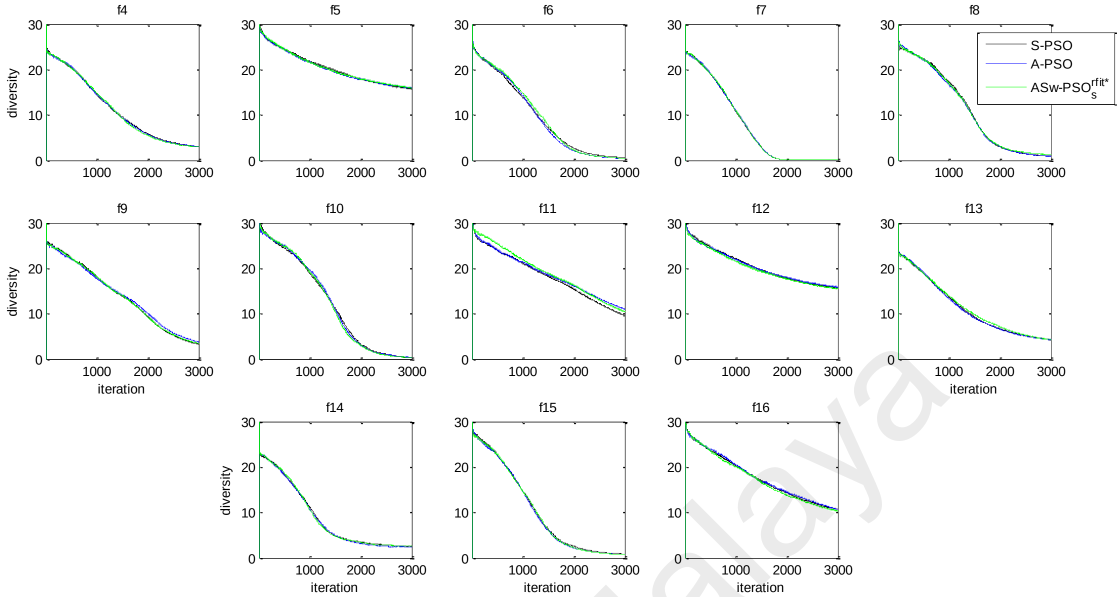

Figure 7.20: Rate of Position Diversity of Simple Multimodal Functions for ASw-PSO $_{s}^{rfit^{*}}$ with $\Delta = \{85\%\}$

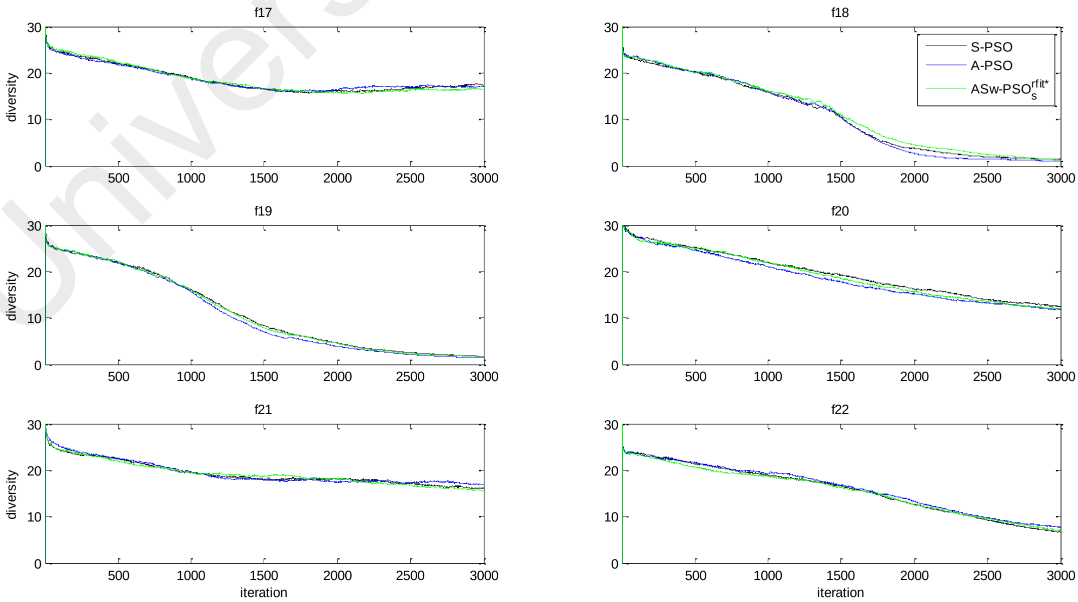

Figure 7.21: Rate of Position Diversity of Hybrid Functions for ASw-PSO $_{s}^{rfit^{*}}$ with $\Delta = \{85\%\}$

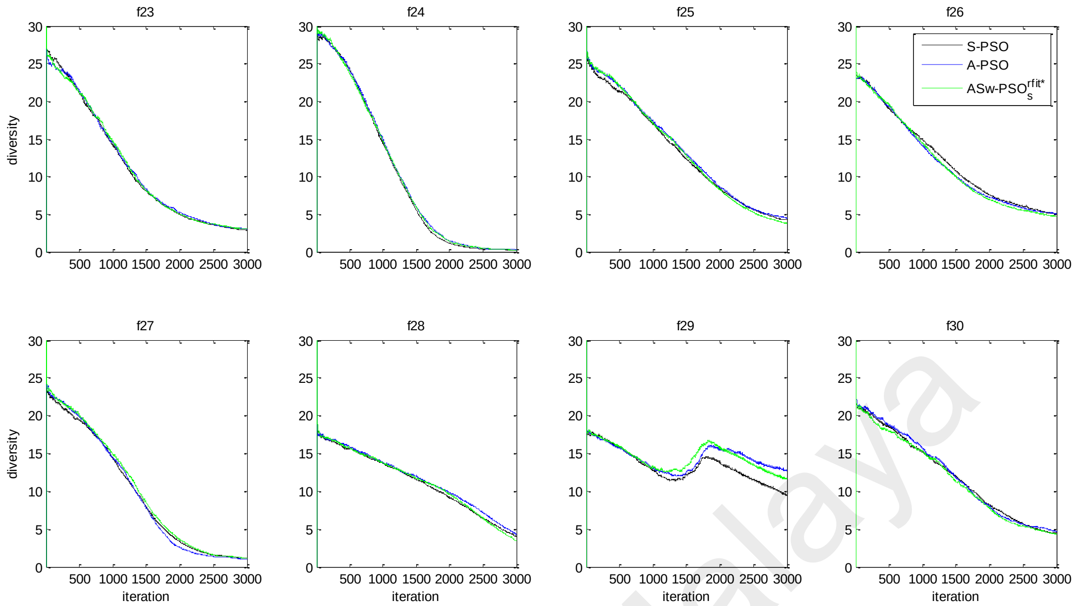

Figure 7.22: Rate of Position Diversity of Composite Functions for ASw-PSO $_s^{rfit^*}$ with Δ= {85%}

7.4.3.2 GSA using Adaptive Switching Iteration Strategy with Randomness

The fitness error distributions of S-GSA, A-GSA and ASw-GSA $_s^{rfit^*}$ with Δ= {65%} are shown in Figure 7.23 to Figure 7.26. It is shown that ASw-GSA $_s^{rfit^*}$ has more number of smaller and lower boxes, this indicates its consistent performance.

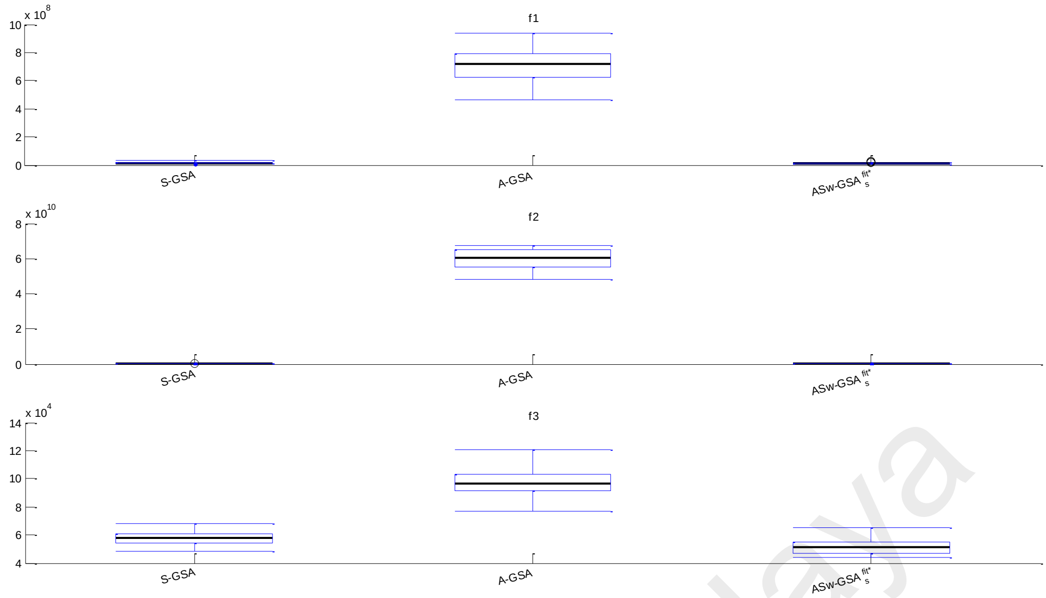

Figure 7.23: Fitness Error Distribution of Unimodal Functions for ASw-GSA $_{s}^{rfit^*}$ with Δ= {65%}

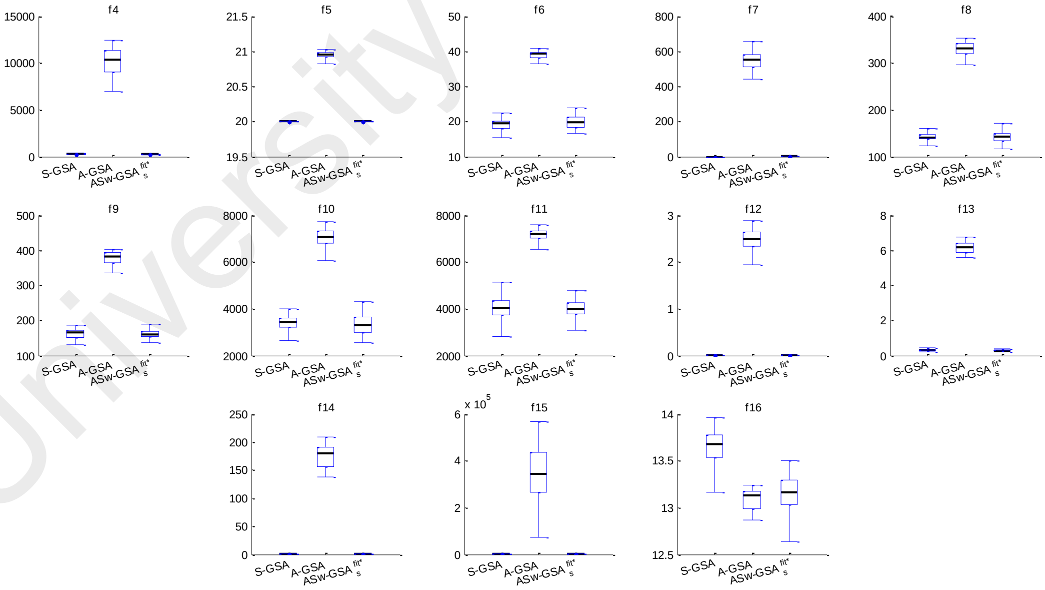

Figure 7.24: Fitness Error Distribution of Simple Multimodal Functions for ASw-GSA $_{s}^{rfit^*}$ with Δ= {65%}

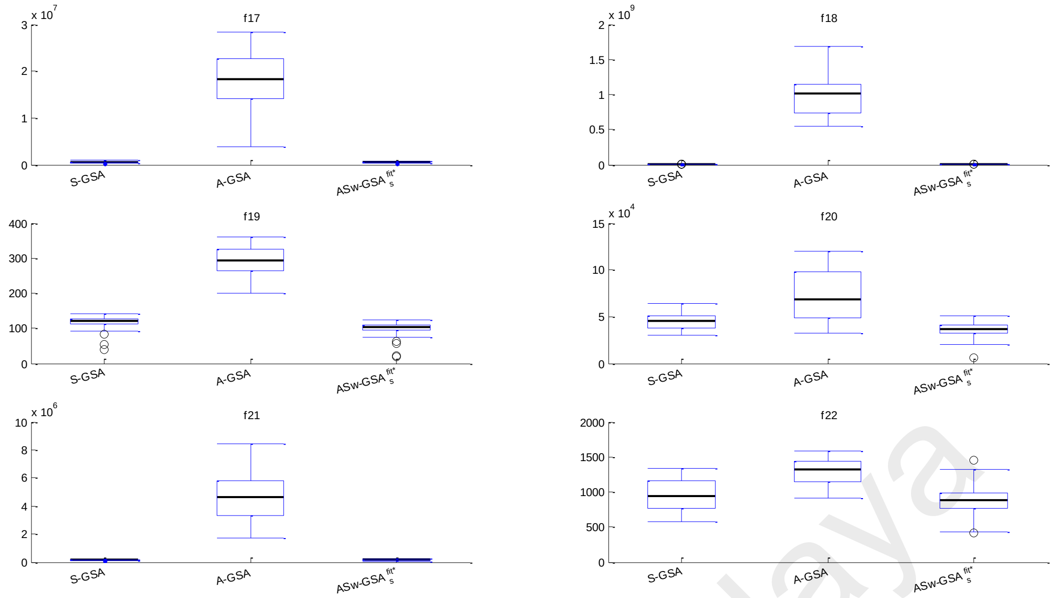

Figure 7.25: Fitness Error Distribution of Hybrid Functions for ASw-GSA $_{s}^{rfit^{*}}$ with Δ= {65%}

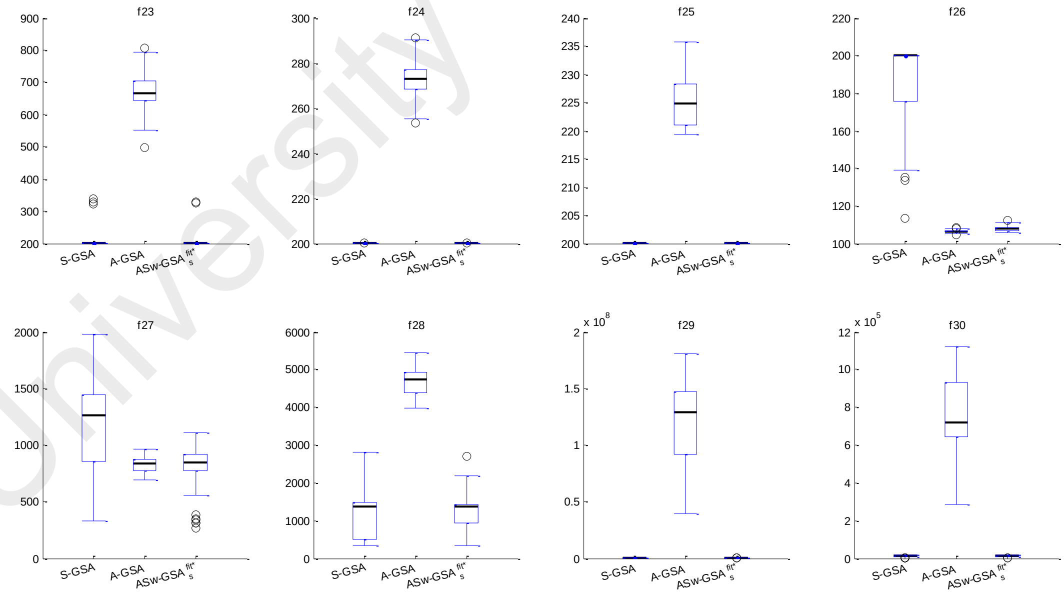

Figure 7.26: Fitness Error Distribution of Composite Functions for ASw-GSA $_{s}^{rfit^{*}}$ with Δ= {65%}

The fitness error rate of selected functions for ASw-GSA $_{s}^{rfit^*}$ with $\Delta = \{65\%\}$, S-GSA and A-GSA are shown in Figure 7.27. The ASw-GSA $_{s}^{rfit^*}$ with $\Delta = \{65\%\}$ executes synchronous update in the initial phase of the optimization. Based on the value of Δ, the switching happens at the earliest after 65% of the maximum iterations. The benefit of switching can be clearly seen in f16, f19, and f27. In f16 and f26, S-GSA performed worse than A-GSA. Before 65% of the total fitness evaluation, i.e. maximum iteration, the fitness error of ASw-GSA $_{s}^{rfit^*}$ is showing similar trend as S-GSA, however, when the switching occurs, ASw-GSA $_{s}^{rfit^*}$ is able to further improved its fitness error.

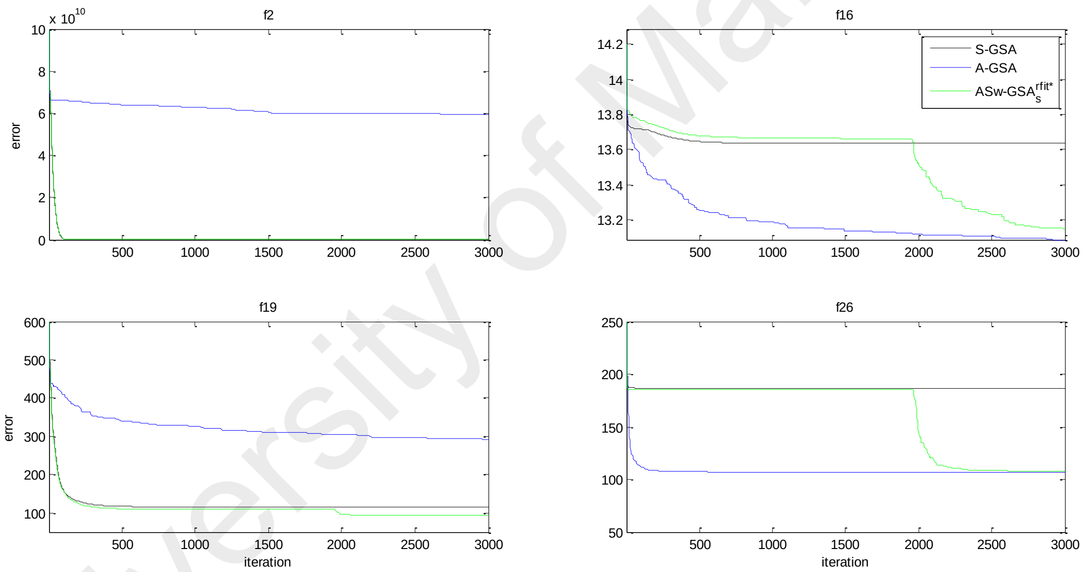

Figure 7.27: Fitness Error Rate of Unimodal Functions for ASw-GSA $_{s}^{rfit^*}$ with $\Delta = \{65\%\}$

The change of the agents' behaviour can be seen through the position diversity as in Figure 7.28 to Figure 7.31. For the first half of the iteration, ASw-GSA $_{s}^{rfit^*}$'s position diversity rapidly decreases. This is similar to the population of S-GSA. As the switching occurs, which is after 65% of the total number of iteration, the diversity of the agents increases significantly. The disturbance to the diversity of the agents is the factor contributing to the better performance of ASw-GSA $_{s}^{rfit^{*}}$.

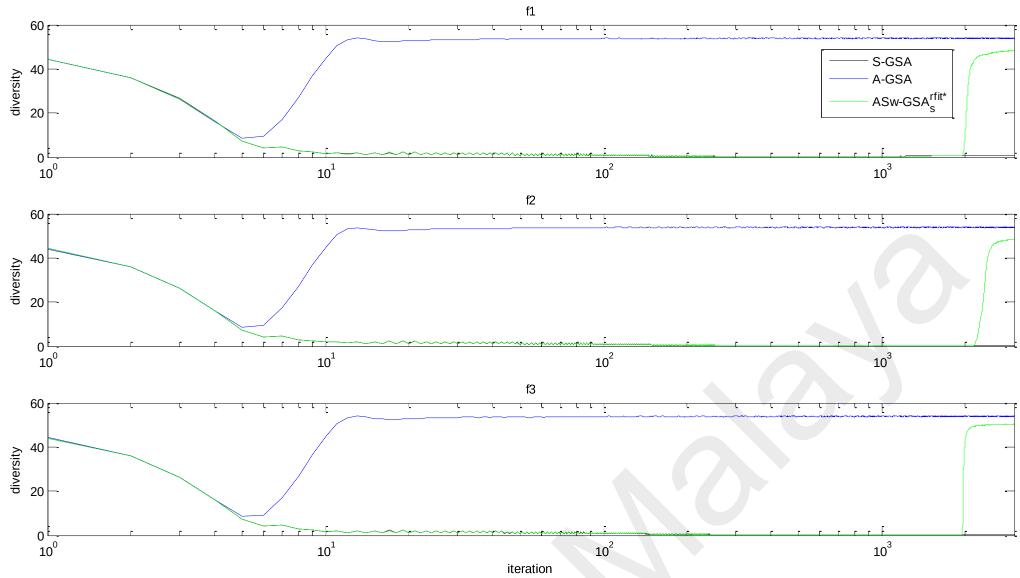

Figure 7.28: Rate of Position Diversity of Unimodal Functions for ASw-GSA $_{s}^{rfit^{*}}$ with $\Delta = \{65\%\}$

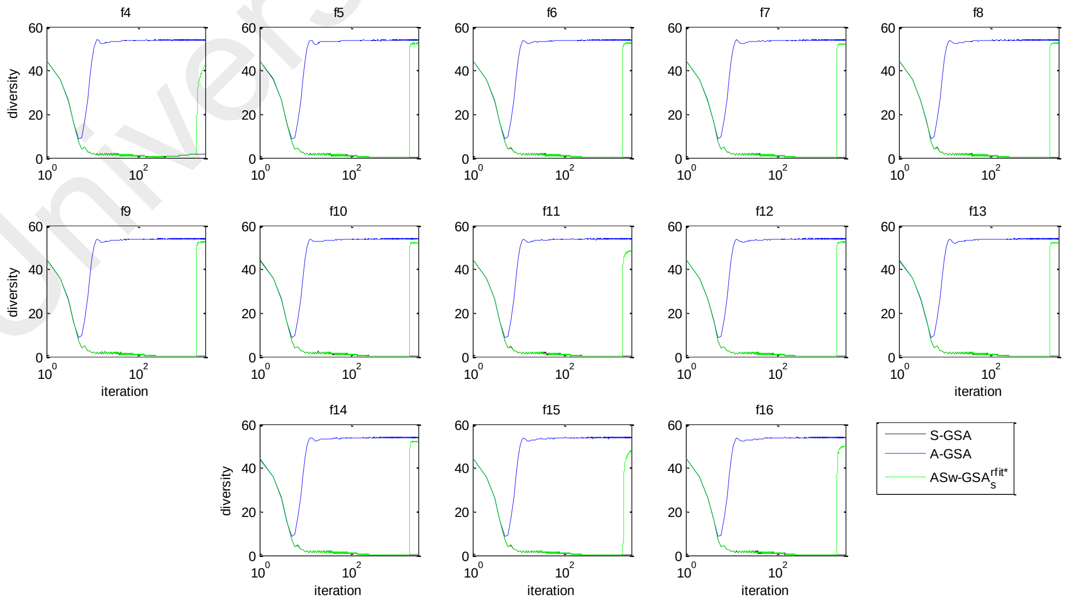

Figure 7.29: Rate of Position Diversity of Simple Multimodal Functions for ASw-GSA $_{s}^{rfit^{*}}$ with $\Delta = \{65\%\}$

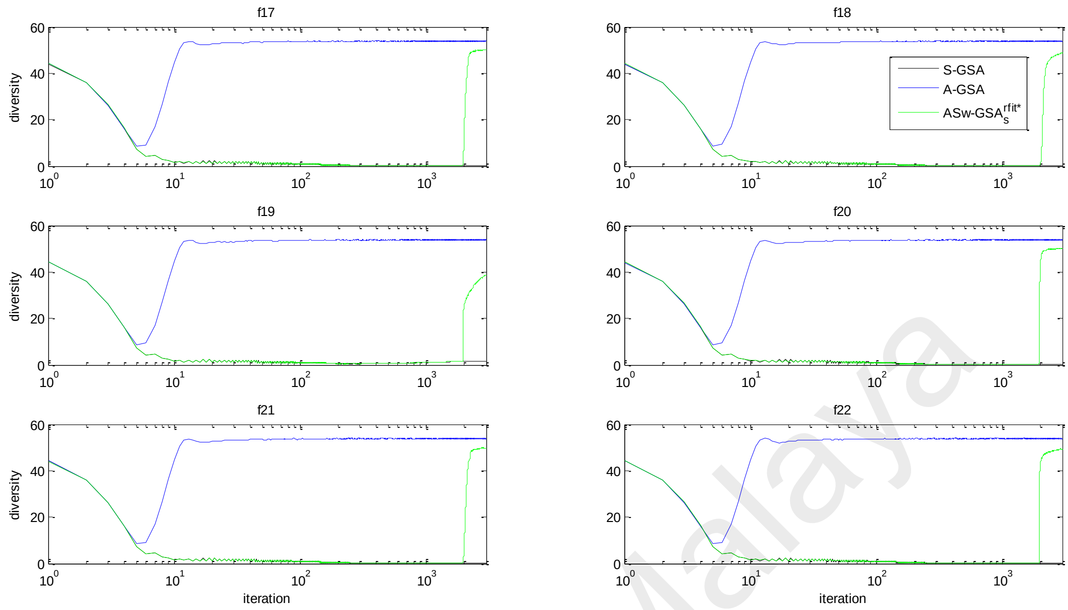

Figure 7.30: Rate of Position Diversity of Hybrid Functions for ASw-GSA $_s^{rfit^*}$ with Δ= {65%}

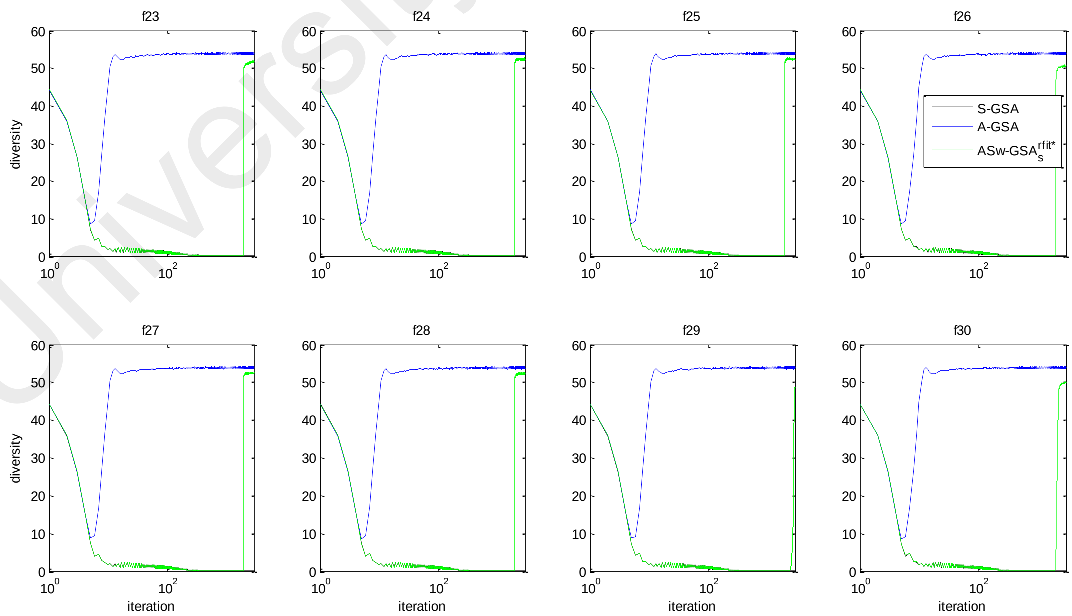

Figure 7.31: Rate of Position Diversity of Composite Functions for ASw-GSA $_s^{rfit^*}$ with Δ= {65%}

7.4.3.3 SKF using Adaptive Switching Iteration Strategy with Randomness

The improved performance of ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$ can be seen through the boxplots in Figure 7.32 to Figure 7.35. The ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$ has more number of smaller and lower boxes than S-GSA's and A-GSA's.

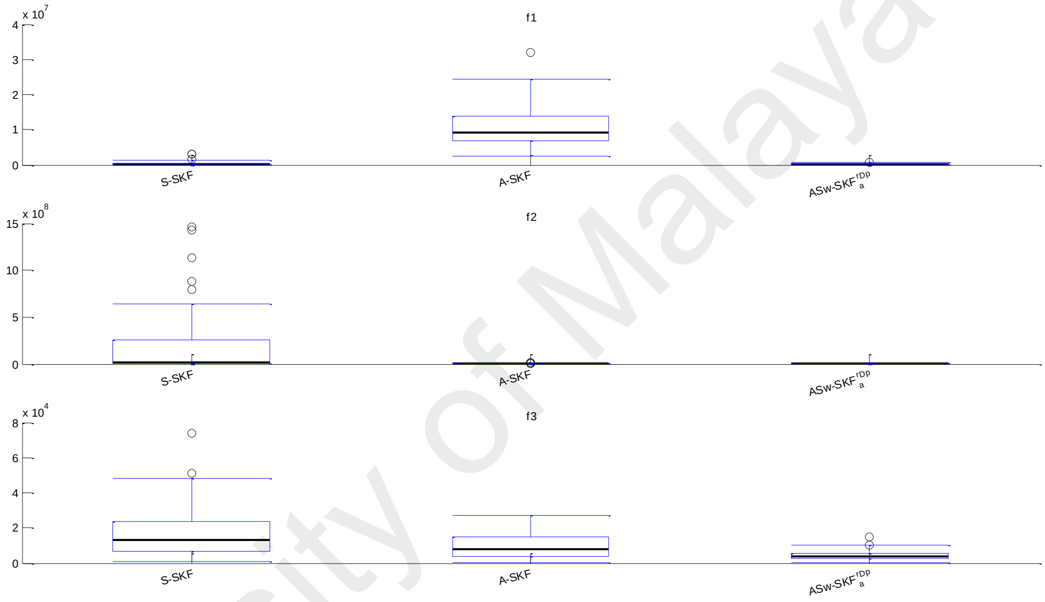

Figure 7.32: Fitness Error Distribution of Unimodal Functions for ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$

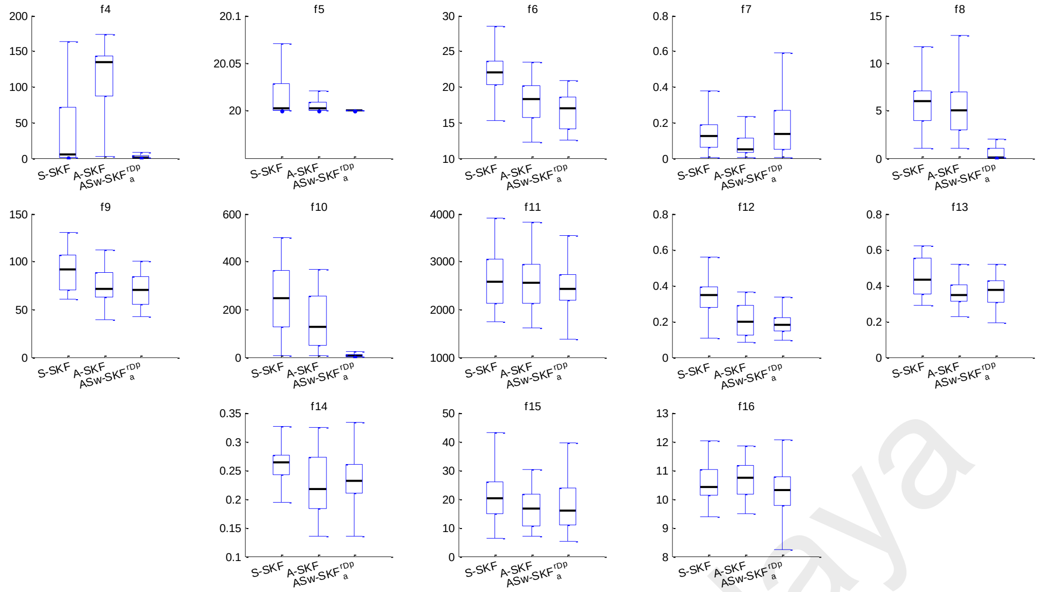

Figure 7.33: Fitness Error Distribution of Simple Multimodal Functions for ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$

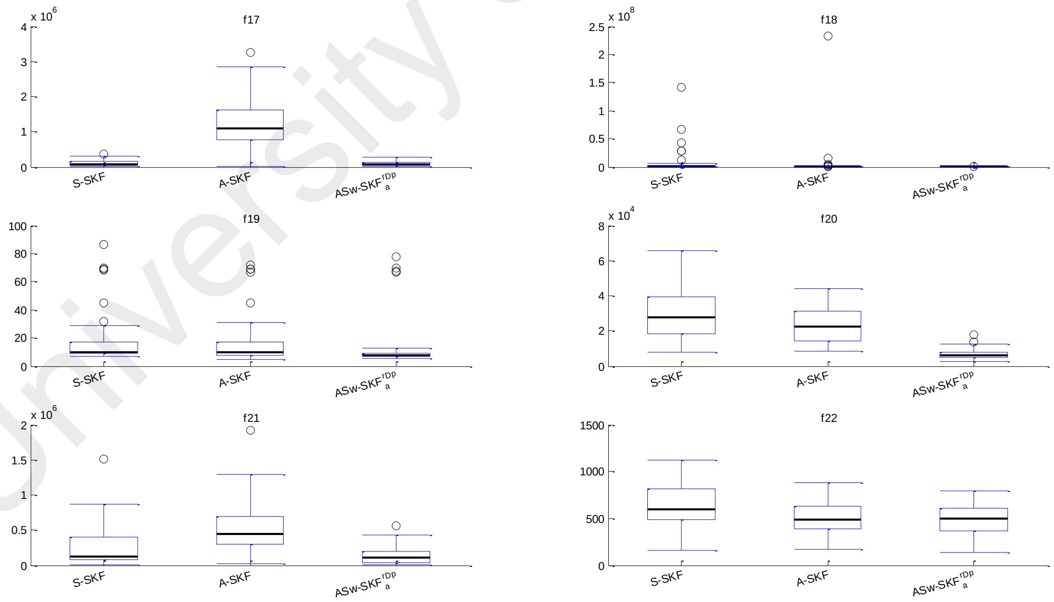

Figure 7.34: Fitness Error Distribution of Hybrid Functions for ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$

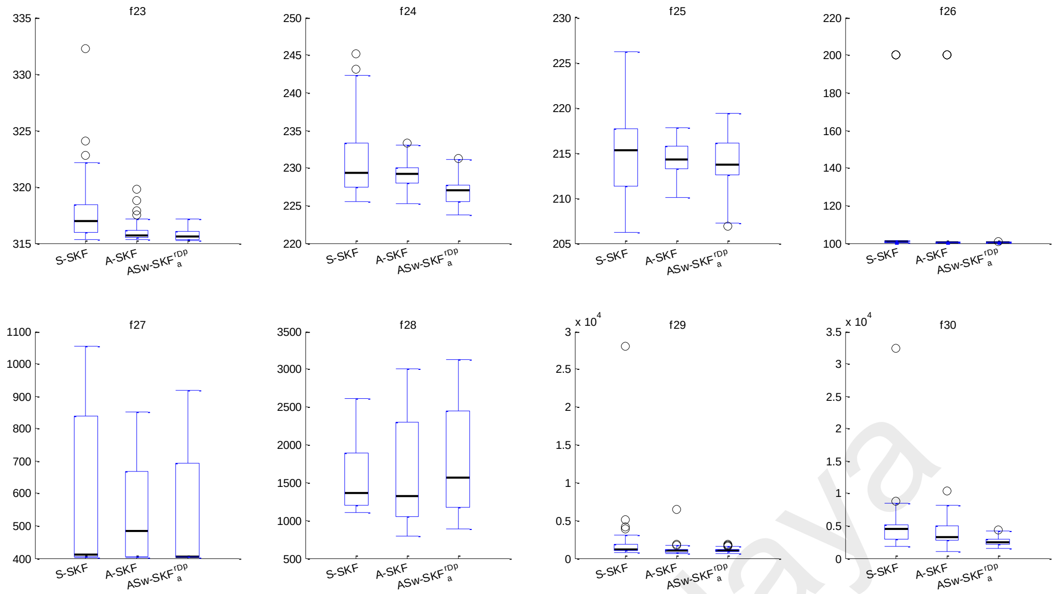

Figure 7.35: Fitness Error Distribution of Composite Functions for ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$

Figure 7.36 shows the fitness error rate of ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$, S-SKF and A-SKF for selected functions. It is observed that the fitness error of ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$ decreases at a slower rate than S-SKF but faster than A-SKF and settles at a smaller value than the two SKF algorithms.

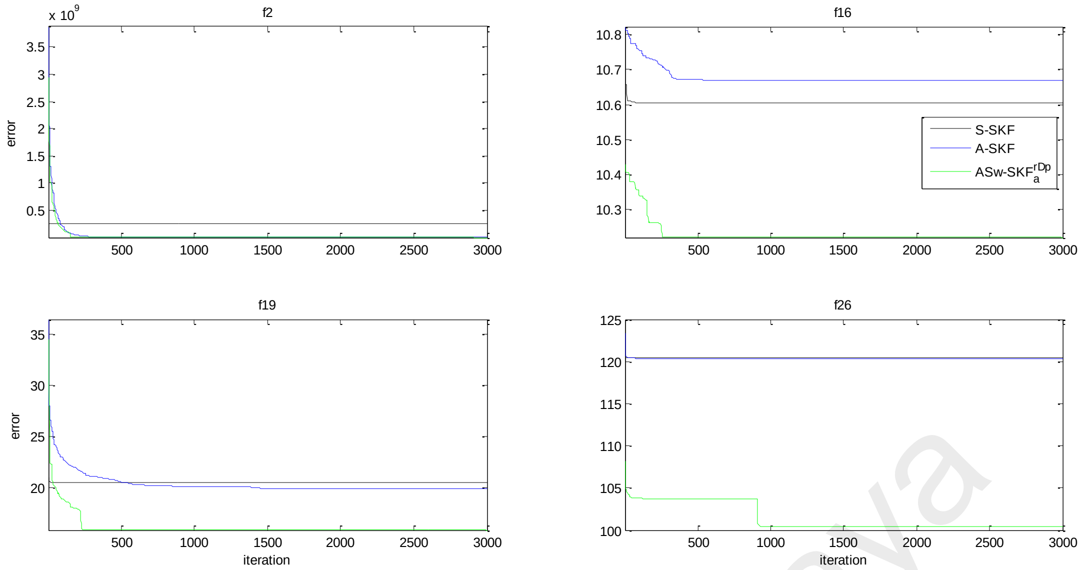

Figure 7.36: Fitness Error Rate of Unimodal Functions for ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$

The change of agents search behaviour due to the change of the iteration strategy can be seen in the graphs of the position diversity over iteration shown in Figure 7.37 to Figure 7.40. The switch caused minor disturbance to the diversity. The adaptive nature of ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$ can be seen in f12, f13 and f26. In these functions, the number of switch is not equal to the maximum number of switch possible.

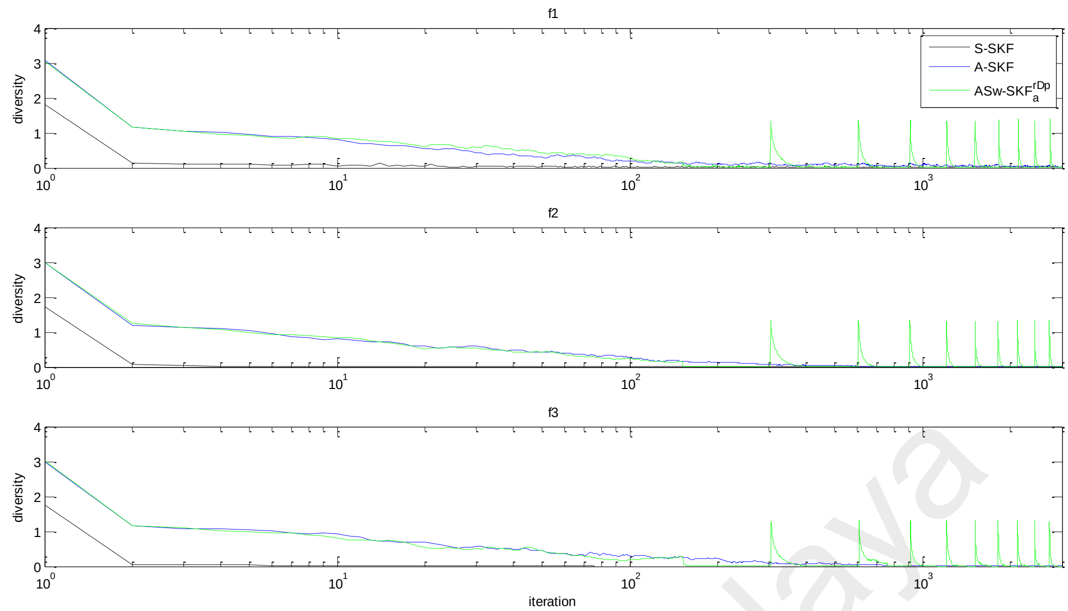

Figure 7.37: Rate of Position Diversity of Unimodal Functions for ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$

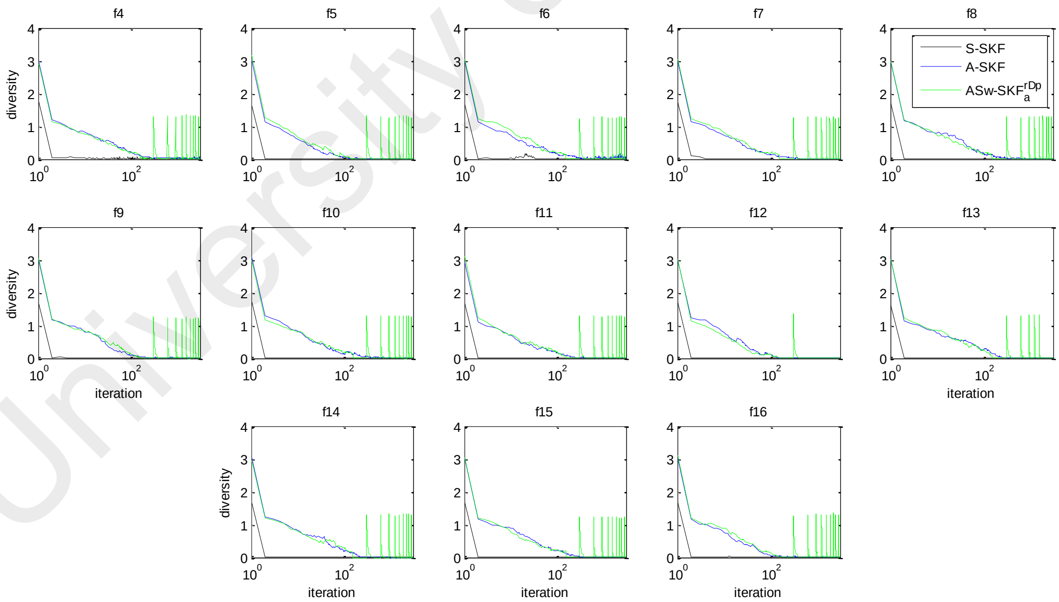

Figure 7.38: Rate of Position Diversity of Simple Multimodal Functions for ASw-SKF$_a^{rD^p}$ with $\Delta = \{5\%\}$

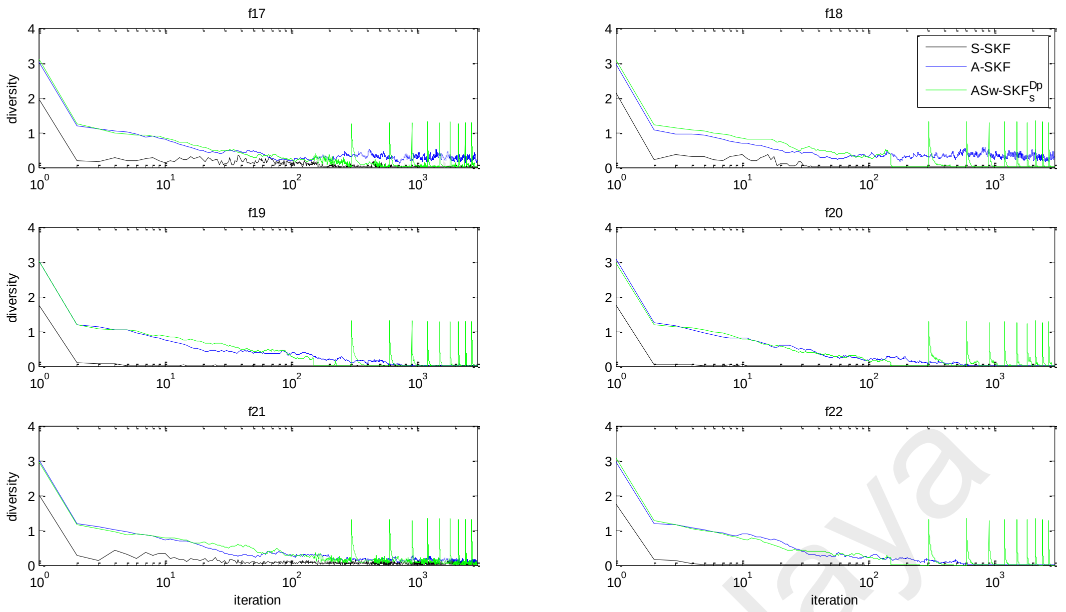

Figure 7.39: Rate of Position Diversity of Hybrid Functions for ASw-SKF$_a^{rDp}$ with $\Delta = \{5\%\}$

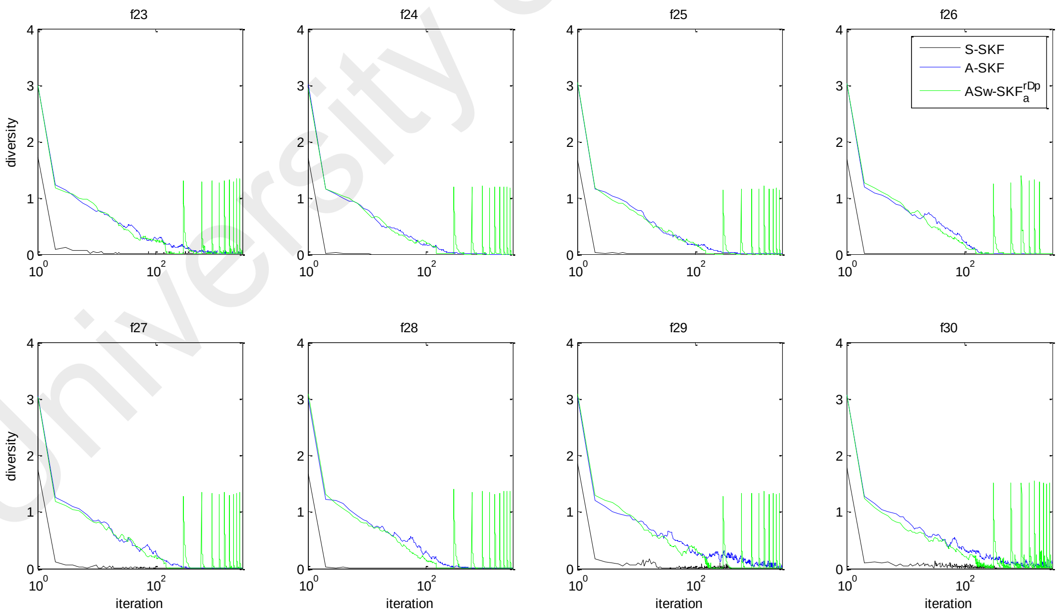

Figure 7.40: Rate of Position Diversity of Composite Functions for ASw-SKF$_a^{rDp}$ with $\Delta = \{5\%\}$

7.4.4 Parameter Control of Adaptive Switching Iteration Strategy with Randomness SKF

A. E. Eiben et al., (1999) defined parameter tuning as identification of good values for the parameters of an algorithm before the run of the algorithm and these values are then used for the entire run. The authors also acknowledged that in reality the best values for the parameters are not fixed for the entire run. Thus, parameter control is a better option, where the execution of an optimizer starts with a set of parameters values which are changed during the run.

There are multiple parameters in ASw-SKF $_{x}^{rb}$; the original SKF parameters; $P(0), Q$ and R, the parameters of the proposed iteration strategy which is the starting strategy, switching indicator and also Δ. The experiments conducted earlier can be seen as parameter tuning. Here, parameter control is conducted using parameter-less SKF is used. The parameter-less SKF search for best parameters setting and then the performance of ASw-SKF $_{x}^{rb}$ using the setting is feed to the parameter-less SKF.

Figure 7.41 shows the fitness of parameter optimization of ASw-SKF $_{x}^{rb}$ by parameter-less SKF over iteration. It could be seen that through parameter control the algorithm's performance can be improved.

The fitness of the solution found by optimal parameter setting is tabulated in table 7.27. The Friedman rank is presented in Table 7.28, where ASw-SKF $_{x}^{rb}$ with optimized parameters is ranked the best followed by A-SKF and S-SKF.

The Holm procedure shows that with significance level of 5%, ASw-SKF$_{x}^{rb}$ is on par with A-SKF, while S-SKF is the worst among the algorithms. The statistical values for Holm procedure are tabulated in Table 7.29.

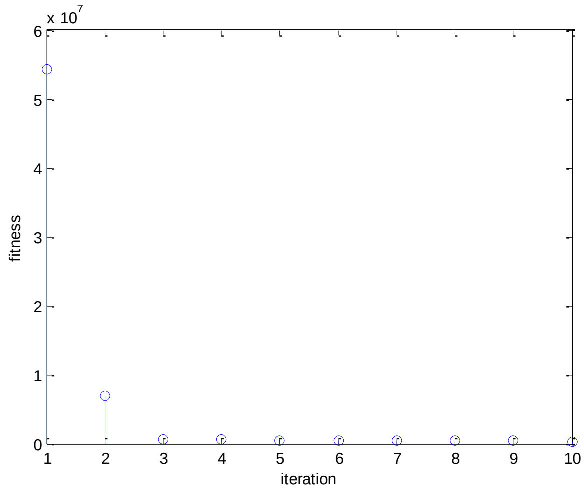

Figure 7.41: Fitness vs Iteration for Parameter Control of ASw-SKF$_x^{rb}$

Table 7.27: Performance of ASw-SKF$_x^{rb}$ vs S-SKF and A-SKF

Function ID	ASw-SKF$_x^{rb}$	S-SKF	A-SKF
f1	2.040E+05	4.860E+05	1.100E+07
f2	4.040E+03	2.450E+08	1.290E+06
f3	1.726E+04	1.841E+04	9.901E+03
f4	2.648E+00	3.646E+01	1.177E+02
f5	2.001E+01	2.002E+01	2.001E+01
f6	2.201E+01	2.195E+01	1.817E+01
f7	1.312E+03	1.635E-01	8.444E-02
f8	7.135E+00	5.878E+00	5.473E+00
f9	9.205E+01	9.087E+01	7.526E+01
f10	3.613E+02	2.263E+02	1.620E+02
f11	2.218E+03	2.640E+03	2.585E+03
f12	2.577E-01	3.592E-01	2.099E-01
f13	2.930E-01	4.443E-01	3.567E-01
f14	1.751E-01	2.593E-01	2.273E-01
f15	2.477E+01	2.192E+01	1.640E+01
f16	1.012E+01	1.060E+01	1.067E+01
f17	7.100E+04	1.050E+05	1.170E+06
f18	9.660E+03	1.150E+07	8.560E+06
f19	6.690E+01	2.050E+01	1.985E+01
f20	2.851E+04	2.984E+04	2.415E+04
f21	2.362E+05	2.610E+05	5.550E+05
f22	1.391E+02	6.217E+02	4.973E+02
f23	3.169E+02	3.181E+02	3.161E+02
f24	2.315E+02	2.310E+02	2.292E+02
f25	2.087E+02	2.151E+02	2.143E+02
f26	1.003E+02	1.204E+02	1.204E+02
f27	4.017E+02	5.985E+02	5.476E+02
f28	1.126E+03	1.574E+03	1.610E+03
f29	8.244E+02	2.477E+03	1.189E+03
f30	4.204E+03	5.438E+03	3.848E+03

Table 7.28: Friedman Rank of ASw-SKF$_x^{rb}$, S-SKF and A-SKF

Algorithm	Rank
ASw-SKF$_x^{rb}$	1.7333
S-SKF	2.5167
A-SKF	1.75
p-value = 0.00245	

Table 7.29: Statistics of Holm Test for ASw-SKF$_x^{rb}$, S-SKF and A-SKF

Algorithms	**p-value**	**Holm Value**
ASw-SKF$_x^{rb}$ vs. A-SKF	0.948533	0.1
S-SKF vs. A-SKF	0.002985	0.05
ASw-SKF$_x^{rb}$ vs. S-SKF	0.002415	0.033333

7.5 Conclusion

In the third hybrid iteration strategy proposed, the information on condition of the population is used to determine the suitable time to switch and randomness is used to increase the chance of switching. The setting of the strategy is algorithm dependent. For example, big Δ is better for PSO, whereas for all variants of SKF employing adaptive switching with randomness iteration strategy, small value of Δ guarantees a performance better than S-SKF and A-SKF. The overall performance of this iteration strategy is tabulated in Table 7.30.

ASw-PSO$_s^{rfit^*}$ is able to outperformed the original PSO, S-PSO, when the strategy is switched towards the end of the search. Similarly, ASw-GSA$_s^{rfit^*}$ is better than S-GSA when switch is done towards the end. These observations indicate disturbance to population's diversity which is provided by asynchronous update is beneficial to the performance of the algorithms.

All variants of SKF employing adaptive switching iteration strategy with randomness are performing better than SKF with the traditional iteration strategies. Parameter controlled ASw-SKF$_x^{rb}$ is ranked better than S-SKF and A-SKF.

Table 7.30: Overall Performance of Adaptive Switching Iteration Strategy with Randomness

	S-PSO	A-PSO
ASw-PSO$_a^{rfit^*}$	On par except for ASw-PSO$_a^{rfit^*}$ with $\Delta = \{15\%\}$	ASw-PSO$_a^{rfit^*}$ with $\Delta = \{5\%, 20\%, 30\%, 40\%, 45\%, 50\%, 55\%, 60\%\ 70\%, 85\%, 90\%, 95\%\}$ on par
ASw-PSO$_s^{rfit^*}$	ASw-PSO$_s^{rfit^*}$ with $\Delta = \{85\%, 95\%\}$ significantly better	ASw-PSO$_s^{rfit^*}$ with $\Delta = \{10\%, 15\%, 25\%, 35\%, 40\%, 45\%, 50\%, 70\%, 75\%, 85\%, 90\%, 95\%\}$ on par
ASw-PSO$_a^{rD^p}$	On par except for ASw-PSO$_a^{rD^p}$ with $\Delta = \{5\%\}$	ASw-PSO$_a^{rD^p}$ with $\Delta = \{10\%, 15\%, 20\%, 25\%, 30\%, 40\%, 45\%, 50\%, 60\%, 70\%, 80\%, 90\%\}$ on par
ASw-PSO$_s^{rD^p}$	On par	ASw-PSO$_a^{rD^p}$ with $\Delta = \{5\%, 10\%, 15\%, 20\%, 30\%, 35\%, 50\%, 60\%, 75\%, 85\%, 95\%\}$ on par

Table 7.27: Overall Performance of Adaptive Switching Iteration Strategy with Randomness (continued…)

	S-GSA	**A-GSA**
ASw-GSA$_a^{rfit^*}$	Not as good	ASw-GSA$_a^{rfit^*}$ with $\Delta = \{5\%, 10\%, 15\%\}$ significantly better
ASw-GSA$_s^{rfit^*}$	ASw-GSA$_s^{rfit^*}$ with $\Delta = \{40\%, 60\%, 65\%, 70\%, 80\%, 90\%\}$ significantly better	Significantly better
ASw-GSA$_a^{Dp}$	Not as good	ASw-GSA$_a^{Dp}$ with $\Delta = \{5\%, 10\%\}$ significantly better
ASw-GSA$_s^{Dp}$	ASw-GSA$_s^{Dp}$ with $\Delta = \{5\%, 10\%, 15\%\}$ not as good ASw-GSA$_s^{Dp}$ with $\Delta = \{20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 50\%\}$ on par	ASw-GSA$_s^{Dp}$ with $\Delta = \{5\%, 10\%, 15\%, 20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 50\%\}$ significantly better

Table 7.27: Overall Performance of Adaptive Switching Iteration Strategy with Randomness (continued…)

	S-SKF	A-SKF
ASw-SKF$_{a}^{rfit^*}$	Significantly better	ASw-SKF$_{a}^{rfit^*}$ with $\Delta=\{5\%, 10\%, 15\%, 20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 60\%, 75\%, 95\%\}$ significantly better
ASw-SKF$_{s}^{rfit^*}$	Significantly better	ASw-SKF$_{s}^{rfit^*}$ with $\Delta=\{5\%, 10\%, 15\%, 25\%\}$ significantly better
ASw-SKF$_{a}^{rD^p}$	Significantly better	ASw-SKF$_{a}^{rD^p}$ with $\Delta=\{5\%, 10\%, 15\%, 20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 55\%, 80\%, 90\%\}$ significantly better
ASw-SKF$_{s}^{rD^p}$	Significantly better except for $\Delta=\{55\%\}$	ASw-SKF$_{s}^{rD^p}$ with $\Delta=\{5\%, 10\%\}$ significantly better

CHAPTER 8: CONCLUSION

8.1 Introduction

The research in this thesis is motivated by how iteration strategy can result in different search behavior in population-based metaheuristics. Therefore, in depth analysis on the effect of iteration strategy towards the performance of population-based metaheuristics and the population diversity is provided in this thesis. The possibilities of using the iteration strategies to improve population-based metaheuristics was also explored. The differences in the agents' search behavior towards different iteration strategies were used to diversify or intensify the agents' search.

Three parent algorithms were used in this study. The algorithms are PSO, GSA and SKF. The algorithms were introduced in chapter 2 together with CEC2014's single objective real-parameter numerical optimization test suite, which is used as the benchmark problems in this study.

In chapter 3, existing works in premature convergence avoidance for the parent algorithms were reviewed. The works categorized as step size based, reinitialization and relearning based, information sharing based, algorithms hybridization based, and combination of two or more of the methods mentioned previously. None of the works reviewed manipulates iteration strategy to achieve their objective of better algorithms.

Traditionally the iteration strategy of population-based metaheuristics is either synchronous or asynchronous update. However, among the three parent algorithms employed in this thesis, only PSO had been reported to be implemented using both synchronous and asynchronous update. Therefore, asynchronously updated GSA and SKF were introduced in chapter 4.

A new class of iteration strategies is introduced in this research. The proposed strategies are hybrid strategies. The hybrid strategies try to achieve premature avoidance and better performance through switching between the traditional iteration strategies. This is a new category for premature convergence avoidance. Figure 8.1 shows the updated categories of premature convergence avoidance methods.

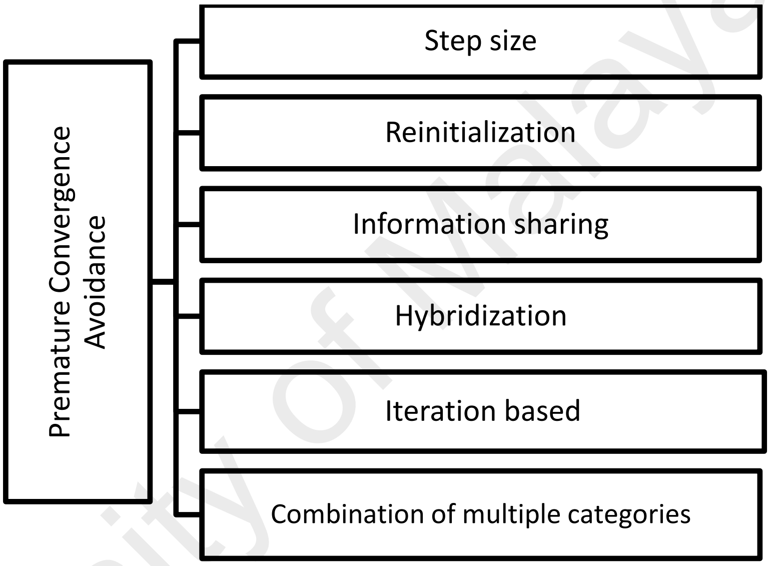

Figure 8.1: Updated Categories of Premature Convergence Avoidance Methods

The first hybrid iteration strategy which is the random switching iteration strategy is introduced and studied in chapter 5. In chapter 6, the second hybrid strategy which is the adaptive switching iteration strategy is presented. The adaptive switching with randomness iteration strategy is discussed in chapter 7. These strategies were implemented using the parent algorithms and the findings are presented in their respective chapter. As a summary, the iteration strategies and their classes are shown in Figure 8.2. The new variations of the parent algorithms are in the shaded box

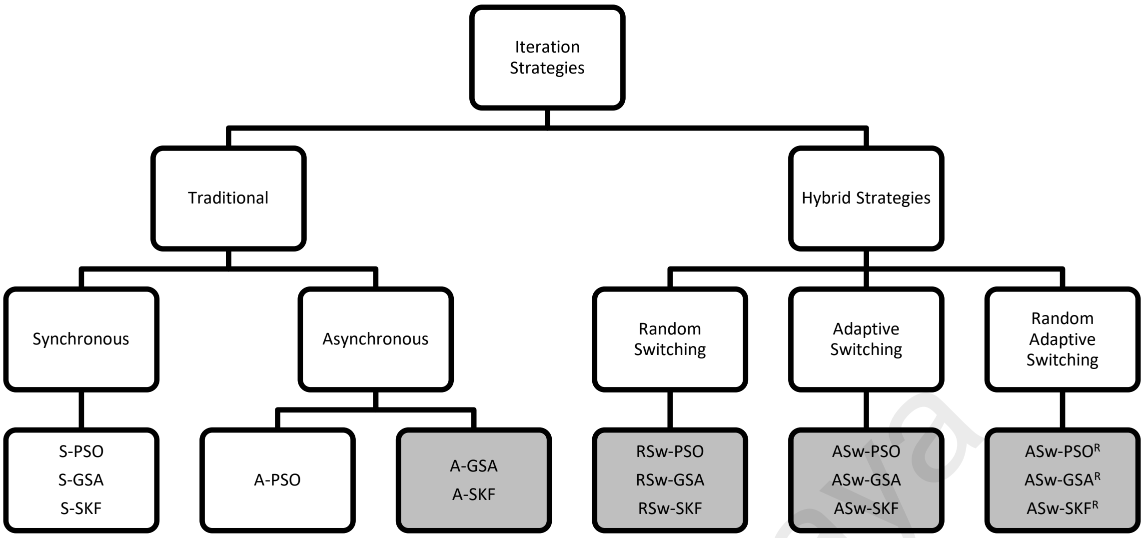

Figure 8.2: Available Iteration Strategies

8.2 Contributions of the Research

Asynchronous update GSA and SKF are considered in this study. It is found that asynchronous update is able to improve SKF algorithm. The A-SKF is significantly better than the original, S-SKF. A-GSA is not performing as good as the original, S-SGSA.

Random switching iteration strategy is found to be able to outperform both S-SKF and A-SKF. Random switching is the simplest among the hybrid strategies suggested. No unique parameter setting is required.

Adaptive switching iteration strategy also benefits SKF. SKF with adaptive switching is found to be able to outperformed S-SKF. The adaptive switching SKF must start as synchronous update population. Both fit^* and D^p can be used as the switching indicator.

The last hybrid strategy suggested, adaptive switching with randomness is able to improve all original version of the parent algorithms, i.e; the synchronous versions.

Switching towards the later stage of the search is able to outperformed S-PSO and S-GSA. On the other hand, high number of switching is found to be better for SKF.

8.3 Limitation

In this research, iteration strategy is proposed as a potential approach for performance enhancement and premature avoidance. The findings show that manipulation of iteration strategy is able to provide improvement to some of the parent algorithms. However, this observation is made based on the three parent algorithms adopted for this study only. No study on the relation of fitness landscape with iteration strategy were performed as it is out of the scope of this research.

8.4 Recommendation for Future Research

For future research, it is recommended that more parent algorithms with various degree of the importance of memory to be analyzed. More number of parent algorithms allow for more observation with regards to the influence of memory towards population's behavior under different iteration strategies.

Another interesting issue to be explored is the relationship of problem's complexity with algorithm's iteration strategy. A problem with higher complexity might have more significant response towards change of iteration strategy compared to less complex problem, thus, the hybrid strategies can be considered for better performance.

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